Worked Solutions for ENGAA Papers by Topic

Section 1

Topic: Matter & Thermal Physics

Section 1 Topic	Number of Questions 2016 - 2020
Algebra	34
Calculus	16
Coordinate geometry	11
Electricity	18
Energy	8
Exponentials and logarithms	9
Forces and equilibrium	7
Geometry	40
Kinematics	15
Materials	2
Matter & thermal physics	5
Mechanics	55
Number	11
Probability	3
Radioactivity	14
Ratio and proportion	7
Sequences and series	8
Trigonometry	6
Waves	13

ENGAA S1 2020 - Question 24

24 What is the area of the region enclosed between the curve $y = \frac{1}{2}x^2$, the line $y = -x$, and the lines $x = 1$ and $x = 3$?

A $\frac{1}{3}$

B 2

C 4

D 6

E $\frac{25}{3}$

F $\frac{28}{3}$

ENGAA S1 2020 - Question 24 - Worked Solution

$$A = \int_{x=1}^{3} \left(\frac{x^2}{2} - (-x)\right) dx$$

$$x = 1$$

$$A = \left[\frac{x^3}{6} + \frac{x^2}{2}\right]_1^3$$

$$A = \frac{27}{6} + \frac{9}{2} - \frac{1}{6} - \frac{1}{2}$$

$$A = \frac{25}{3}$$

Answer is E

ENGAA S1 2020 - Question 34

34 The curve

$$y = x^3 + 3\sqrt{5}px^2 + 3px + 13$$

has two distinct turning points.

What are all the possible values of p?

A $p < 0$, $p > 0.2$

B $p \leq 0$, $p \geq 0.2$

C $0 < p < 0.2$

D $0 \leq p \leq 0.2$

E $p < 0$, $p > 1.2$

F $p \leq 0$, $p \geq 1.2$

G $0 < p < 1.2$

H $0 \leq p \leq 1.2$

ENGAA S1 2020 - Question 34 - Worked Solution

$$\frac{dy}{dx} = 3x^2 + 6\sqrt{5}px + 3p$$

$$= ax^2 + bx + c$$

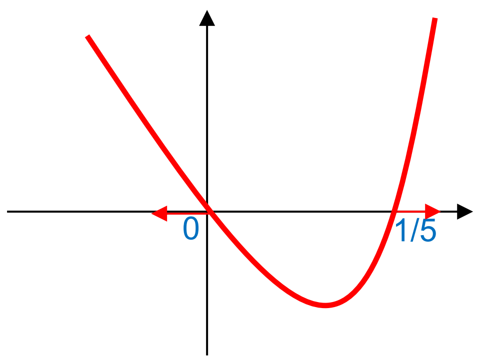
For definite turning point

$$b^2 - 4ac > 0$$

$$180p^2 - 36p > 0$$

$$p = 0 \ \ or \ \ p = \frac{1}{5}$$

$$we\ need\ 36p(5p - 1) > 0$$



$$p < 0 \quad , \qquad p > \frac{1}{5}$$

Answer is A

ENGAA S1 2020 - Question 40

40 Find the maximum value of the gradient of the curve with equation

$$y = 2 - 4x + 4x^{\frac{3}{2}} - x^2$$

where $x > 0$

A -4

B $-\frac{8}{9}$

C $\frac{1}{2}$

D 2

E 4

ENGAA S1 2020 - Question 40 - Worked Solution

$$y = 2 - 4x + 4x^{\frac{3}{2}} - x^2$$

$$\frac{dy}{dx} = -4 + 6x^{\frac{1}{2}} - 2x$$

$$\frac{dy}{dx} = \frac{3}{\sqrt{x}} - 2 = 0$$

$$\sqrt{x} = \frac{3}{2}$$

$$x = \frac{9}{4}$$

$$\frac{dy}{dx}\left(x = \frac{9}{4}\right) = \frac{1}{2}$$

Answer is C

ENGAA S1 2019 - Question 23

23 The curve

$$y = x^3 + px^2 + qx + r$$

has a local maximum when $x = -1$ and a local minimum when $x = 3$

What is the value of p?

A –9

B –3

C –1

D 1

E 3

F 9

ENGAA S1 2019 - Question 23 - Worked Solution

$$y' = 3x^2 + 2px + q$$
$$y' = 0 \ when \ x = -1 \ and \ x = 3 \ , sub \ in$$
$$3 - 2p + q = 0$$
$$27 + 6p + q = 0$$
$$27 - 3 + 6p + 2p + q - q = 0$$
$$p = -3$$

Answer is B

ENGAA S1 2019 - Question 33

33 For a particular function $f(x)$, it is given that:

$$\int_{-2}^{2} 2f(x)\mathrm{d}x + \int_{2}^{4} f(x)\mathrm{d}x = 4$$

and also:

$$\int_{-2}^{2} 5f(x)\mathrm{d}x - \int_{-2}^{4} f(x)\mathrm{d}x = 7$$

Find the value of $\int_{2}^{4} f(x)\mathrm{d}x$

A $\frac{1}{3}$

B $\frac{11}{7}$

C $\frac{11}{6}$

D $\frac{13}{6}$

E $\frac{13}{3}$

ENGAA S1 2019 - Question 33 - Worked Solution

$$\int_{-2}^{2} 2f(x)dx + \int_{2}^{4} f(x)dx = 4 \quad ----①$$

and also

$$\int_{-2}^{2} 5f(x)dx - \int_{-2}^{4} f(x)dx = 7$$

$$\int_{-2}^{2} 4f(x)dx + \int_{-2}^{2} fdx + \int_{4}^{-2} fdx = 7$$

$$\int_{-2}^{2} 4fdx - \int_{2}^{4} fdx = 7 \quad (taking\ 2(①)\ off\ this$$

$$\int_{-2}^{2} 4fdx - 2\int_{-2}^{2} 2fdx - \int_{2}^{4} fdx - 2\int_{2}^{4} fdx = 7 - 2 \times 4$$

$$0 - 3\int_{2}^{4} fdx = -1$$

$$\therefore \int_2^4 \quad f dx = \frac{-1}{-3} = \frac{1}{3}$$

Answer is A

ENGAA S1 2019 - Question 35

35 Given that

$$f(x) = \int_0^x (3 + 2t)^7 \, dt$$

what is the coefficient of x^4 in the expansion of $f(x)$ in powers of x?

A 70

B 162

C $\frac{2835}{4}$

D 3024

E 5670

F 15 120

G 22 680

ENGAA S1 2019 - Question 35 - Worked Solution

$$f(x) = \int_0^x \quad (3 + 2t)^7 dt$$

$$f(x) = \int \quad (C_0 + C_1 t + C_2 t^2 + C_3 t^3 + \cdots + +C_7 t^7) dt$$

$$t^3 \; term \; will \; go \; to \; x^4$$

$$C_3 t^3 = \frac{7!}{3!\,4!} \, . \, 3^4 (2t)^3 = 35 \times 81 \times 8t^3$$

$$= 5670 \times 4t^3$$

$$5670 \times \int_0^x \quad 4t^3 = 5670x^4$$

Answer is E

ENGAA S1 2018 - Question 29

29 Curve C has equation $y = 9 - x^2$

Line L has equation $y = 5$

What is the area enclosed between C and L?

A $\frac{32}{3}$

B $\frac{62}{3}$

C $\frac{92}{3}$

D $\frac{122}{3}$

E $\frac{152}{3}$

ENGAA S1 2018 - Question 29 - Worked Solution

$$y = 9 - x^2 = (3 - x)(3 + x)$$

Intersect at:

$$5 = 9 - x^2$$
$$x^2 = 4$$
$$x = \pm 2$$

$$A = \int_{-2}^{2} 9 - x^2 dx - (4 \times 5)$$
$$= \left[9x - \frac{x^3}{3}\right]_{-2}^{2} - 20$$
$$= \frac{92}{3} - 20$$
$$= \frac{32}{3}$$

Answer is A.

ENGAA S1 2018 - Question 33

33 The line $y = x + k$, where k is a constant, is a tangent to the curve $y = 3x^2 - 2x + 1$

What is the value of k?

A -2

B -1

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{1}{2}$

F $\frac{3}{4}$

G 1

H 2

ENGAA S1 2018 - Question 33 - Worked Solution

$$\text{①}\ y = x + k$$

$$\text{②}\ y = 3x^2 - 2x + 1$$

$$y = 3x^2 - 2x + 1$$

$$\frac{dy}{dx} = 6x - 2$$

Gradient of y-axis is 1

So when tangent

$$6x - 2 = 1$$

$$x = \frac{1}{2}$$

$$y = \frac{3}{4} \quad \left(sub\ x = \frac{1}{2}\ into\ \text{②}\right)$$

$$= \frac{1}{2} + k \quad \left(sub\ x = \frac{1}{2}\ into\ \text{①}\right)$$

$$k = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$$

Answer is C.

ENGAA S1 2018 - Question 49

49 Given that

$$\int_0^2 x^m \mathrm{d}x = \frac{16\sqrt{2}}{7}$$

and

$$\int_0^2 x^{m+1} \mathrm{d}x = \frac{32\sqrt{2}}{9}$$

what is the value of m?

A $-\frac{11}{2}$

B $-\frac{9}{2}$

C $-\frac{22}{29}$

D $\frac{7}{22}$

E $\frac{5}{2}$

F $\frac{7}{2}$

ENGAA S1 2018 - Question 49 - Worked Solution

$$\int_0^2 x^m dx = \frac{16\sqrt{2}}{7}$$

$$\left[\frac{x^{m+1}}{m+1}\right]_0^2 = \frac{16\sqrt{2}}{7}$$

$$\frac{2^{m+1}}{m+1} = \frac{16\sqrt{2}}{7} \quad ①$$

$$\int_0^2 x^{m+1} dx = \frac{32\sqrt{2}}{9}$$

$$\left[\frac{x^{m+2}}{m+2}\right]_0^2 = \frac{32\sqrt{2}}{9}$$

$$\frac{2^{m+2}}{m+2} = \frac{32\sqrt{2}}{9} \quad ②$$

② (①)

$$2 \times \frac{m+1}{m+2} = \frac{32}{9} \times \frac{7}{16}$$
$$(m+1) = \frac{1}{2} \times \frac{14}{9}(m+2)$$
$$9m + 9 = 7m + 14$$
$$2m = 5$$
$$m = 5/2$$

Answer is E.

ENGAA S1 2018 - Question 51

51 The two functions f and g satisfy

$$f'(x) = ax + g(x)$$

where a is a constant.

Given that

$$\int_2^4 g(x)\,dx = 12$$

and

$$f(4) = 18 + f(2)$$

what is the value of a?

A 1

B 3

C 5

D 6

E 15

ENGAA S1 2018 - Question 51 - Worked Solution

$$f'(x) = ax + g(x)$$

$$\frac{df}{dx} = ax + g(x)$$

$$\int_2^4 \quad g(x)dx = 12$$

$$\int_2^4 \quad \frac{dt}{dx}dx = \left[\frac{ax^2}{2}\right]_2^4 + \int_2^4 \quad g(x)dx$$

$$\int_2^4 \quad dt = \frac{16a}{2} - \frac{4a}{2} + 12 \quad ①$$

$$f(4) - f(2) = 6a + 12$$

$$f(4) - f(2) = 18 \quad ②$$

$$8 = 6a + 12$$

$$6 = 6a$$

$$a = 1$$

Answer is A.

ENGAA S1 2017 - Question 39

39 The graph of the function $y = x^3 + px^2 + qx + 6$, where p and q are real constants, has a local maximum when $x = 2$ and a local minimum when $x = 4$.

What are the values of p and q?

A $p = -3$ and $q = -8$

B $p = -3$ and $q = 8$

C $p = 3$ and $q = -8$

D $p = -9$ and $q = 24$

E $p = 9$ and $q = 24$

F $p = 9$ and $q = -24$

ENGAA S1 2017 - Question 39 - Worked Solution

$$\frac{dy}{dx} = 3x^2 + 2px + q$$

$$At\ maximum: \frac{dy}{dx} = 0$$

$$x = 2:$$

$$3(2)^2 + 2p \times 2 + q = 0$$

$$\Rightarrow 4p + q + 12 = 0 \quad ------ \text{①}$$

$$x = 4:$$

$$3(4)^2 + 2p \times 4 + q = 0$$

$$\Rightarrow 48 + 8p + q = 0 \quad ----\text{②}$$

② - ①

$$4p + 36 = 0$$

$$\Rightarrow p = -\frac{36}{4} = -9$$

Sub p = -9 into ①

$$q = -12 - 4p$$

$$q = -12 + (4 \times 9)$$

$$q = 24$$

Answer is D

ENGAA S1 2017 - Question 43

43 Given that $y=(2+3x)^6$, what is the coefficient of x^3 in $\frac{dy}{dx}$?

A 240

B 4320

C 4860

D 12 960

E 19 440

ENGAA S1 2017 - Question 43 - Worked Solution

$$\frac{d}{dx}[x^n] = nx^{n-1}, \textit{so look at } x^4 \textit{ term in } y$$

$$x^4 \textit{ term in } y = \binom{6}{2}.2^2.(3x)^4$$

$$= 4860x^4$$

$$\textit{Differentiate}: 4 \times 4860x^3$$

$$= 19440x^3$$

Answer is E

ENGAA S1 2017 - Question 51

51 The curve $y = \sin x$ is stretched by a scale factor of $\frac{1}{2}$ parallel to the x-axis and then translated by $\frac{\pi}{4}$ in the negative x-direction.

What is the equation of the new curve?

A $y = \sin\left(\frac{x}{2} - \frac{\pi}{4}\right)$

B $y = \sin\left(\frac{x}{2} + \frac{\pi}{4}\right)$

C $y = \sin\left(\frac{x}{2} - \frac{\pi}{8}\right)$

D $y = \sin\left(\frac{x}{2} + \frac{\pi}{8}\right)$

E $y = \sin\left(2x - \frac{\pi}{4}\right)$

F $y = \sin\left(2x + \frac{\pi}{4}\right)$

G $y = \sin\left(2x - \frac{\pi}{2}\right)$

H $y = \sin\left(2x + \frac{\pi}{2}\right)$

ENGAA S1 2017 - Question 51 - Worked Solution

Initially:

$$y = sin\ sin\ (x)$$

Stretch in x-axis

$$x \to 2x$$

Translation :

$$x \to x + \frac{\pi}{4}$$

$$\Rightarrow y = sin\ sin\ \left(2\left(x + \frac{\pi}{4}\right)\right)$$

$$y = sin\ sin\ \left(2x + \frac{\pi}{2}\right)$$

Answer is H

ENGAA S1 2016 - Question 43

43 $f(x) = x^3 - a^2x$ where a is a positive constant.

Find the complete set of values of x for which $f(x)$ is an increasing function.

A $x \le -a,\ x \ge a$

B $-a \le x \le a$

C $x \le -a,\ 0 \le x \le a$

D $-a \le x \le 0,\ x \ge a$

E $x \le -\frac{a}{3},\ x \ge \frac{a}{3}$

F $-\frac{a}{3} \le x \le \frac{a}{3}$

G $x \le -\frac{a}{\sqrt{3}},\ x \ge \frac{a}{\sqrt{3}}$

H $-\frac{a}{\sqrt{3}} \le x \le \frac{a}{\sqrt{3}}$

ENGAA S1 2016 - Question 43 - Worked Solution

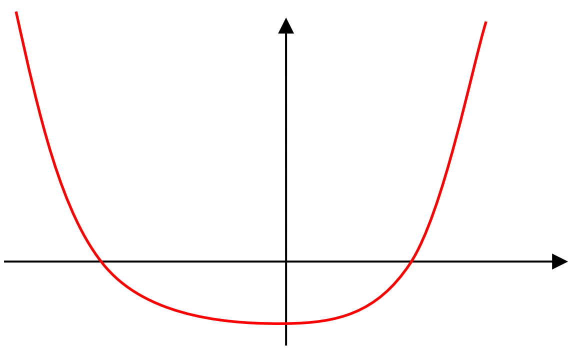
$$f(x) = x^3 - a^2x$$

$$f'x = 3x^2 - a^2$$

$$f'x = 3\left(x^2 - \left(\frac{a}{\sqrt{3}}\right)^2\right) > 0$$

$$3\left(x^2 - \left(\frac{a}{\sqrt{3}}\right)^2\right) > 0$$

$$3\left(x + \frac{a}{\sqrt{3}}\right)\left(x - \frac{a}{\sqrt{3}}\right) > 0$$



$$x \leq -\frac{a}{\sqrt{3}} \ , x \geq \frac{a}{\sqrt{3}}$$

Answer is G

ENGAA S1 2016 - Question 45

45 The curve $y = x^2$ is translated by the vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and then reflected in the line $y = -1$

Which one of the following is an equation of the resulting curve?

A $y = -3 - (x-4)^2$

B $y = -3 + (x+4)^2$

C $y = 3 - (x+4)^2$

D $y = 3 + (x-4)^2$

E $y = -5 - (x-4)^2$

F $y = -5 + (x+4)^2$

G $y = 5 - (x+4)^2$

H $y = 5 + (x-4)^2$

ENGAA S1 2016 - Question 45 - Worked Solution

First the translation

This transforms the curve into

$y = (x-4)^2 + 3$ As this is a shift to the right by 4 and up by 3.

Then the reflection, the bottom point of the curve is 4 units above the line y = - 1 and it is a positive quadratic.

After the reflection it will be 4 units below the line and a negative quadratic.

$$y = -(x-4)^2 - 5$$

Answer is E

ENGAA S1 2016 - Question 51

51 What is the area enclosed by the line $x=7$ and the curve $x=3(y-1)^2+4$?

A 4

B 8

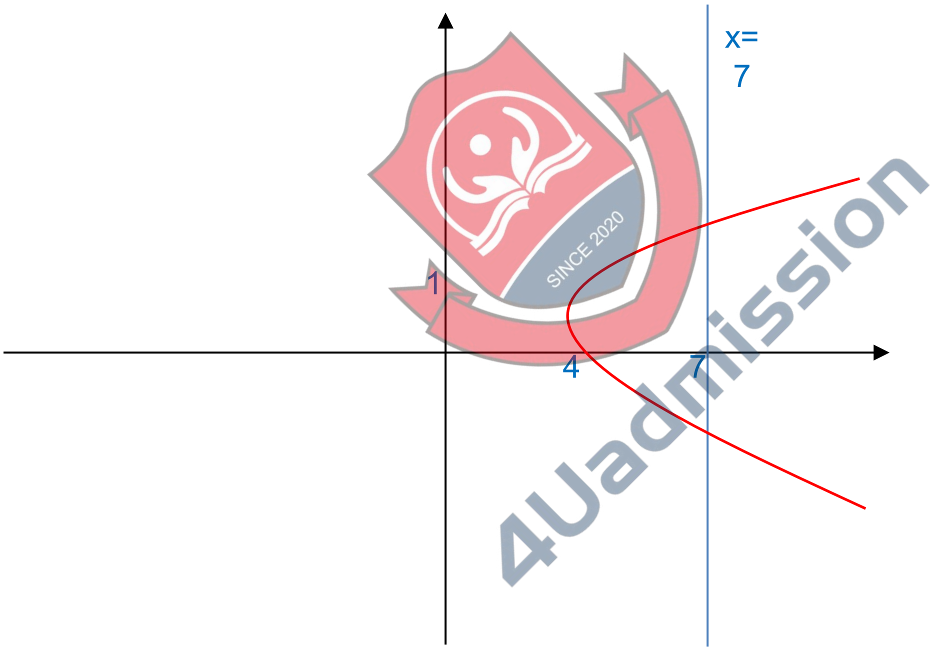
C 10

D 11

E 14

F 20

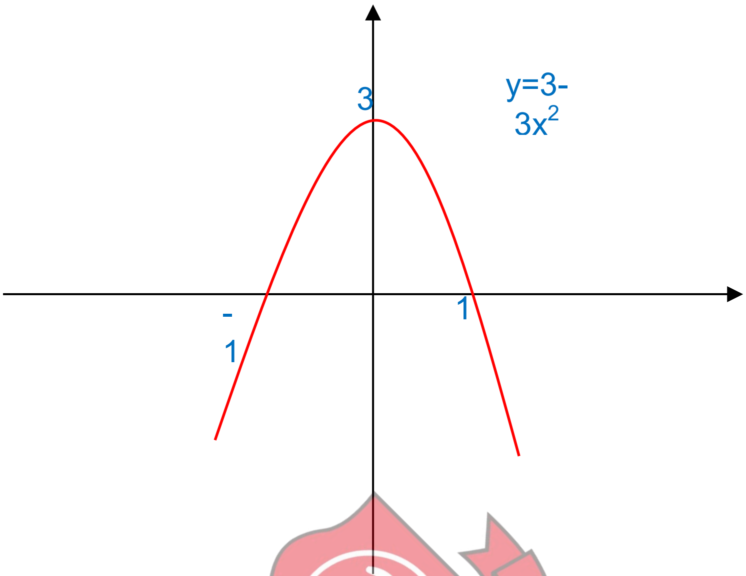
ENGAA S1 2016 - Question 51 - Worked Solution



$$3(y-1)^2+4=7$$
$$3(y-1)^2=3$$
$$(y-1)^2=1$$
$$y=1\pm1$$
$$y=0\ ,\qquad y=2$$

Imagine the line x = 7 as the x axis.

The area enclosed by the parabola $x=3(y-1)^2+4$ and x = 7 is the same as the area enclosed by the x axis and the parabola $y=3-3x^2$



$$A = \int_{-1}^{1} 3 - 3x^2 dx$$

$$A = [3x - x^3]_{-1}^{1}$$

$$A = (3 - 1) - \left(-3 - (-1)\right)$$

$$A = 2 - (-2)$$

$$A = 4$$

Answer is A