Worked Solutions for ENGAA Papers by Topic

Section 1

Topic: Coordinate geometry

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ENGAA S1 2020 - Question 26

26 A line with non-zero gradient m is reflected in the line $y = x$

What is the gradient of the reflected line?

A m

B $-m$

C $\frac{1}{m}$

D $-\frac{1}{m}$

ENGAA S1 2020 - Question 26 - Worked Solution

$$\text{Any inverse of a function is reflected in line } y = x$$

$$f(x) = m \times x$$

$$f^{-1}(x) = \frac{1}{m}x$$

Answer is C

ENGAA S1 2020 - Question 30

30 The line L with equation $y = mx + c$, where $m > 0$ and $c \geq 0$, passes through the point $(2, 4)$.

A line is drawn through the point $(2, 4)$ perpendicular to L.

The triangle enclosed between the two lines and the y-axis has area 5 square units.

What is the **larger** of the two possible values of m?

A –0.5

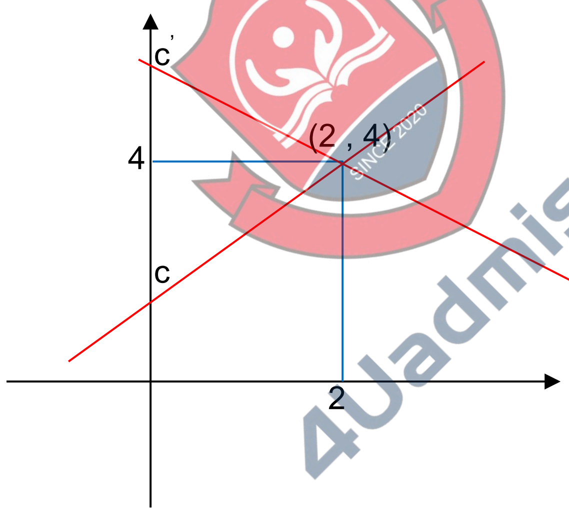
B 0.5

C 1.25

D 2

E 5

ENGAA S1 2020 - Question 30 - Worked Solution



$$\frac{1}{2}(C' - C) \times 2 = 5$$

$$C' - C = 5$$

$$y = mx + C$$

$$y = -\frac{1}{m}x + C'$$

$$2m + \frac{2}{m} = 5$$

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$$m = 2$$

$$m = \frac{1}{2}$$

Answer is D

ENGAA S1 2019 - Question 21

21 Find the area of the shape bounded by the four lines:

$$2y + x = 4$$

$$x = -6$$

$$x = 0$$

$$y = 0$$

A 4

B 12

C 21

D 25

E 27

F 30

ENGAA S1 2019 - Question 21 - Worked Solution

$$y = -\frac{x}{2} + 2$$
$$A_2 = 6 \times 2 = 12$$
$$P_2 = (-6\,,2)$$
$$P_3 = (0\,,2)$$
$$P_1 = (-6\,,5)\ via\ equqtion{:}\ \ y = -\frac{x}{2} + 2$$
$$A_1 = \frac{1}{2} \times 6 \times 3 = 9$$
$$A_1 + A_2 = 9 + 12 = 21$$

Answer is C

ENGAA Specimen S1 - Question 19

19 The square PQRS is positioned so that its vertices are at the points with coordinates: (1, 1), (−1, 1), (−1, −1) and (1, −1).

The square is rotated clockwise through 90° about the origin and then reflected in the line $y = x$.

Which transformation will return the square to its original orientation?

A A reflection in the x-axis.

B A reflection in the y-axis.

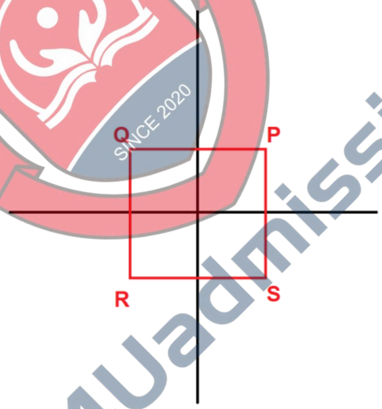
C A reflection in the line $y = -x$.

D A rotation of 90° clockwise about the origin.

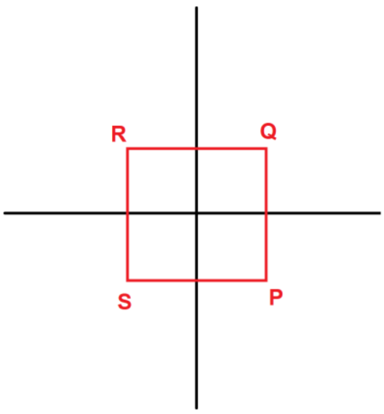
E A rotation of 90° anticlockwise about the origin.

ENGAA Specimen S1 - Question 19 - Worked Solution

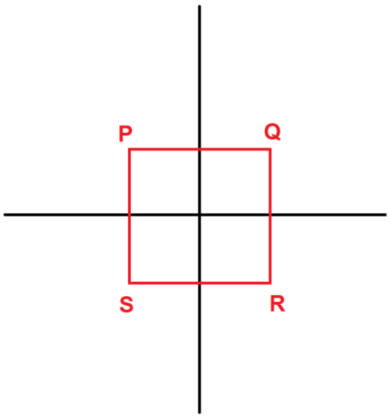
Originally, the square looks like this:



After the rotation of 90 degrees clockwise:



After a reflection in the line $y = x$:



A reflection of this in the y-axis will return this square to the original position.

Answer is B

ENGAA S1 2018 - Question 5

5 The line joining the points with coordinates $(p, p-1)$ and $(1-p, 2p)$ is parallel to the line with equation $2x + 3y + 1 = 0$

What is the value of p?

A -1

B $-\frac{1}{7}$

C $\frac{1}{9}$

D $\frac{1}{8}$

E 1

F $\frac{5}{4}$

G 2

H 5

ENGAA S1 2018 - Question 5 - Worked Solution

Gradient ①

$$m = \frac{\Delta y}{\Delta x} = \frac{2p - (p-1)}{(1-p) - p}$$

$$= \frac{p+1}{-2p+1}$$

Gradient ②

$$2x + 3y + 1 = 0$$

$$y = -\frac{2}{3}x - \frac{1}{3}$$

$$m = -\frac{2}{3} = \frac{p+1}{-2p+1}$$

$$3p + 3 = -2 + 4p$$

$$p = 5$$

Answer is H.

ENGAA S1 2018 - Question 11

11 The straight lines

$$5x + 2y = 20$$

$$y = 3x - 23$$

$$x = 0$$

enclose a region with area K square units.

What is the value of K?

A 39

B 78

C 99

D 129

E 198

F 258

ENGAA S1 2018 - Question 11 - Worked Solution

$$① \; 5x + 2y = 20$$

$$② \; y = 3x - 23$$

$$③ \; x = 0$$

$$①: y = -\frac{5}{2}x + 10$$

Finding intersection : ② → ①

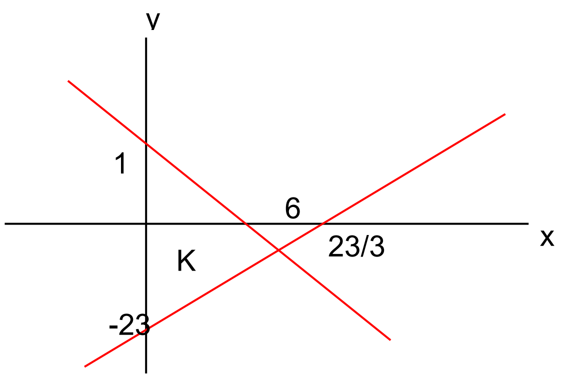
$$5x + 2(3x - 23) = 20$$

$$5x + 6x - 46 = 20$$

$$11x = 66$$

$$x = 6$$

Sketch lines:



$$x = \frac{1}{2} \times 6 \times 33 = 99$$

Answer is C.

ENGAA S1 2018 - Question 43

43 Circle C has equation $(x+3)^2 + (y-2)^2 = 5$

The length of the tangent from the circle C to the point P is $5\sqrt{3}$

What is the shortest distance from P to C?

A $5\sqrt{3}$

B $5\sqrt{3} + \sqrt{5}$

C $3\sqrt{5}$

D 5

E 10

ENGAA S1 2018 - Question 43 - Worked Solution

$$Centre : (-3,2)$$

$$Radius : \sqrt{5}$$

$$Radius \perp tangent$$

Pythagoras:

$$(S+r)^2 = \left(5\sqrt{3}\right)^2 + r^2$$

$$\left(S+\sqrt{5}\right) = \sqrt{75+5}$$

$$S = \sqrt{80} - \sqrt{5}$$

$$= 4\sqrt{5} - \sqrt{5}$$

$$= 3\sqrt{5}$$

Answer is C.

ENGAA S1 2018 - Question 45

45 The points $A\,(-3,2)$, $B\,(1,3)$ and $C\,(-1,u)$ are such that the distances AC and AB are related by:

$$AC = 2AB$$

What are the possible values of u?

A 2 and −6

B −2 and 6

C 6 and −10

D −6 and 10

E $2+2\sqrt{13}$ and $2-2\sqrt{13}$

F $-3+2\sqrt{13}$ and $-3-2\sqrt{13}$

ENGAA S1 2018 - Question 45 - Worked Solution

$$AB = \sqrt{\left(1-(-3)\right)^2 + (3-2)^2}$$

$$= \sqrt{17}$$

$$AC = \sqrt{(-1+3)^2 + (u-2)^2} = \sqrt{4+(u-2)^2}$$

$$= 2\sqrt{17}$$

$$4 + (u-2)^2 = 4 \times 17$$

$$(u-2)^2 = 64$$

$$u - 2 = \pm 8$$

$$u = 10 \ or \ -6$$

Answer is D.

ENGAA S1 2017 - Question 53

53 The equations of two straight lines are $y = 3 + (2p^2 - p)x$ and $y = 7 + (p - 2)x$, where p is a real constant.

For certain values of p, the two lines are perpendicular.

Which of the following numbers is closest to the greatest such value of p?

A 2.00

B 1.75

C 1.50

D 1.00

E −0.25

F −0.50

ENGAA S1 2017 - Question 53 - Worked Solution

$$perpendicular \Rightarrow (2p^2 - p)(p - 2) = -1$$

Product of gradients is -1

$$\Rightarrow 2p^3 - 5p^2 + 2p + 1 = 0 \quad - - - - - ①$$

Try p =1

$$LHS = 2 - 5 + 2 + 1 = 0 \Rightarrow p = 1 \; is \; a \; root, \quad (p - 1) is \; a \; factor$$

Use this to factorize ①

$$2p^3 - 5p^2 + 2p + 1 = (p - 1)(2p^2 - 3p - 1) = 0$$

Now need to solve:

$$(p - 1)(2p^2 - 3p - 1) = 0$$
$$p = 1 \quad , \quad (2p^2 - 3p - 1) = 0$$

Solving quadratic:

$$p = \frac{3 \pm \sqrt{9 - 8}}{4}$$
$$= \frac{3}{4} \pm \frac{\sqrt{17}}{4}$$

$$\sqrt{16} = 4 \Rightarrow \frac{\sqrt{17}}{4} > 1, so \; the \; greatest \; value \; at \; p \; is$$

$$p = \frac{3}{4} + \frac{\sqrt{17}}{4} \qquad \langle \sqrt{17} \approx 4 \rangle$$
$$\approx \frac{3}{4} + 1$$
$$= 1.75$$

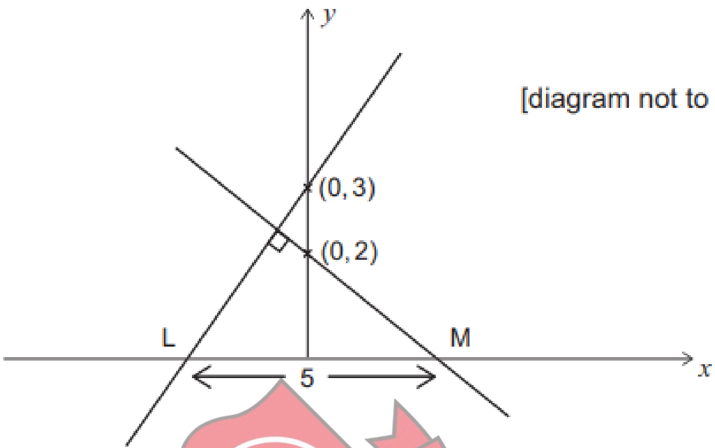
Answer is B.

ENGAA S1 2016 - Question 41

41 The straight line with equation $y = mx + 3$, where $m > 0$, $m \neq 1$, is perpendicular to the line with equation $y = px + 2$

The lines cut the x-axis at the points L and M respectively. The length of LM is 5 units.

What is the value of $m + p$ given that $m > 1$?



[diagram not to scale]

A $-\frac{8}{3}$

B $-\frac{13}{6}$

C $-\frac{5}{6}$

D $\frac{5}{6}$

E $\frac{13}{6}$

F $\frac{8}{3}$

ENGAA S1 2016 - Question 41 - Worked Solution

Since the lines are perpendicular , the product of their gradients is -1.

$$mp = -1 \quad ----①$$

To find the x intercepts, set y to 0.

$$0 = mx + 3 \; => \; x = -\frac{3}{m} => L \gg \left(-\frac{3}{m}, 0\right)$$

$$0 = px + 2 \; => x = -\frac{2}{p} \; => \; M => \left(-\frac{2}{p}, 0\right)$$

$$LM = 5$$

$$-\frac{2}{p}+\frac{3}{m}=5$$

$$\frac{3p-2m}{mp}=5$$

$$3p-2m=-5$$

$$3p+\frac{2}{p}=-5$$

$$3p^2+5p=2=0$$

$$(3p+2)(p+1)=0$$

$$p=-\frac{2}{3}\ , \qquad p=-1$$

$$=> \quad m=\frac{3}{2}\quad , m=1\ \ but\ m\neq 1$$

$$\therefore p=-\frac{2}{3}\quad , \qquad m=\frac{3}{2}$$

$$m+p=\frac{5}{6}$$

Answer is D

ENGAA S1 2016 - Question 49

49 A cursor starts at the point (0, 10) and moves parallel to the x-axis in the negative direction.

What is the minimum distance parallel to the y-axis between the cursor and the graph of $y = 4x^3 - 12x^2 - 36x - 15$?

A 0

B 5

C 25

D 69

E 133

ENGAA S1 2016 - Question 49 - Worked Solution

The cursor is at the point , (x , 10) when $x \leq 0$

The distance parallel to the y axis from the curve to the cursor

$$s = 10 - (4x^3 - 12x^2 - 36x - 15)$$

$$s = -4x^3 + 12x^2 = 36x + 25$$

$$\frac{ds}{dx} = -12x^2 + 24x + 36$$

$$to\ find\ s_{min}\ , set\ \frac{ds}{dx}\ to\ 0$$

$$-12x^2 + 24x + 36 = 0$$

$$-12(x^2 - 2x - 3) = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3\ ,\ \ x = -1$$

$$x = 3\ can't\ be\ the\ solution\ as\ the\ cursor\ moves\ in\ negative\ x$$

Verify x = -1 is in fact a minimum by evaluating the second derivative

$$\frac{d^2s}{d^2x} = -24x + 24 = 48 > 0$$

$$s = -4(-1)^3 + 12(-1)^2 + 36(-1) + 25$$

$$s = 4 + 12 - 36 + 25 = 5$$

Answer is B