Worked Solutions for ENGAA Papers by Topic

Section 1

Topic: Algebra

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ENGAA S1 2020 - Question 2

2 The admission charge to a cinema is different for adults and children.

Admission for 2 adults and 3 children costs £20.

Admission for 4 adults and 4 children costs £34.

What does admission cost for 6 adults and 2 children?

A £27

B £29

C £33

D £39

E £44

F £48

G £72

ENGAA S1 2020 - Question 2 - Worked Solution

$$2\alpha + 3\beta = 20$$
$$4\alpha + 4\beta = 34$$
$$\alpha = £5.50$$
$$\beta = £3.00$$
$$6\alpha + 2\beta = £39$$

Answer is D

ENGAA S1 2020 - Question 6

6 Consider the four lines with the following equations.

1 $2x + 6y = 3$

2 $9y = 3x - 4$

3 $2y = 6x + 3$

4 $4x + 6y - 9 = 0$

Which two lines are perpendicular?

A 1 and 2

B 1 and 3

C 1 and 4

D 2 and 3

E 2 and 4

F 3 and 4

ENGAA S1 2020 - Question 6 - Worked Solution

1)

$$2x + 6y = 3$$

$$y = -\frac{1}{3}x + \frac{1}{2}$$

2)

$$9y = 3x - 4$$

$$y = \frac{1}{3}x - \frac{4}{9}$$

3)

$$y = 3x + \frac{3}{2}$$

$$y = 3x + \frac{3}{2}$$

4)

$$4x + 6y - 9 = 0$$

$$y = -\frac{3}{2}x + \frac{9}{4}$$

Gradients are negative reciprocal

1 & 3

Answer is B

ENGAA S1 2020 - Question 8

8 Find the sum of the solutions of

$$2\left(\frac{x}{4}+3\right)^2-\left(\frac{x}{4}+3\right)-36=0$$

A 2

B $\frac{3}{2}$

C $\frac{1}{2}$

D −4

E −13

F −22

G −26

H −34

ENGAA S1 2020 - Question 8 - Worked Solution

$$2\left(\frac{x}{4}+3\right)^2-\left(\frac{x}{4}+3\right)-36=0$$

$$let\ y=\frac{x}{4}+3 \rightarrow x=4y-12$$

$$2y^2-y-36=0$$

$$y^2-\frac{1}{2}y-18=0$$

The second coefficient is the negative of the sum of roots $\rightarrow y_1+y_2=\frac{1}{2}$

$$x_1+x_2=4(y_1)-12+4(y_2)-12$$

$$=4(y_1+y_2)-24=2-24$$

$$x_1+x_2=-22$$

Answer is F

ENGAA S1 2020 - Question 10

10 When the expression

$$(2x + 3)^2 - (x - 3)^2$$

is written in the form $p(x + q)^2 + r$, where p, q and r are constants, what is the value of r?

A -27

B -9

C 0

D 3

E 15

ENGAA S1 2020 - Question 10 - Worked Solution

$$\begin{aligned}
&(2x + 3)^2 - (x - 3)^2 \\
&= 4x^2 + 12x + 9 - (x^2 - 6x + 9) \\
&= 3x^2 + 18x + 0 \\
&= 3(x^2 + 6x) \\
&= 3((x + 3)^2 - 3^2) \\
&= 3(x + 3)^2 - 27
\end{aligned}$$

Answer is A

ENGAA S1 2020 - Question 12

12 The number of pairs of winter boots sold on a day is inversely proportional to the cube of the outside temperature on that day, measured in °C.

On a day when the outside temperature is 8 °C, 250 pairs of boots are sold.

The next day, when the outside temperature is x °C, the number of pairs of boots sold is 700% more than on the previous day.

What is the value of x?

A 2

B 4

C $\frac{8}{\sqrt[3]{7}}$

D $8\sqrt[3]{7}$

E 16

ENGAA S1 2020 - Question 12 - Worked Solution

$$N \propto \frac{1}{T^3}$$

$$\therefore NT^3 = \text{constant}$$

$$250 \times 8^3 = 8 \times 250 \times x^3$$

$$x = \sqrt[3]{8^2} = 4$$

Answer is B

ENGAA S1 2020 - Question 14

14 In a sale, all prices are reduced by 25%.

A customer calculates the pre-sale price of a bicycle incorrectly by increasing the marked sale price by 25%.

The customer's calculated pre-sale price is incorrect by £15.

What is the correct pre-sale price of the bicycle?

A £180

B £195

C £210

D £225

E £240

ENGAA S1 2020 - Question 14 - Worked Solution

$$\beta = 0.75\alpha$$

$$Y = 1.25\beta = 1.25 \times 0.75 \times \alpha$$

$$\gamma = \frac{15}{16}\alpha$$

Y incorrect by E15

$$\frac{1}{16}\alpha = E15 \rightarrow \alpha = 15 \times 16 = E240$$

Answer is E

ENGAA S1 2020 - Question 20

20 The quadratic equation $2x^2 - px - 4 = 0$, where p is a positive constant, has two solutions that differ by 6.

What is the value of p?

A 2

B $4\sqrt{7}$

C 12

D $4\sqrt{11}$

E $4\sqrt{34}$

F $6\sqrt{30}$

ENGAA S1 2020 - Question 20 - Worked Solution

$$ax^2 + bx + c = 0$$

$$x = -\frac{b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac}$$

$$\text{let a} = 2\,,\ \text{b} = -\text{p}\,,\ \text{c} = 4$$

$$3 = \frac{1}{4}\sqrt{p^2 + 4\,.\,2\,.\,4}$$

$$144 = \text{p}^2 + 32$$

$$\text{p} = \sqrt{112} = \sqrt{16}\sqrt{7} = 4\sqrt{7}$$

Answer is B

ENGAA S1 2020 - Question 22

22 $(x-1)$ and $(x-2)$ are both factors of $x^4 + ax^3 + bx^2 - 12x + 4$

What are the values of a and b?

A $a = -6$ and $b = -23$

B $a = -6$ and $b = 13$

C $a = 6$ and $b = -11$

D $a - 6$ and $b - 1$

ENGAA S1 2020 - Question 22 - Worked Solution

$$x^4 + ax^3 + bx^2 - 12x + 4 = f(x)$$
$$f(1) = 0 \rightarrow a + b - 8 = 0$$
$$f(2) = 0 \rightarrow 8a + 4b - 20 = 0$$
$$a = -6$$
$$b = 13$$

Answer is B

ENGAA S1 2020 - Question 36

36 Find the number of solutions of the equation

$$14\cos^3 x + 10\sin^2 x \cos x = 13\cos x$$

in the range $-2\pi \leq x \leq 2\pi$

A 4

B 6

C 8

D 10

E 12

F 14

ENGAA S1 2020 - Question 36 - Worked Solution

Recalling $\sin^2 x = 1 - \cos^2 x$

$14c3x + 10cx(1 - c2x) = 13cx$

$4\cos^3 x - 3\cos\cos x = 0$

$\cos\cos x\,(4\cos^2 x - 3) = 0$

$$\cos\cos x = 0 \quad or \quad \cos\cos x = \pm\sqrt{\frac{3}{4}}$$

4 zeros for each cosine solution

$\cos\cos x = 0$

$$\cos\cos x - \sqrt{\frac{3}{4}}$$

$$\cos\cos x = -\sqrt{\frac{3}{4}}$$

12 solutions

Answer is E

ENGAA S1 2019 - Question 3

3 The equation gives y in terms of x:

$$y = 3 - 4\left(1 - \frac{x}{2}\right)^2$$

Which one of the following is a rearrangement for x in terms of y?

A $x = -2 \pm 2\sqrt{\frac{3-y}{4}}$

B $x = -2 \pm 2\sqrt{\frac{4-y}{3}}$

C $x = 1 \pm \sqrt{\frac{3-y}{4}}$

D $x = 1 \pm 2\sqrt{\frac{3-y}{4}}$

E $x = 2 \pm 2\sqrt{\frac{3-y}{4}}$

F $x = 2 \pm 2\sqrt{\frac{4-y}{3}}$

G $x = 2 \pm 2\sqrt{\frac{3+y}{4}}$

ENGAA S1 2019 - Question 3 - Worked Solution

$$y = 3 - 4\left(1 - \frac{x}{2}\right)^2$$

$$\Rightarrow \quad 1 - \frac{x}{2} = \pm\sqrt{3 - 4}$$

$$2 \mp 2\sqrt{3 - 4} = x$$

Answer is E

ENGAA S1 2019 - Question 7

7 The equation of a curve is $y = px^2 + qx$ where p and q are constants.

The curve passes through the points $(2, 6)$ and $(4, -4)$.

What is the value of $q - p$?

A 1

B 2

C 5

D 6

E 9

F 16

ENGAA S1 2019 - Question 7 - Worked Solution

substituting in equ

$6 = 4p + 2q$

$-4 = 16p + 4q$

$p = -2$

$q = 7$

$\therefore q - p = 9$

Answer is E

ENGAA S1 2019 - Question 9

9 Which of the following is a simplification of

$$4-\frac{x(3x+1)}{x^2(3x^2-2x-1)}$$

A $\dfrac{12x^3-8x^2-7x-1}{x(3x-1)(x-1)}$

B $\dfrac{4x^2+4x-1}{x(x+1)}$

C $\dfrac{4x^2+4x+1}{x(x+1)}$

D $\dfrac{4x^2-4x-1}{x(x-1)}$

E $\dfrac{4x^2-4x+1}{x(x-1)}$

F $\dfrac{12x^3-8x^2-x+1}{x(3x-1)(x-1)}$

ENGAA S1 2019 - Question 9 - Worked Solution

Trick : the diameter probably has one of the top factors is 1.

$$3x2 - 2x - 1 = (3x + 1)(x - 1)$$

$$4-\frac{1}{x(x-1)}=\frac{4x(x-1)-1}{x(x-1)}$$

Answer is D

ENGAA S1 2019 - Question 15

15 *PR* and *QS* are the diagonals of a rhombus *PQRS*.

$PR = (3x + 2)$ cm

$QS = (8 - 2x)$ cm

The area of *PQRS* is 11 cm^2.

What is the difference, in cm, between the two possible lengths of *PR*?

A $2\frac{2}{3}$

B $4\frac{1}{2}$

C $5\frac{1}{3}$

D 8

E 14

ENGAA S1 2019 - Question 15 - Worked Solution

$$A = \frac{1}{2} \times PR \times QS = 11cm^2$$

$$\frac{1}{2} \times (3x + 2)(8 - 2x) = 11$$

$$(3x - 1)(x - 3) = 0$$

$$x = \frac{1}{3}, x = 3$$

$$PR = 3x + 2$$

$$: PR = 3 \ , PR = 11$$

$$11 - 3 = 8$$

Answer is D

ENGAA S1 2019 - Question 25

25 When simplified, $\frac{1}{(1-\sqrt{2})^3}$ is written in the form $a+b\sqrt{2}$ where a and b are integers.

What is the value of b?

A –7

B –5

C –1

D 1

E 5

F 7

ENGAA S1 2019 - Question 25 - Worked Solution

$$\frac{1}{(1-\sqrt{2})^3} \times \left(\frac{1+\sqrt{2}}{1+\sqrt{2}}\right)^3 = \left(\frac{1+\sqrt{2}}{1^2-\sqrt{2}^2}\right)^3$$

$$= \frac{(1+\sqrt{2})^3}{(-1)^3} = -(1+3\sqrt{2}+3(2)+2\sqrt{2})$$

$$= -7-5\sqrt{2}$$

$$b = -5$$

Answer is B

ENGAA S1 2019 - Question 39

39 Find the complete set of values of x for which

$$x^3 - 2x^2 - 7x - 4 > 0$$

A $x < -1$

B $x > -1$

C $-1 < x < 4$

D $x < -1$ or $x > 4$

E $x < 4$

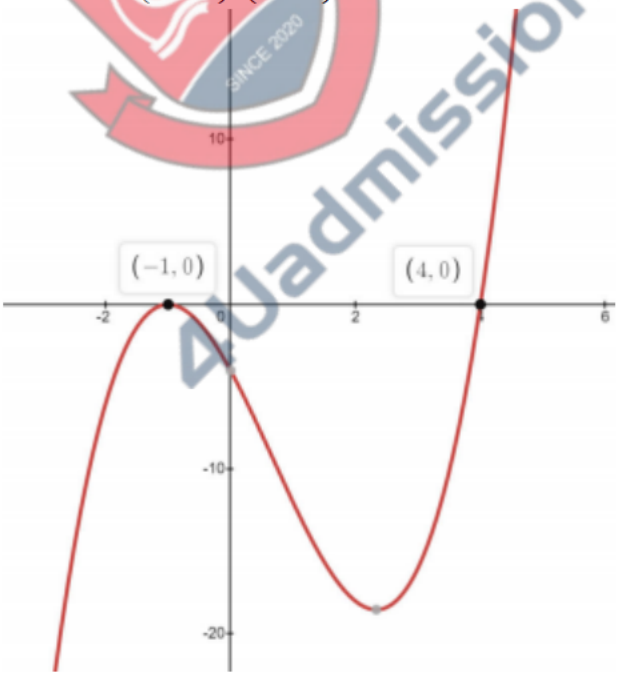
F $x > 4$

ENGAA S1 2019 - Question 39 - Worked Solution

$$f = x^3 - 2x^2 - 7x - 4 > 0$$

Use factor theorem to find that -1 is a root. This helps to factorise the expression.

$$(x + 1)^2 (x - 4) > 0$$



Drawing the graph helps see where it is above 0.

Answer is F

ENGAA Specimen S1 - Question 5

5 Which of the expressions below has the largest value for $0 < x < 1$?

A $\frac{1}{x}$

B x^2

C $\frac{1}{(1+x)}$

D $\frac{1}{\sqrt{x}}$

E $\sqrt{x}$

ENGAA Specimen S1 - Question 5 - Worked Solution

Let's first consider the range of each of the options:

$$0 < x < 1$$
$$0 < x^2 < 1$$
$$0 < \sqrt{x} < 1$$
$$\frac{1}{x} > 1$$
$$\frac{1}{\sqrt{x}} > 1$$
$$\frac{1}{1+x} < 1$$

We can see from this that it is either **Option A** or **Option D**

$x < \sqrt{x}$ as $\sqrt{x}$ is less than 1. Thus, $\frac{1}{x} > \frac{1}{\sqrt{x}}$

$\frac{1}{x}$ has the greatest value over $0 < x < 1$

Answer is A

ENGAA Specimen S1 - Question 15

15 Solve the inequality $x^2 \geq 8 - 2x$

A $x \geq 4$

B $x \leq 2$ and $x \geq -4$

C $x \geq -2$ and $x \leq 4$

D $x \geq 2$ or $x \leq -4$

ENGAA Specimen S1 - Question 15 - Worked Solution

$$x^2 \geq 8 - 2x$$
$$x^2 + 2x - 8 \geq 0$$
$$(x + 4)(x - 2) \geq 0$$

$$x \leq -4, x \geq 2$$

Answer is D

ENGAA Specimen S1 - Question 21

21 Which one of the following is a simplification of $\frac{x^2-4}{x^2-2x}$ where $x \neq 2$ and $x \neq 0$?

A $\frac{x-4}{x-2}$

B $\frac{x-2}{x}$

C $\frac{2}{x}$

D $\frac{x+2}{x}$

E $\frac{x+2}{x+1}$

ENGAA Specimen S1 - Question 21 - Worked Solution

$$\frac{x^2-4}{x^2-2x} = \frac{(x+2)(x-2)}{x(x-2)} = \frac{x+2}{x} \quad if \ x \neq 2, x \neq 0$$

Answer is D

ENGAA Specimen S1 - Question 29

29 How many real roots does the equation $x^4 - 4x^3 + 4x^2 - 10 = 0$ have?

A 0

B 1

C 2

D 3

E 4

ENGAA Specimen S1 - Question 29 - Worked Solution

The turning points are:

$$(0\,,-10)\,,(1\,,-9) \text{ and } (2\,,-10)$$

Due to the shape of quartics, we can know $(1\,,—9)$ is a local maximum and $(2\,,—10)$ are minimum.

The curve therefore only intersects the x axis twice and have 2 real roots.

Answer is C

ENGAA S1 2018 - Question 47

47 What is the coefficient of x^3 in the expansion of $(1 - 2x)^5(1 + 2x)^5$?

A −6400

B −640

C −80

D 0

E 80

F 800

G 960

ENGAA S1 2018 - Question 47 - Worked Solution

$$(1 - 2x)^5 (1 + 2x)^5$$

$$= [1 - (2x) \times 5 \times 1_3 \times (2x)^2 \times 10 \times 1_4 - (2x)^3 \times 10 \times 1_5 + \cdots][1 + (2x) \times 5 + (2x_2) \times 10 + (2x)^3 \times 10 + (2x)^3 \times 10 + \cdots]$$

$$= [1 - 10x + 40x^2 - 80x^3 + \cdots][1 + 10x + 40x^2 + 80x^3 + \cdots]$$

Find coefficient of x^3:

$$(1 \times 80) - (10 \times 4) + (40 \times 10) - (80 \times 1)$$

$$= 0$$

Answer is D.

ENGAA S1 2017 - Question 3

3 Solve fully the inequality

$$2x^2 \geq 15 - x$$

A $x \leq -3$

B $x \geq 2.5$

C $x \leq -1.5\,,\ x \geq 5$

D $-1.5 \leq x \leq 5$

E $x \leq -3\,,\ x \geq 2.5$

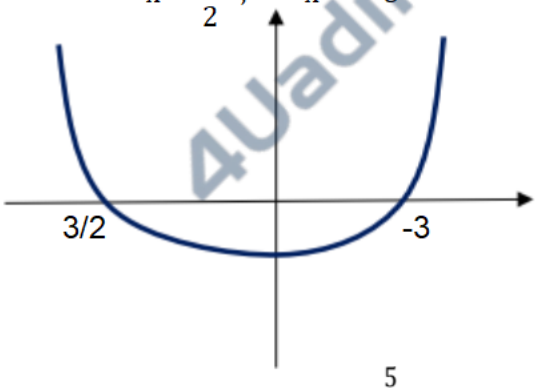
F $-3 \leq x \leq 2.5$

ENGAA 2017 - Question 3 - Worked Solution

$$2x^2 \geq 15 - x$$
$$2x^2 + x - 15 \geq 0$$
$$(2x - 5)(x + 3) \geq 0$$

Critical values are then:

$$x = \frac{5}{2}\ , \qquad x = -3$$



$$x \leq -3\ , \qquad x \geq \frac{5}{2}$$

Answer is F

ENGAA S1 2017 - Question 5

5 The equation gives y in terms of x:

$$y = 3\left(\frac{x}{2} - 1\right)^2 - 5$$

Which one of the following is a rearrangement for x in terms of y?

A $x = 2 \pm 2\sqrt{\frac{y-5}{3}}$

B $x = 2 \pm 2\sqrt{\frac{y+5}{3}}$

C $x = 2 \pm 3\sqrt{\frac{y+5}{3}}$

D $x = -2 \pm 2\sqrt{\frac{y+5}{3}}$

E $x = -2 \pm 3\sqrt{\frac{y+5}{2}}$

F $x = 2 + 2\left(\frac{y+5}{3}\right)^2$

G $x = -2 + 2\left(\frac{y+5}{3}\right)^2$

ENGAA 2017 - Question 5 - Worked Solution

$$y = 3\left[\frac{x}{2} - 1\right]^2 - 5$$

$$y + 5 = 3\left[\frac{x}{2} - 1\right]^2$$

$$\frac{y+5}{3} = \left[\frac{x}{2} - 1\right]^2$$

$$\pm\sqrt{\frac{y+5}{3}} = \frac{x}{2} - 1$$

$$1 \pm \sqrt{\frac{y+5}{3}} = \frac{x}{3}$$

$$2 \pm 2\sqrt{\frac{y+5}{3}} = x$$

Answer is B

ENGAA S1 2017 - Question 7

7 A fruit stall sells apples costing £x each, and pears costing £y each.

Sam bought 2 apples and 5 pears, and the total cost of these was £P.

Lesley bought 3 apples and 2 pears, and the total cost of these was £Q.

Which of the following is an expression for the cost, in pounds (£), of a pear?

A $\frac{2Q-3P}{3}$

B $\frac{2Q-3P}{11}$

C $\frac{Q-P}{3}$

D $\frac{Q-P}{11}$

E $\frac{P-Q}{3}$

F $\frac{3P-2Q}{3}$

G $\frac{3P-2Q}{11}$

ENGAA 2017 - Question 7 - Worked Solution

$$\text{sam} : 2x + 5y = P \quad ---①$$
$$\text{Lesley} : 3x + 2y = Q \quad ---②$$

Want to find value at y

$$3 \times ① : 6x + 15y = 3P$$
$$2 \times ② : 6x + 4y = 2Q$$

Subtract to eliminate x:

$$15y - 4y = 3P - 2Q$$
$$11y = 3P - 2Q$$
$$y = \frac{3P - 2Q}{11}$$

Answer is G

ENGAA S1 2017 - Question 11

11 Which one of the following is a simplification of

$$2-\frac{x^2\left(9x^2-4\right)}{x^3(2-3x)}$$

A $-1-\frac{2}{x}$

B $-1+\frac{2}{x}$

C $5-\frac{2}{x}$

D $5+\frac{2}{x}$

E $5-\frac{3}{x}$

F $5+\frac{3}{x}$

ENGAA 2017 - Question 11 - Worked Solution

$$2-\frac{x^2(9x^2-4)}{x^3(2-3x)}$$
$$=2-\frac{(9x^2-4)}{x(2-3x)}$$
$$=2-\frac{(3x-2)(3x-2)}{x(2-3x)}$$
$$=2+\frac{(3x-2)(3x-2)}{x(2-3x)}$$
$$=2+\frac{3x+2}{x}$$
$$=2+3+\frac{2}{x}$$
$$=5+\frac{2}{x}$$

Answer is D

ENGAA S1 2017 - Question 13

13 What is the value of x that makes the following expression correct?

$$2^{3+2x}4^x8^{-x} = 4\sqrt{2}$$

A -2.25

B -1.75

C -1.5

D -0.5

E -0.25

ENGAA 2017 - Question 13 - Worked Solution

$$2^{3+2x}4^x8^{-x} = 4\sqrt{2}$$
$$2^{3+2x}(2^2)^x(2^3)^{-x} = 2^2 2^{1/2}$$
$$2^{3+2x}2^{2x}2^{-3x} = 2^2 2^{1/2}$$
$$2^{3+2x+2x-3x} = 2^{2+1/2}$$
$$2^{x+3} = 2^{2+1/2}$$
$$x + 3 = 2 + 0.5$$
$$x = -0.5$$

Answer is D

ENGAA S1 2017 - Question 49

49 Find the complete set of values of x for which

$$\frac{x^3-6x^2+9x-4}{x}>0$$

A $x<0,\ x>4$

B $0<x<4$

C $0<x<1,\ x>4$

D $x<0,\ 1<x<4$

E $x<1,\ x>4$

F $1<x<4$

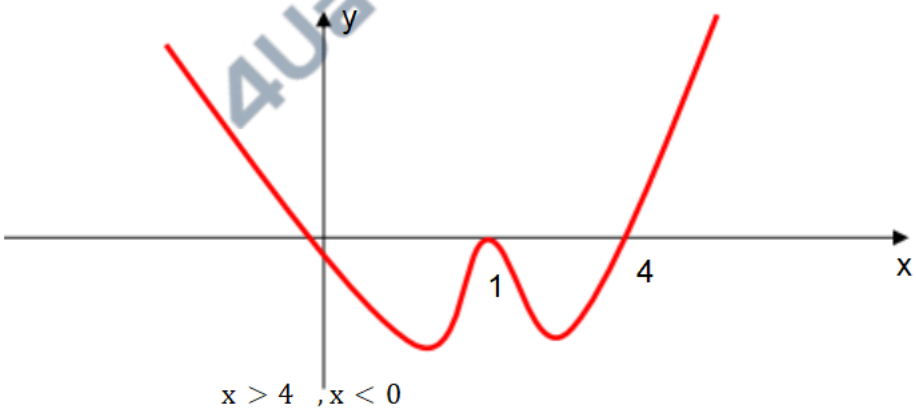
ENGAA S1 2017 - Question 49 - Worked Solution

$$(x^3-6x2+9x-4)x>0$$
$$(x-1)^2(x-4)x>0$$

The cuitical points are

$$x=1 \text{ (Repeated Roots)}, x=4, x$$

$$\text{sketch } (x-1)^2(x-4)x=y$$



$$x>4 \quad , x<0$$

Answer is A

ENGAA S1 2016 - Question 1

Find the complete set of solutions to $-8 < 6 - \frac{x}{2}$

A $x < 4$

B $x > 4$

C $x < 20$

D $x > 20$

E $x < 22$

F $x > 22$

G $x < 28$

H $x > 28$

ENGAA S1 2016 - Question 1 - Worked Solution

$$-8 < 6 - \frac{x}{2}$$

$$-14 < -\frac{x}{2}$$

$$-28 < -x$$

$$x < 28$$

Answer is G

ENGAA S1 2016 - Question 13

13 The quantities x and y are positive.

x is inversely proportional to the square root of y.

When $x = 8$, $y = 9$.

What is the value of y when $x = 6$?

A $\frac{3}{2}$

B 2

C $\frac{81}{16}$

D $\frac{27}{14}$

E 12

F 16

ENGAA S1 2016 - Question 13 - Worked Solution

$$x \propto \frac{1}{\sqrt{y}}$$

$$x = \frac{K}{\sqrt{y}}$$

$$8 = \frac{K}{\sqrt{9}}$$

$$8 = \frac{K}{3}$$

$$K = 24$$

$$6 = \frac{K}{\sqrt{y}}$$

$$6 = \frac{24}{\sqrt{y}}$$

$$y = \left(\frac{24}{6}\right)^2 = 16$$

Answer is F

ENGAA S1 2016 - Question 17

17 Make b the subject of the formula:

$$a = \frac{b^2 + 2}{3b^2 - 1}$$

A $b = \pm\sqrt{\left(\frac{a+2}{3a+1}\right)}$

B $b = \pm\sqrt{\left(\frac{a+2}{3a-1}\right)}$

C $b = \pm\sqrt{\left(\frac{2-a}{3a+1}\right)}$

D $b = \pm\sqrt{\left(\frac{2-a}{3a-1}\right)}$

E $b = \pm\sqrt{\left(\frac{3}{3a+1}\right)}$

F $b = \pm\sqrt{\left(\frac{3}{3a-1}\right)}$

ENGAA S1 2016 - Question 17 - Worked Solution

$$a = \frac{b^2 + 2}{3b^2 - 1}$$
$$a(3b^2 - 1) = b^2 + 2$$
$$3ab^2 - a = b^2 + 2$$
$$(3a - 1)b^2 - a - 2 = 0$$
$$(3a - 1)b^2 = a + 2$$
$$b^2 = \frac{a + 2}{3a - 1}$$
$$b = \pm\sqrt{\frac{a + 2}{3a - 1}}$$

Answer is B

ENGAA S1 2016 - Question 21

21 Which one of the following is a simplification of $4+\frac{4-x^2}{x^2-2x}$?

A $3-\frac{2}{x}$

B $3+\frac{2}{x}$

C $4-\frac{2}{x}$

D $4+\frac{2}{x}$

E $5-\frac{2}{x}$

F $5+\frac{2}{x}$

ENGAA S1 2016 - Question 21 - Worked Solution

$$4+\frac{4-x^2}{x^2-2x}$$
$$=4+\frac{(2+x)(2-x)}{x(x-2)}$$
$$=4-\frac{2+x}{x}$$
$$=4-\frac{2}{x}-1$$
$$=3-\frac{2}{x}$$

Answer is A

ENGAA S1 2016 - Question 29

29 When $x = 2$ is substituted in the expression $x^3 + px^2 + qx + p^2$ the result is 0.

When $x = 1$ is substituted into the same expression, the result is -3.5.

Find all possible value(s) of p.

A $p = -1 \pm \frac{\sqrt{6}}{3}$

B $p = 1$ or $p = -3$

C $p = 1$

D $p = 1 \pm \sqrt{7}$

E there are no values for p

ENGAA S1 2016 - Question 29 - Worked Solution

$$2^3 + p \times 2^2 + 2q + p^2 = 0 \quad \text{———} \ (1)$$
$$1^3 + p \times 1^2 + 1q + p^2 = -3.5 \quad \text{———} \ (2)$$
$$(1) - 2 \times (2)$$
$$6 + 2p - p^2 = 7$$
$$p^2 - 2p + 1 = 0$$
$$(p - 1)^2 - 1^2 + 1 = 0$$
$$(p - 1)^2 = 0$$
$$p = 1$$

Answer is C

ENGAA S1 2016 - Question 39

39 The complete set of values of a for which the equation $3x^2 = (a+2)x - 3$ has two real distinct roots is

A no values of a

B $-4\sqrt{2} < a < 4\sqrt{2}$

C $a < -4\sqrt{2},\ a > 4\sqrt{2}$

D $-4 < a < 8$

E $a < -4,\ a > 8$

F $-8 < a < 4$

G $a < -8,\ a > 4$

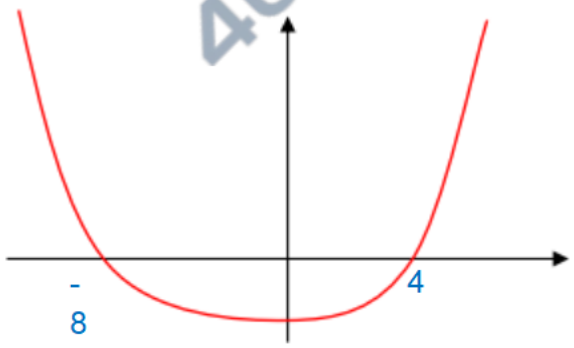
H all values of a

ENGAA S1 2016 - Question 39 - Worked Solution

$$3x^2 = (a+2)x - 3$$
$$3x^2 - (a+2)x + 3 = 0$$

For $\alpha x^2 + \beta x + Y = 0$ to have two real roots

$$\beta^2 - 4\alpha Y > 0$$
$$(a+2)^2 - 4(3)(3) > 0$$
$$a^2 + 4a + 4 - 36 > 0$$
$$a^2 + 4a - 32 > 0$$



$$(a-4)(a+8) > 0$$

Answer is G

ENGAA S1 2016 - Question 47

47 The complete set of values of x for which $2x^4 - 9x^2 + 4 > 0$ is

A $x < \frac{1}{2},\ x > 4$

B $\frac{1}{2} < x < 4$

C $x < -2,\ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}},\ x > 2$

D $-2 < x < \frac{-1}{\sqrt{2}},\ \frac{1}{\sqrt{2}} < x < 2$

E $-2 < x < 2$

ENGAA S1 2016 - Question 47 - Worked Solution

$$2x^4 - 9x^2 + 4 > 0$$

$$\text{Let } u = x^2$$

$$2u^2 - 9u + 4 > 0$$

$$(2u - 1)(u - 4) > 0$$

$$u = \frac{1}{2},\ u = 4 \text{ when the function equals } 0$$

$$x^2 = \frac{1}{2},\ x^2 = 4$$

$$x = \frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}},\ 2,\ -2$$

$$x < -2,\ -\frac{1}{\sqrt{2}},\ x > 2$$

Answer is C

ENGAA S1 2016 - Question 53

53 A curve has equation $y = 3x^4 - 4x^3 - 12x^2 + 20$

What is the complete set of values of the constant k for which the equation

$$3x^4 - 4x^3 - 12x^2 + 20 = k$$

has exactly four distinct real roots?

A no values of k

B $-12 < k < 15$

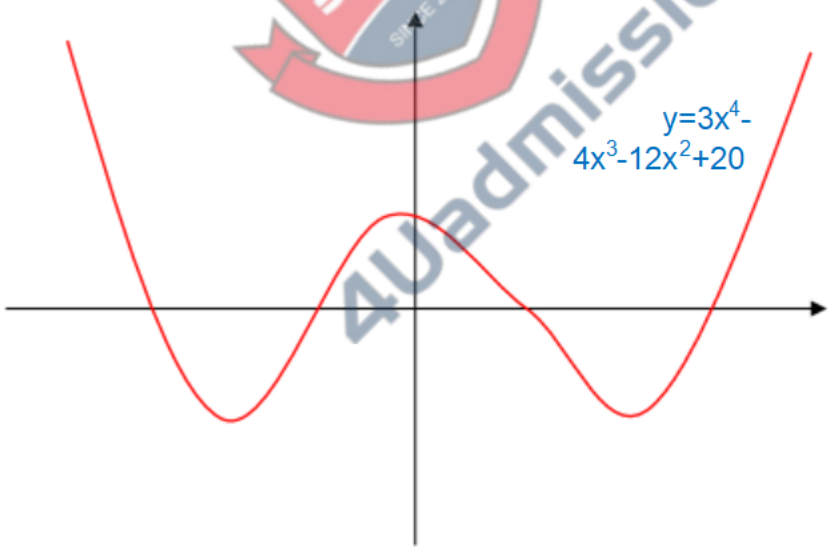
C $15 < k < 20$

D $k > 20$

E $7 < k < 20$

F all values of k

ENGAA S1 2016 - Question 53 - Worked Solution



$$y = 3x^4 - 4x^3 - 12x^2 + 20$$

As the curve is a quadratic it has this form with 2 minima and one maximum. The exact shapes does not matter only that it is positive.

For y = k to have 4 real roots, the line y = k must intersect $y = 3x^4 - 4x^3 - 12x^2 + 20$ 4 times. K must be higher than both of the minimum values and lower than the maximum value.

To find the extrema, set $\frac{dy}{dx}$ to 0

$$\frac{dy}{dx} = 12x^3 - 12x^2 - 24x = 0$$

$$x(12x^2 - 12x - 24) = 0$$

$$12x(x + 1)(x - 2) = 0$$

$$x = 0, x = -1, x = 2$$

The y values of these points are : 20 , 15 , and -12 respectively.

$$: k > -12, k > 15, k < 20$$

$$15 < k < 20$$

Answer is C

ENGAA 2019 - Section 1 - Question 6

Insert

ENGAA 2019 - Section 1 - Question 6 - Worked Solution

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