Worked Solutions for ENGAA Papers by Topic

Section 1

Topic: Geometry

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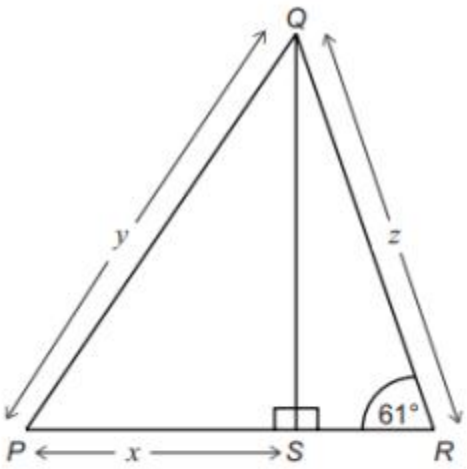
ENGAA S1 2020 - Question 16

16



[diagram not to scale]

In the diagram, QS is perpendicular to PR.

$PS = x$ cm
$PQ = y$ cm
$QR = z$ cm

angle $QRS = 61°$

PSR is a straight line.

Which one of the following is an expression for the length z, in cm?

A $\sqrt{y^2 + x^2}\sin 61°$

B $\sqrt{y^2 - x^2}\sin 61°$

C $\sqrt{y^2 + x^2}\cos 61°$

D $\sqrt{y^2 - x^2}\cos 61°$

E $\dfrac{\sqrt{y^2 + x^2}}{\sin 61°}$

F $\dfrac{\sqrt{y^2 - x^2}}{\sin 61°}$

G $\dfrac{\sqrt{y^2 + x^2}}{\cos 61°}$

H $\dfrac{\sqrt{y^2 - x^2}}{\cos 61°}$

ENGAA S1 2020 - Question 16 - Worked Solution

$$d = \sqrt{y^2 - x^2}$$

By Pythagoras' theorem

$$\frac{d}{Z} = sin\ sin\ (61°)$$

$$\therefore Z = \frac{d}{sin\ sin\ 61} = \frac{\sqrt{y^2 - x^2}}{sin\ sin\ (61°)}$$

Answer is F

ENGAA S1 2020 - Question 18

18 Two vertices of a square are at (1, 1) and (3, 5).

What is the difference between the perimeters of the largest and smallest possible squares that can be drawn with these points as two of their vertices?

A 0

B $4\sqrt{3}(2-\sqrt{2})$

C $4\sqrt{3}(\sqrt{2}-1)$

D $4\sqrt{5}(2-\sqrt{2})$

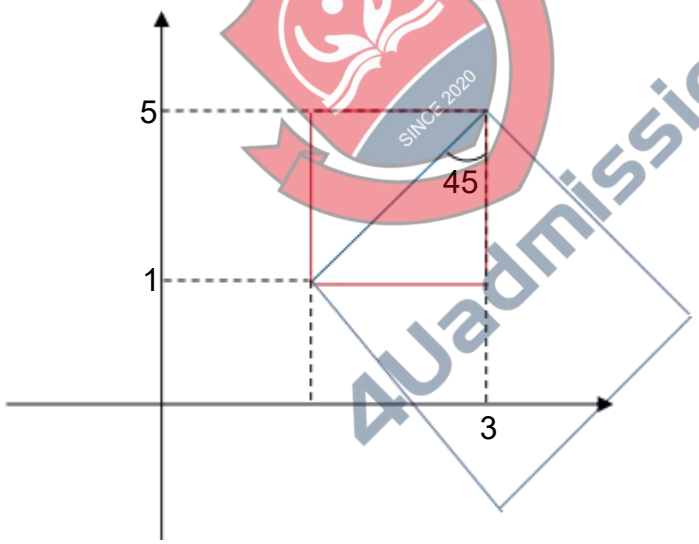
E $4\sqrt{5}(\sqrt{2}-1)$

F $4\sqrt{13}(2-\sqrt{2})$

G $4\sqrt{13}(\sqrt{2}-1)$

H $4\sqrt{3}\sqrt{5}(2-\sqrt{2})$

ENGAA S1 2020 - Question 18 - Worked Solution



$$\alpha = \sqrt{(3-1)^2 + (5-1)^2} = \sqrt{20}$$

$$\beta = \alpha \cos\cos(45^\circ) = \frac{1}{\sqrt{2}}\sqrt{20} = \sqrt{10}$$

$$4\alpha - 4\beta = 4\sqrt{20} - 4\sqrt{10}$$

$$= 4\sqrt{5}(2-\sqrt{2})$$

Answer is D

ENGAA S1 2019 - Question 11

11 The ball for a garden game is a solid sphere of volume 192 cm^3.

For the children's version of the game the ball is a solid sphere made of the same material, but the radius is reduced by 25%.

What is the volume, in cm^3, of the children's ball?

A 48

B 81

C 96

D 108

E 144

ENGAA S1 2019 - Question 11 - Worked Solution

$$V_1 = \frac{4}{3}\pi r^3$$

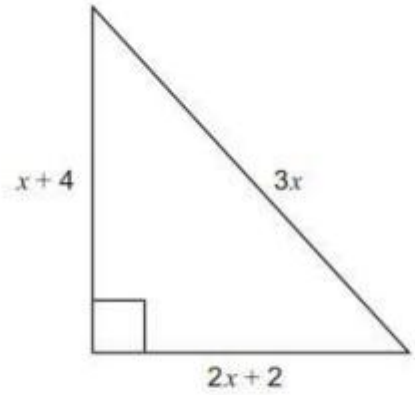
$$V_2 = \frac{4}{3}\pi\left(\frac{4}{3}r\right)^3 = V_1 \cdot \frac{3^3}{4^3}$$

$$192 \cdot \frac{3^3}{4^3} = 81 cm^3$$

Answer is B

ENGAA S1 2019 - Question 13

13 The diagram shows a right-angled triangle, with sides of length $x + 4$, $2x + 2$ and $3x$, all in cm.



[diagram not to scale]

What is the area, in cm^2, of the triangle?

A 10

B 12

C 28

D 36

E 40

F 54

G 70

ENGAA S1 2019 - Question 13 - Worked Solution

Using Pythagoras' theorem

$$(x + 4)2 + (2x + 2)^2 = (3x)^2$$

$$(x - 5)(x + 1) = 0$$

Lengths must be positive

$$x = 5, x \neq 1$$

$$A = \frac{1}{2} \text{ base} \times \text{height}$$

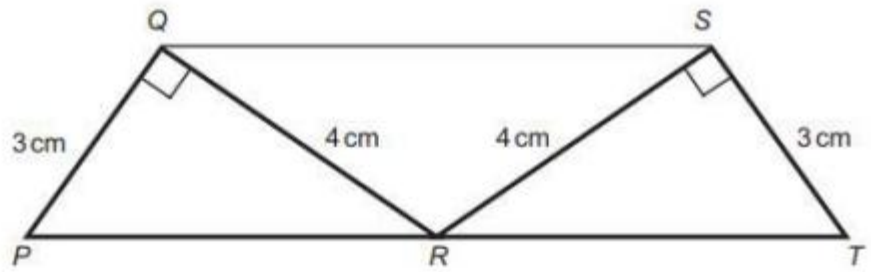
$$A = \frac{1}{2}(5 + 4)(2 \times 5 + 2)$$

$$A = 54$$

Answer is F

ENGAA S1 2019 - Question 17

17



[diagram not to scale]

The diagram shows two congruent right-angled triangles *PQR* and *TSR* with right angles at *Q* and *S*, respectively.

$PQ = TS = 3\,\text{cm}$

$QR - SR - 4\,\text{cm}$

PRT is a straight line.

What is the length, in cm, of *QS*?

A 4

B $3\sqrt{2}$

C 5.2

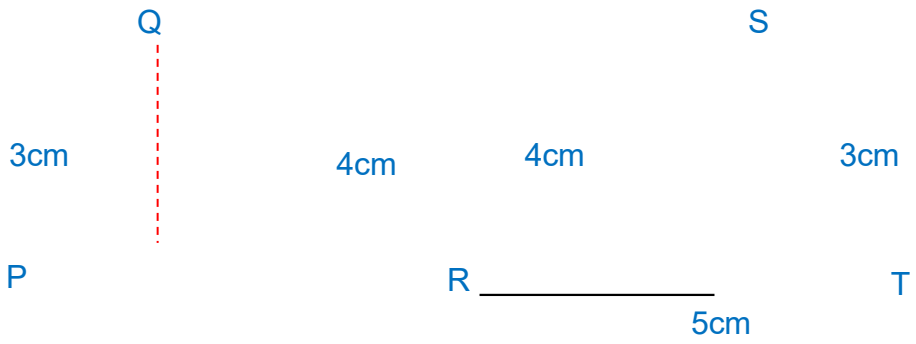
D $4\sqrt{2}$

E 6.4

F 8.2

G 10

ENGAA S1 2019 - Question 17 - Worked Solution



$$Qs = 2(5\text{cm} - \beta)$$

By Pythagoras' theorem

$$\beta^2 + \alpha 2 = 32 ----①$$

$$\alpha 2 + (5 - \beta)^2 = 42 ------②$$

Subtracting ① from ②

$$16 - 9 = 25 - 10\beta + \beta^2$$

$$7 = 25 - 10\beta$$

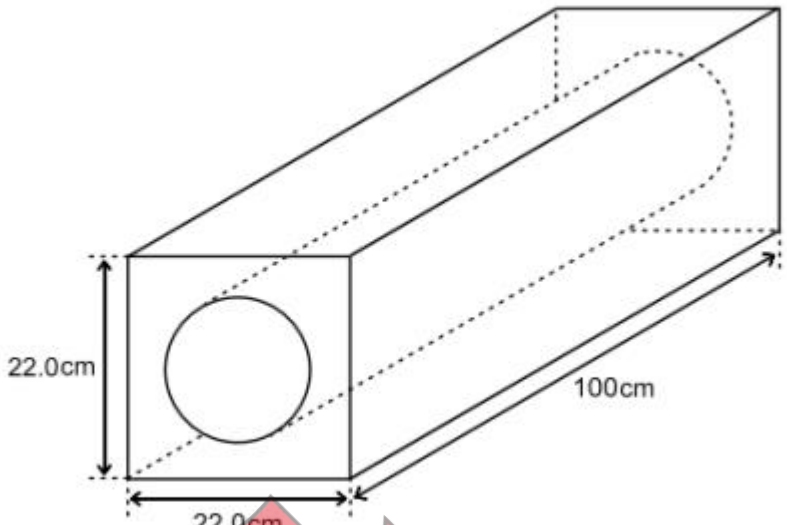
$$\frac{7}{5} = 5 - 2\beta$$

$$QS = 10 - 2\beta = \frac{7}{5} + 5 = \frac{32}{5} = 6.4$$

Answer is E

ENGAA S1 2019 - Question 18

18 A block is designed with a cylindrical channel to accommodate a hot-water pipe. The block is 100 cm long and it has a square cross-section of side 22.0 cm with a cylindrical hole in the middle, as shown in the diagram:



[diagram not to scale]

The diameter of the cylindrical hole is 14.0 cm and the density of the material from which the block is made is $0.100\,g\,cm^{-3}$.

What is the mass of the block?

(take π to be $\frac{22}{7}$)

A 1.32 kg

B 3.30 kg

C 13.2 kg

D 33.0 kg

E 132 kg

F 330 kg

G 1320 kg

H 3300 kg

ENGAA S1 2019 - Question 18 - Worked Solution

$10^{-1} gcm^3 = 10^{-4}$

$V_{block} = V_{cuboid} - V_{cylinder}$

$$V_{block} = (22 \times 22 \times 100) - \left(\frac{22}{7} \times 7 \times 7 \times 100\right)$$

$V_{block} = 22 \times (22 - 7) \times 100$

$V_{block} = 22 \times 1500cm$

$$\rho = \frac{m}{V}$$

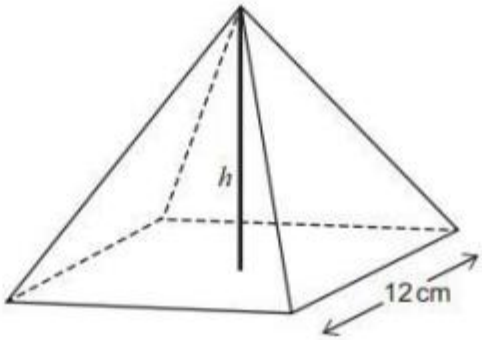
$$10^{-4} = \frac{m}{V}$$

$m = 3.3kg$

Answer is B

ENGAA S1 2019 - Question 19

19



[diagram not to scale]

A solid pyramid has a square base of side length 12 cm and a vertical height of h cm.

The volume of the pyramid, in cm^3, is equal to the total surface area of the pyramid, in cm^2.

What is the value of h?

(volume of pyramid = $\frac{1}{3}$ × area of base × vertical height)

A $\frac{72}{35}$

B $2\sqrt{3}$

C 6

D $\frac{144}{23}$

E 8

F $2\sqrt{21}$

ENGAA S1 2019 - Question 19 - Worked Solution

$$A_b = 12_2 = 144$$

$$SA = A_b + 4\,.A_T = 144 + 4 \times \left(\frac{1}{2} \times 12 \times \sqrt{36 + h_2}\right)$$

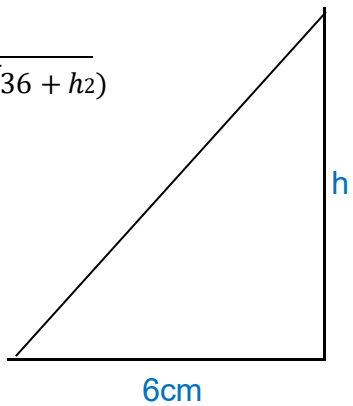
$$Vol = \frac{1}{3} \times A_b \times h = \frac{144}{3}h$$

Let Vol = SA

$$144 + 24\sqrt{36 + h^2} = \frac{144}{3}h$$

$$3h(h - 8) = 0$$

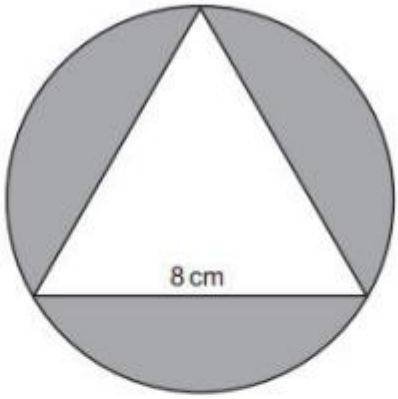
$$h \neq 0 \qquad \therefore h = 8\text{cm}$$



Answer is E

ENGAA S1 2019 - Question 29

29 An equilateral triangle of side 8 cm is drawn so that its vertices lie on the circumference of a circle, as shown in the diagram.



What is the total of the three areas shaded in the diagram, in cm^2?

A $8(2\pi - 3)$

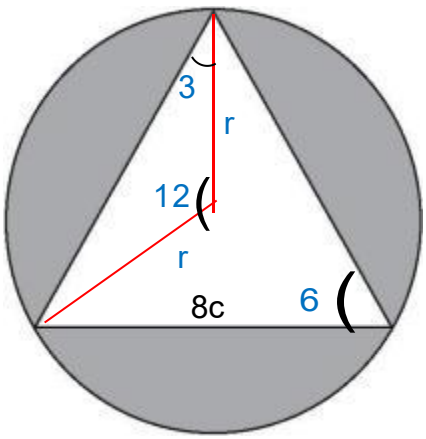
B $24(\pi - \sqrt{3})$

C $48(4\pi - \sqrt{3})$

D $\frac{16}{3}(4\pi - 6 - 3\sqrt{3})$

E $\frac{16}{3}(4\pi - 3\sqrt{3})$

ENGAA S1 2019 - Question 29 - Worked Solution



By Area law

$$A_{triangle} = \frac{1}{2} AB\ sin\ sin\ C$$

$$= \frac{1}{2} \times 8 \times 8 \times sin\ sin\ 60$$

$$\text{Atri} = 16\sqrt{3}$$

$$By\ Sin\ law: \frac{r}{sin\ sin\ 30} = \frac{8}{sin\ sin\ 120}$$

$$r = \frac{8\sqrt{3}}{3}$$

$$A_{circ} = \pi r^2 = \pi \cdot \frac{64}{9} \cdot 3 = \pi \frac{64}{3}$$

$$A_{tri} = 16\sqrt{3}$$

$$A_{shade} = A_c - A_t = \frac{16}{3}(4\pi - 3\sqrt{3})$$

Answer is E

ENGAA S1 2019 - Question 37

37 The three internal angles in a triangle are α, β and θ, and

$$3\tan\alpha - 2\sin\beta = 2$$

$$5\tan\alpha + 6\sin\beta = 8$$

What is the value of θ in degrees?

A 15

B 45

C 75

D 105

E 135

ENGAA S1 2019 - Question 37 - Worked Solution

a =tan tan @

b =tan tan @

3a —2b = 2

5a + 6b = 8

14a = 14

tan tan @ = 1

α = 45。

$b = \frac{1}{2} = sin\ sin$?

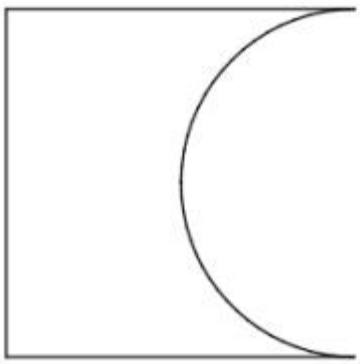
β = 30。

θ = 180—(45。 + 35。) = 105。

Answer is D

ENGAA Specimen S1 - Question 1

1 A square piece of metal has a semicircular piece cut out of it as shown. The area of the remaining metal is 100 cm^2.



[diagram not to scale]

Which one of the following is a correct expression for the length of the side of the square in centimetres?

A $20\sqrt{\frac{2}{8-\pi}}$

B $10\sqrt{\frac{2}{4-\pi}}$

C $20\sqrt{\frac{2}{8+\pi}}$

D $10\sqrt{\frac{1}{8-\pi}}$

E $20\sqrt{\frac{1}{4-\pi}}$

ENGAA Specimen S1 - Question 1 - Worked Solution

Let the side length of the square $= S$

The radius of the semicircle $= \frac{S}{2}$

$$S^2 - \frac{\pi\left(\frac{S}{2}\right)^2}{2} = 100$$

$$S^2\left(1 - \frac{\pi}{8}\right) = 100$$

$$S^2 = \frac{100}{1-\frac{\pi}{8}}$$

$$= \frac{800}{8-\pi}$$

$$S = \sqrt{\frac{800}{8-\pi}} = 20\sqrt{\frac{2}{8-\pi}}$$

Answer is A

ENGAA Specimen S1 - Question 3

3 If you look at a clock and the time is 9.45, what is the angle between the hour and the minute hands?

A 0°

B 7.5°

C 15°

D 22.5°

E 30°

ENGAA Specimen S1 - Question 3 - Worked Solution

The minute hand is pointing at 9. The hour hand is somewhere between 9 and 10.

The angle between 9 and 10 is 30°. It takes 60 minutes for the hour hand to travel from 9 to 10.

Therefore in 45 minutes, it travels:

$$\frac{45}{60} \times 30°$$

$$= 22.5°$$

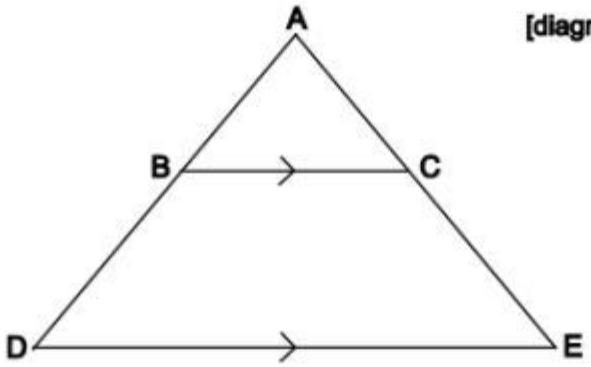
Answer is D

ENGAA Specimen S1 - Question 7

7 A shape is formed by drawing a triangle ABC inside the triangle ADE.

BC is parallel to DE.

AB = 4 cm BC = x cm DE = $x + 3$ cm DB = $x - 4$ cm



[diagram not to scale]

Calculate the length of DE.

A 5 cm

B 7 cm

C 9 cm

D $4+2\sqrt{7}$ cm

E $7+2\sqrt{7}$ cm

ENGAA Specimen S1 - Question 7 - Worked Solution

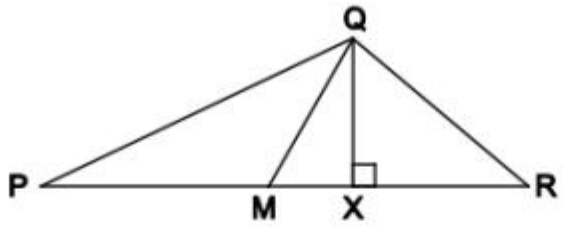
Triangle ADE and Triangle ABC are similar so $\frac{DE}{AD} = \frac{BC}{AB}$

$$AD = AB + DB$$
$$= 4 + (x - 4)$$
$$= x \text{ cm}$$
$$\frac{x - 3}{x} = \frac{x}{4}$$
$$x^2 = 4(x + 3)$$
$$x^2 - 4x - 12 = 0$$
$$(x + 2)(x - 6) = 0$$
$$x = 6 \text{ cm} \quad (x \text{ cannot be negative})$$
$$DE = x + 3 = 9\text{cm}$$

Answer is C

ENGAA Specimen S1 - Question 13

13 In the triangle PQR shown below:



[diagram not to scale]

X lies on PR

$\angle QXR$ is $90°$

$\frac{QX}{PX} = \frac{1}{6}$

$\frac{QX}{XR} = \frac{2}{3}$

M is the midpoint of PR.

What is $\frac{QX}{MX}$?

A $\frac{1}{9}$

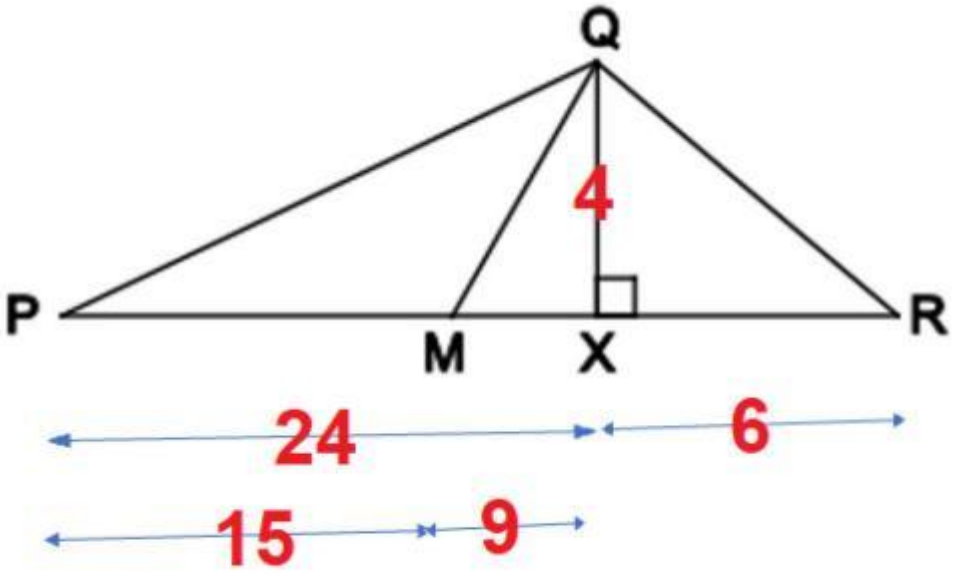
B $\frac{5}{12}$

C $\frac{4}{9}$

D $\frac{1}{2}$

E $\frac{5}{6}$

ENGAA Specimen S1 - Question 13 - Worked Solution



Let QX be 4 (You can set any number for this. 4 is just used here to ensure all the other lengths are integers.)

From the information given in the question, we can see that PX is 24 and XR is 6. Therefore, PR is 30. Since M is the midpoint of PR, PM = 30/2 = 15. Therefore, as seen in the diagram. $MX = 24 - 15 = 9$. Therefore the $\frac{QX}{MX} = \frac{4}{9}$

Answer is C

ENGAA Specimen S1 - Question 17

17 The total surface area of a cylinder, measured in square centimetres, is numerically the same as its volume, measured in cubic centimetres.

The radius of the cylinder is r cm, the height is h cm.

Express h in terms of r.

A $h = \dfrac{2r}{r-2}$

B $h = \dfrac{2r}{r+2}$

C $h = r+2$

D $h = r-2$

E $h = 2r(r-2)$

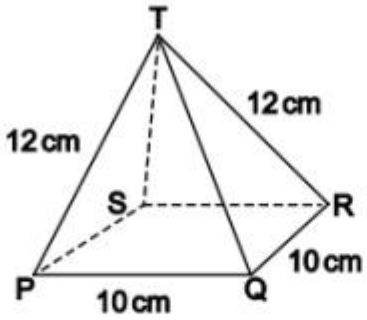
ENGAA Specimen S1 - Question 17 - Worked Solution

$$V = \pi r^2 h$$
$$SA = 2\pi r^2 + 2\pi rh = 2\pi r(r+h)$$
$$2\pi r(r+h) = \pi r^2 h$$
$$2(r+h) = hr$$
$$2r + 2h = rh$$
$$(2-r)h + 2r = 0$$
$$h = -\frac{2r}{2-r} = \frac{2r}{r-2}$$

Answer is A

ENGAA Specimen S1 - Question 31

31 A box is a hollow pyramid. The base of the box is a square with sides 10 cm and all the slant edges of the box are 12 cm long.



[diagram not to scale]

What is the angle made by the slant edge TP with the base PQRS?

A $\sin^{-1}\frac{2\sqrt{5}}{12}$

B $\sin^{-1}\frac{5}{12}$

C $\sin^{-1}\frac{5\sqrt{2}}{12}$

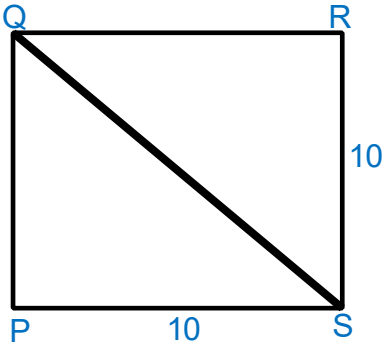
D $\cos^{-1}\frac{2\sqrt{5}}{12}$

E $\cos^{-1}\frac{5}{12}$

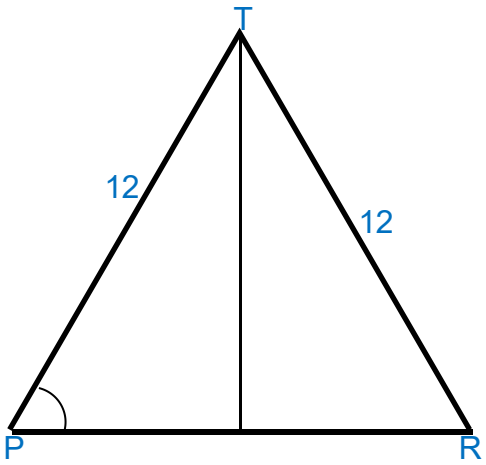
F $\cos^{-1}\frac{5\sqrt{2}}{12}$

ENGAA Specimen S1 - Question 31 - Worked Solution

Consider the base



Consider the pyramid stood up and one of the corners pointing



$$cos\ cos\ \theta = \frac{5\sqrt{2}}{12}$$

$$\theta = \left(\frac{5\sqrt{2}}{12}\right)$$

Answer is F

ENGAA Specimen S1 - Question 35

35 A triangle is to be drawn with sides that are integer lengths in centimetres, and a total perimeter of 12 cm.

How many different (non-congruent) triangles can be drawn?

A 1

B 2

C 3

D 10

E 12

ENGAA Specimen S1 - Question 35 - Worked Solution

In a triangle, the length of the longest side cannot exceed nor equal the sum of the lengths of the other 2 sides.
Therefore the longest possible side length is 5cm
This gives 2 - 5 - 5 and 3 - 4 - 5.
There is also 4 - 4 -4.
There are no other combinations
3 possible triangles

Answer is C

ENGAA Specimen S1 - Question 37

37 The triangle PQR has a right angle at R.

The length of PQ is 4 cm, correct to the nearest centimetre.

The length of PR is 2 cm, correct to the nearest centimetre.

Find the minimum possible length, in centimetres, of QR.

A $\sqrt{6} - \frac{1}{2}$

B $2\sqrt{3} - \frac{1}{2}$

C $2\sqrt{5} - \frac{1}{2}$

D $2\sqrt{5}$

E $2\sqrt{3}$

F $\sqrt{6}$

ENGAA Specimen S1 - Question 37 - Worked Solution

$$(PR)_2 + (QR)^2 = (PQ)^2$$

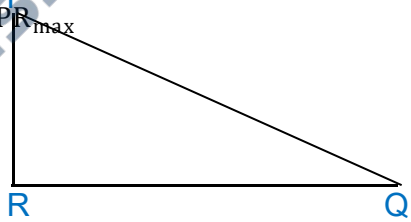
To find QR_{min} : use PQ_{min} and PR_{max}

$$PR_{max} = 2.5\text{cm}$$

$$PQ_{min} = 4.5\text{cm}$$

$$2.5_2 + (RQ)^2 = 4.5_2$$

$$RQ = \sqrt{6}\text{cm}$$



Answer is F

ENGAA S1 2018 - Question 3

3 A cuboid has sides of length x, $\sqrt{2}x$ and $2x$, measured in cm.

The volume, in cm^3, of the cuboid is numerically equal to twice the total surface area, in cm^2, of the cuboid.

What is the value of x?

A 10

B $6 + 2\sqrt{2}$

C 5

D $3 + \sqrt{2}$

E $\frac{5}{2}$

F $\frac{3}{2} + \frac{1}{2}\sqrt{2}$

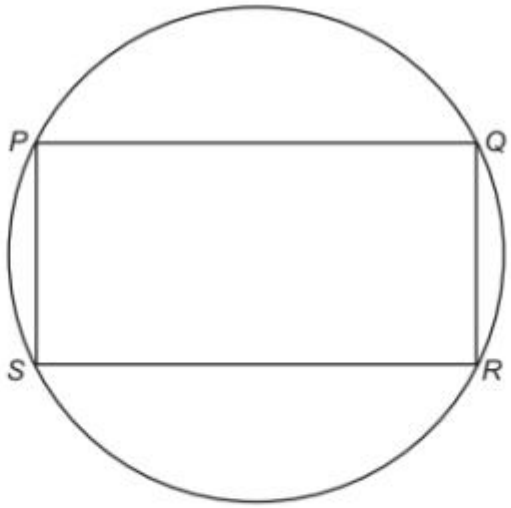
ENGAA S1 2018 - Question 3 - Worked Solution

$$V = 2A$$
$$V = 2\sqrt{2}x^3$$
$$A = 2 \times [\sqrt{2}x^2 + 2\sqrt{2}x^2 + 2x^2]$$
$$= 2x^2[3\sqrt{2} + 2]$$
$$2\sqrt{2}x^3 = 4x^2[3\sqrt{2} + 2]$$
$$\sqrt{2}x = 2[3\sqrt{2} + 2]$$
$$x = 2\left[3 + \frac{2}{\sqrt{2}}\right]$$
$$x = 6 + 2\sqrt{2}$$

Answer is B.

ENGAA S1 2018 - Question 7

7 A rectangle *PQRS* is drawn inside a circle, with its vertices on the circumference of the circle.



[diagram not to scale]

The ratio of the length of *PQ* to the length of *QR* is 2 : 1

The area of the rectangle *PQRS* is 96 cm^2.

What is the radius, in cm, of the circle?

A $\sqrt{6}$

B 3

C $3\sqrt{2}$

D $2\sqrt{15}$

E $4\sqrt{6}$

F 12

G $12\sqrt{2}$

H $8\sqrt{15}$

ENGAA S1 2018 - Question 7 - Worked Solution

Let QR = y

Then PQ = 2y

$$A = 2y^2 = 96$$

$$y = 4\sqrt{3}$$

$$PQ = 8\sqrt{3}, QR = 4\sqrt{3}$$

Using Pythagoras:

$$\left(\frac{PQ}{2}\right)^2 + \left(\frac{QR}{2}\right)^2 = r^2$$

$$16 \times 3 + 4 \times 3 = r^2$$

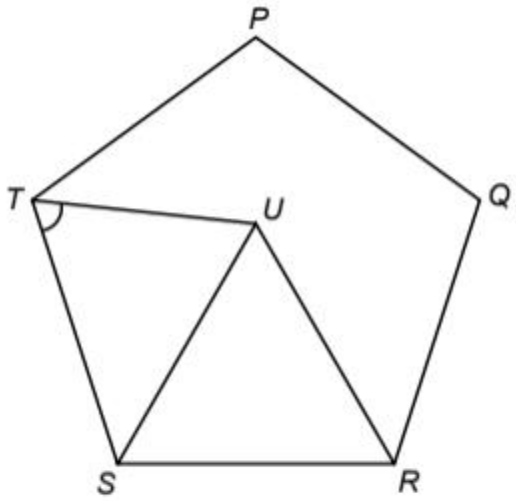
$$r = \sqrt{60}$$

$$r = 2\sqrt{15}$$

Answer is D.

ENGAA S1 2018 - Question 15

15



[diagram not to scale]

PQRST is a regular pentagon.

RSU is an equilateral triangle.

What is the size of angle *STU*?

A 48°

B 54°

C 60°

D 66°

E 84°

ENGAA S1 2018 - Question 15 - Worked Solution

Equilateral triangle:

$$SUR = \frac{180}{3} = 60°$$

Sum of internal angles of pentagon:

$$S = 180(n-2) \quad (n = 5)$$
$$= 540$$

$$TSR = \frac{540}{5} = 108°$$

TS = SR = SU → TUS is isosceles

$$2x + TUS = 180°$$
$$2x + (180-60) = 180$$
$$2x = 132$$
$$x = 66°$$

Answer is D.

ENGAA S1 2018 - Question 19

19 Q is 5 km away from P on a bearing of 065°

R is 5 km away from Q on a bearing of 155°

What is the bearing of P from R?

A 070°

B 110°

C 225°

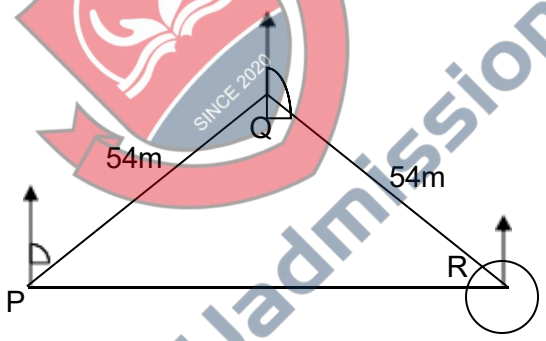
D 270°

E 290°

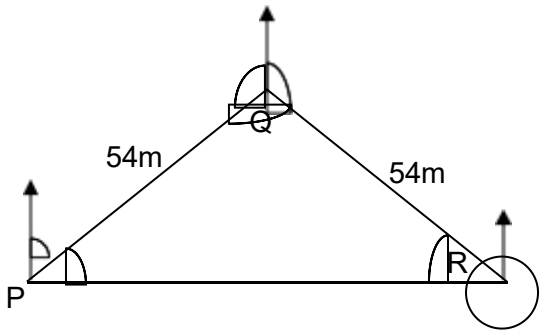
F 315°

G 335°

ENGAA S1 2018 - Question 19 - Worked Solution



- PQR is isosceles
- Using laws for corresponding and alternative angles and above point:

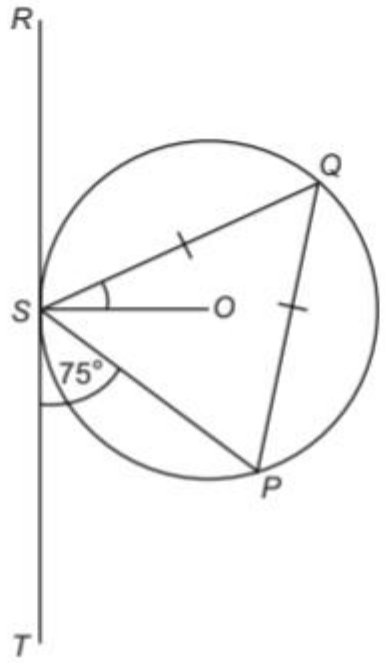


$$\theta = 360 - 45 - 25$$
$$\theta = 290^{\circ}$$

Answer is E.

ENGAA S1 2018 - Question 21

21



[diagram not to scale]

The line segment RT is a tangent at the point S to a circle with centre O

Q and P are points on the circumference of the circle such that $QS = QP$

Angle $PST = 75°$

What is the size of angle QSO?

A 15°

B 30°

C 37.5°

D 45°

E 52.5°

F 60°

G 67.5°

H 75°

ENGAA S1 2018 - Question 21 - Worked Solution

OS $\perp$ ST as a tangent is always perpendicular to the radius.

$OSP = 90 - 75 = 15$

By alternate angle theorem:

$PST = SQP$

ΔSPQ is isosceles:

$180 = 75 + 2(\theta + 15)$

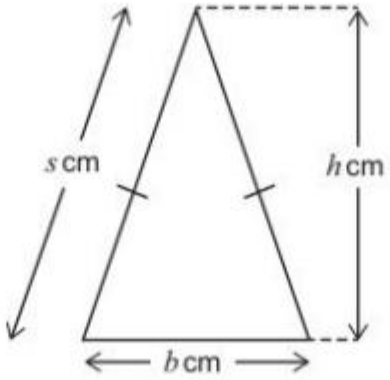
$105 = 2(\theta + 15)$

$\theta = 37.5$

Answer is C.

ENGAA S1 2018 - Question 23

23



[diagram not to scale]

The vertical height h cm of an isosceles triangle is 3 cm longer than the base length of b cm.

The sloping side is of length s cm.

The area of the triangle is 14 cm^2.

There is one value of s which satisfies these conditions.

Within which range does this value of s lie?

A $5 < s < 6$

B $6 < s < 7$

C $7 < s < 8$

D $8 < s < 9$

E $9 < s < 10$

F $10 < s < 11$

ENGAA S1 2018 - Question 23 - Worked Solution

$$h = b + 3$$

$$A = 14$$

$$\frac{1}{2}bh = 14$$

$$(b + 3)b = 28$$

$$b^2 + 3b - 28 = 0$$

$$b = 4 \quad \text{or} \quad -7$$

$$b = 4 \quad \text{as} \quad b > 0$$

$$h = 7$$

using Pythagoras:

$$S^2 = \left(\frac{b}{2}\right)^2 + 4^2$$

$$S = \sqrt{4 + 49}$$

$$S = \sqrt{53}$$

$$\sqrt{49} = 7, \sqrt{64} = 8 \rightarrow 7 < S < 8$$

Answer is C.

ENGAA S1 2018 - Question 35

35 A sector S of a circle has area $10\pi\,\text{cm}^2$.

The angle of sector S is increased by $\frac{\pi}{20}$ radians to form sector T.

The total area of sector T is $\frac{25}{2}\pi\,\text{cm}^2$.

What is the total arc length, in cm, of sector T?

A $\frac{9\sqrt{5}}{10}\pi$

B $\frac{5\sqrt{2}}{4}\pi$

C 2π

D $\frac{5}{2}\pi$

ENGAA S1 2018 - Question 35 - Worked Solution

r = radius of circle

$$A_S = \frac{1}{2}r^2\theta = 10\pi$$

$$A_T = \frac{1}{2}r^2\left(\theta + \frac{\pi}{20}\right) = \frac{25}{2}\pi \quad ①$$

$$A_S + r^2\frac{\pi}{40} = \frac{25}{2}\pi$$

$$10\pi + r^2\frac{\pi}{40} = \frac{25}{2}\pi$$

$$r^2\frac{\pi}{40} = \frac{5}{2}\pi$$

$$r^2 = 100$$

$$r = 10$$

$$S_T = r\left(\theta + \frac{\pi}{20}\right)$$

Sub into ①

$$\frac{1}{2}rS_t = \frac{25}{2}\pi$$

$$\frac{10}{2}S_T = \frac{25}{2}\pi$$

$$S_T = \frac{5}{2}\pi$$

Answer is D.

ENGAA S1 2018 - Question 53

53 The dimensions of a solid cuboid, in cm, are x, $2x$ and y

The volume of the cuboid is 576 cm^3.

At this volume, the surface area of the cuboid has its maximum value.

What is the area, in cm^2, of the face that has the largest area?

A $2(288)^{\frac{2}{3}}$

B 72

C 96

D 432

E $4(144)^{\frac{2}{3}}$

ENGAA S1 2018 - Question 53 - Worked Solution

$$V = 2x^2y = 576$$

$$xy = \frac{288}{x} \quad ①$$

$$A = 2(2x^2 + xy + 2xy)$$

$$= 4x^2 + 6xy \quad ②$$

① → ②

$$A = 4x^2 + \frac{1728}{x}$$

At maximum surface area

$$\frac{dA}{dx} = 0$$

$$8x - \frac{1728}{x^2} = 0$$

$$x^3 = 216$$

$$x = 6$$

$$② \rightarrow 6y = \frac{288}{6} = 48$$

$$y = 8$$

Areas of faces:

$$2x^2 = 72$$

$$2xy = 96$$

$$xy = 48$$

So area of largest surface $= 96cm^2$

Answer is C.

ENGAA S1 2017 - Question 16

16 The radius of an iron-56 atom is 3.0×10^4 times greater than the radius of an iron-56 nucleus.

What is the value of $\frac{\text{density of an iron atom}}{\text{density of an iron nucleus}}$?

A $(3.0 \times 10^4)^{-3}$

B $(3.0 \times 10^4)^{-2}$

C $(3.0 \times 10^4)^{-1}$

D $(3.0 \times 10^4)^{1}$

E $(3.0 \times 10^4)^{2}$

F $(3.0 \times 10^4)^{3}$

ENGAA 2017 - Question 16 - Worked Solution

$$R_a = 3.0 \times 10_4 R_n$$

$$\frac{\rho_a}{\rho_n} = \frac{M_a}{M_n} \times \frac{V_n}{V_a}$$

$$= \frac{V_n}{V_A}$$

$$= \frac{R_{n^3}}{R_{A^3}}$$

$$= (3.0 \times 10_4)^{-3}$$

Answer is A

ENGAA S1 2017 - Question 17

17 An exterior angle of a regular polygon with n sides is 4° larger than an exterior angle of a regular polygon with $(n+3)$ sides.

What is the value of n?

A 10

B 12

C 15

D 18

E 21

F 24

G 27

ENGAA 2017 - Question 17 - Worked Solution

External angles all add up to 360^0

$$\text{one external angle} = \frac{360}{n}$$

$$\frac{360}{n} = 4 + \frac{360}{(n+3)}$$

$$360(n+3) = 4n(n+3) + 360n$$

$$1080 = 4n^2 + 12n$$

$$n^2 + 3 - 270 = 0$$

$$(n+18)(n-15) = 0$$

$$n = -18 \quad \text{or} \quad 15 \qquad (n > 0)$$

$$n = 15$$

Answer is C

ENGAA S1 2017 - Question 19

19 The bearing of a ship *R* from a lighthouse *L* is 220°

A canoe *C* is due North of *R*.

C is the same distance from the ship and the lighthouse.

What is the bearing of *L* from *C*?

A 070°

B 080°

C 090°

D 100°

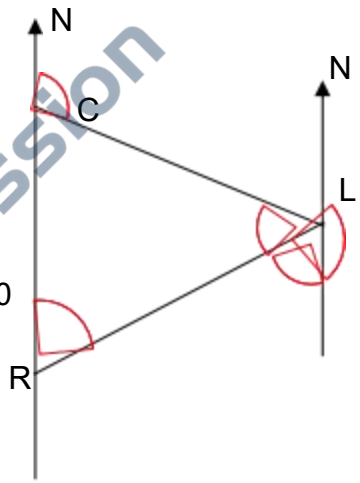
E 140°

ENGAA 2017 - Question 19 - Worked Solution

- Alternative angles rule: $CRL = 40°$
- Isosceles $CLR = 40°$
- In triangle all angles = $180°$

$\rightarrow (180 - \theta) + 40 + 40 = 180$

$\rightarrow \theta = 80°$



Answer is B

ENGAA S1 2017 - Question 21

21 The hands of a 12-hour analogue clock move continuously. When the time on the clock is 4:00, the angle between the minute hand and the hour hand is 120°.

What is the angle between the two hands at 4:40?

A 80°

B 100°

C 110°

D 120°

E 140°

ENGAA 2017 - Question 21 - Worked Solution

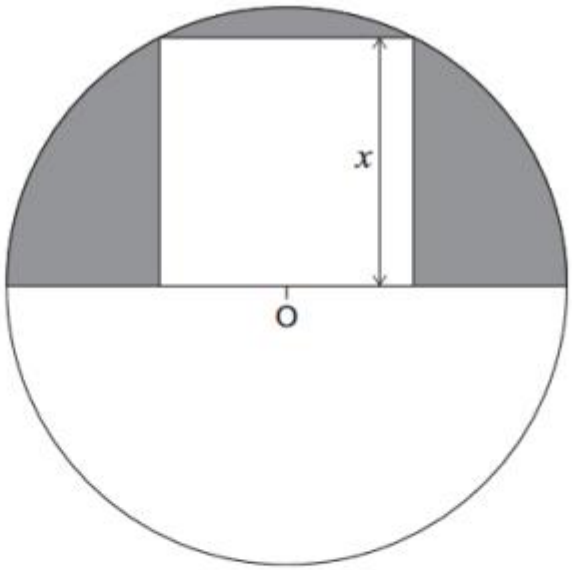
- In 1hr, hour hand moves $\frac{360°}{12} = 30°$
- So in 40 min, moves $\frac{40}{60} \times 30 = 20°$
- Minute hand moves $\frac{40}{60} \times 360 = 240°$

$\rightarrow \theta = 240 - (120 + 20) = 100°$

Answer is B

ENGAA S1 2017 - Question 25

25



The diagram shows a square with side of length x cm. A circle is drawn with centre O which lies at the mid-point of one of the sides of the square. This side forms part of a diameter of the circle. The circle passes through two corners of the square as shown.

What is the area, in cm^2, of the shaded part of the semi-circle?

A $(\pi-1)x^2$

B $\left(\frac{\pi-2}{2}\right)x^2$

C $\left(\frac{3\pi-2}{2}\right)x^2$

D $\left(\frac{3\pi-4}{4}\right)x^2$

E $\left(\frac{5\pi-4}{4}\right)x^2$

F $\left(\frac{5\pi-8}{8}\right)x^2$

ENGAA S1 2017 - Question 25 - Worked Solution

Using Pythagoras

$$r = \sqrt{x^2 + \left(\frac{x}{2}\right)^2}$$

$$r = \sqrt{\frac{5}{4}x^2}$$

$$r = \frac{\sqrt{5}x}{2}$$

$$A = \frac{1}{2} \times circle - square$$

$$A = \frac{1}{2}\pi r^2 - x^2$$

$$A = \frac{1}{2} \times \frac{5}{4}x^2\pi - x^2$$

$$A = \left(\frac{5}{8}\pi - 1\right)x^2$$

$$A = \left(\frac{5\pi - 8}{8}\right)x^2$$

Answer is F

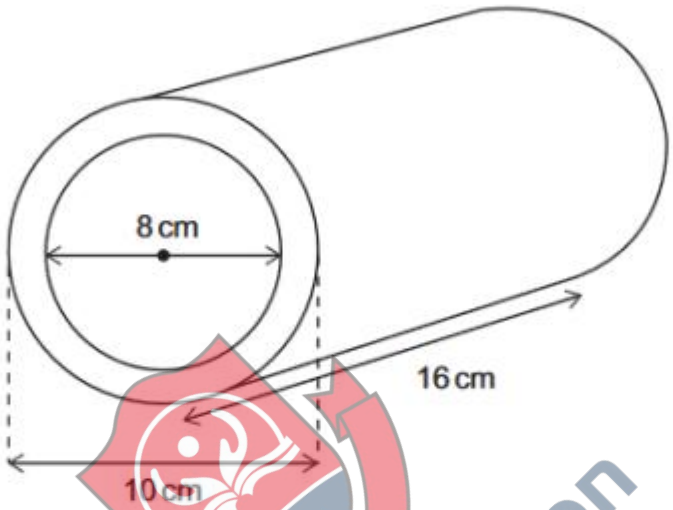
ENGAA S1 2017 - Question 27

27 A cylindrical hollow metal pipe is 16 cm long.

It has an external diameter of 10 cm and an internal diameter of 8 cm.

The density of the metal from which the pipe is made is 8 grams per cm^3.

[diagram not to scale]



What is the mass of the pipe in grams?

A 8π

B 16π

C 18π

D 72π

E 128π

F 512π

G 1152π

H 4608π

ENGAA S1 2017 - Question 27 - Worked Solution

mass = density x volume

volume = volume of outer cylinder - volume of inner cylinder

$$= (5^2\pi \times 16) - (4^2\pi \times 16)$$

$$= 144\pi \text{ cm}^3$$
$$\text{mass} = 8 \times 144\pi$$
$$\text{mass} = 1152\ \pi g$$

Answer is G

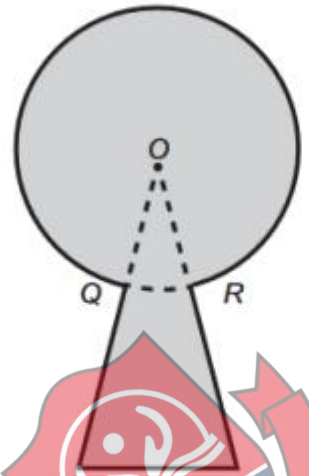
ENGAA S1 2017 - Question 35

35 The diagram shows the outline of a keyhole consisting of three straight sides and an arc from a circle.

The sides PQ and RS are both 18 mm in length and when extended meet at the centre of the circle O forming an angle of $\frac{\pi}{6}$ radians.

The longer arc from Q to R has length 22π mm.

[diagram not to scale]



A $121\pi + \frac{841}{4}$

B $121\pi + \frac{841\sqrt{3}}{4}$

C $132\pi + 225$

D $132\pi + 225\sqrt{3}$

E $144\pi + 225$

F $144\pi + 225\sqrt{3}$

ENGAA S1 2017 - Question 35 - Worked Solution

$$22\pi = r\theta$$
$$= r\left(2\pi - \frac{\pi}{6}\right)$$
$$= \frac{11}{6}\pi r$$
$$r = 12$$

A = circle + triangle - QOR sector

$$= \pi r^2 + \frac{1}{2} PQ \times QS \times sin\ sin\left(\frac{?}{6}\right) - \frac{1}{2} r^2 \left(\frac{\pi}{6}\right)$$

$$= \pi \times 12^2 + \frac{1}{2} \times (18 + 12)^2 \times \frac{1}{2} - \frac{1}{2} \times (12)^2 \times \left(\frac{\pi}{6}\right)$$

$$= 144\pi - 12\pi + 225$$

$$= 132\pi + 225$$

Answer is C

ENGAA S1 2017 - Question 41

41 In triangle PQR

$$PQ = 4x \text{ cm}$$

$$QR = (8 - 3x) \text{ cm}$$

$$\angle PQR = 60°$$

What is the maximum value of the area, in cm^2, of triangle PQR?

A $\frac{8\sqrt{3}}{3}$

B $\frac{16}{3}$

C $\frac{69\sqrt{3}}{16}$

D $\frac{16\sqrt{3}}{3}$

E $\frac{32}{3}$

F $\frac{32\sqrt{3}}{3}$

ENGAA S1 2017 - Question 41 - Worked Solution

$$A = \frac{1}{2} PQ \times PR \text{ Sin Sin } (60)$$

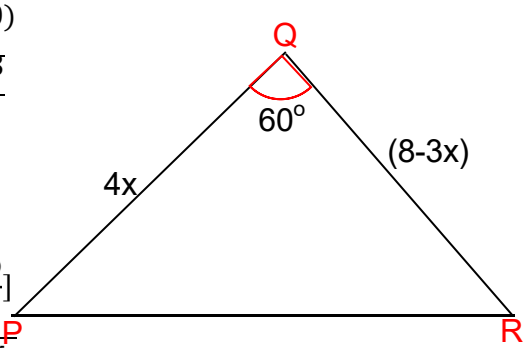
$$= \frac{1}{2} \times 4x(8 - 3x) \times \frac{\sqrt{3}}{2}$$

$$= \sqrt{3}x(8 - 3x)$$

$$= -3\sqrt{3}\left[x^2 - \frac{8}{3}x\right]$$

$$= -3\sqrt{3}\left[\left(x - \frac{4}{3}\right)^2 - \frac{16}{9}\right]$$

$$= -3\sqrt{3}\left[\left(x - \frac{4}{3}\right)^2 + \frac{16\sqrt{3}}{3}\right]$$



Answer is D

ENGAA S1 2016 - Question 11

11 The point A is 4 km due East of the point B.

The bearing of the point C from A is 330° and the bearing of C from B is 060°

Find the distance BC.

A 2 km

B $2\sqrt{3}$ km

C 4 km

D $2\sqrt{5}$ km

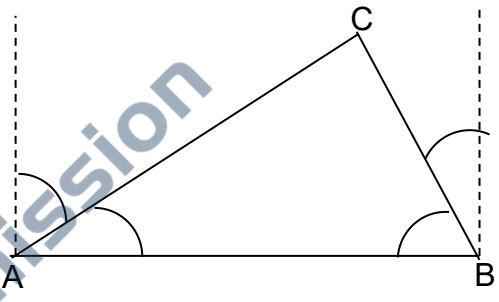
E $4\sqrt{2}$ km

ENGAA S1 2016 - Question 11 - Worked Solution

$$BC = AB \text{ Sin Sin } 60$$

$$BC = 4km \times \frac{\sqrt{3}}{2}$$

$$BC = 2\sqrt{3}km$$

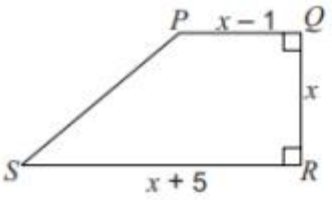


Answer is B

ENGAA S1 2016 - Question 15

15 In a trapezium $PQRS$, the parallel sides are PQ and RS.

$PQ = (x-1)$ cm, $RS = (x+5)$ cm and the vertical height $QR = x$ cm.



[diagram not to scale]

The area of the trapezium is 120 cm^2.

What is the length of RS?

A 9 cm

B 10 cm

C 11 cm

D 12 cm

E 15 cm

F 17 cm

ENGAA S1 2016 - Question 15 - Worked Solution

$$A = \frac{1}{2} \times (x + 5 + x - 1)$$

$$A = \frac{1}{2} \times (2x + 4)$$

$$A = x(x + 2)$$

$$x^2 + 2 = 120$$

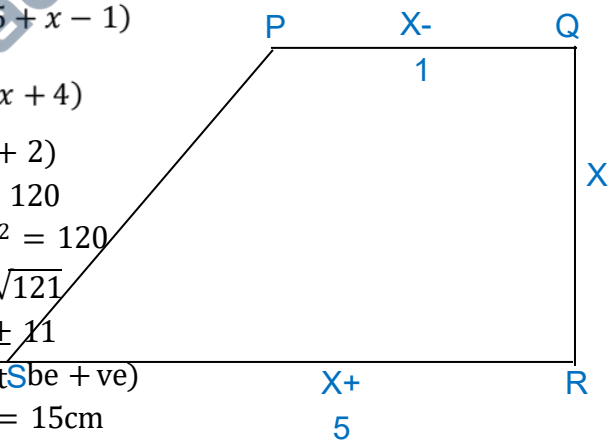
$$(x + 1)^2 - 1^2 = 120$$

$$x + 1 = \sqrt{121}$$

$$x = -1 \pm 11$$

$$x = 10 \quad (\text{must be +ve})$$

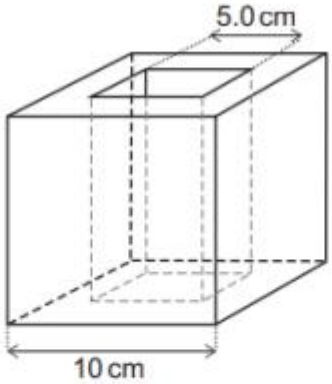
$$RS = x + 5 = 15\text{cm}$$



Answer is E

ENGAA S1 2016 - Question 18

18 A cubic block has a hole through it with a square cross-section. The dimensions are shown on the diagram. The weight of the block is 30 N.



What is the density of the material from which the block is made?

(The gravitational field strength g is 10 N kg^{-1}.)

A 0.30 g cm^{-3}

B 0.40 g cm^{-3}

C 0.60 g cm^{-3}

D 1.2 g cm^{-3}

E 3.0 g cm^{-3}

F 4.0 g cm^{-3}

G 6.0 g cm^{-3}

H 12 g cm^{-3}

ENGAA S1 2016 - Question 18 - Worked Solution

$$v_{cube} = 10^3 - 5^2 \times 10 = 750 cm^3$$

$$750 cm^3 \times P \times 10 \frac{N}{kg} = 30 N$$

$$P = 0.004 kg cm^{-3}$$

$$P = 4 g cm^{-3}$$

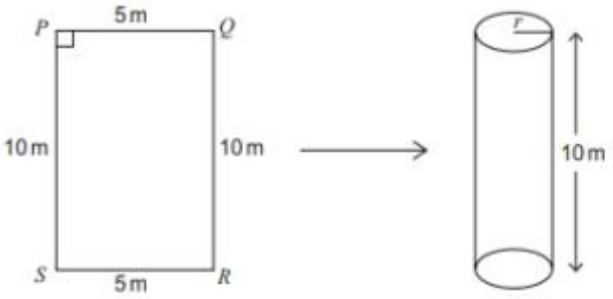
Answer is F

ENGAA S1 2016 - Question 19

19 A thin rectangular sheet of metal 10 m by 5 m is made into an open ended cylinder by joining the edges PS and QR.

The height of the cylinder is 10 m.

What is the volume, in cubic metres, enclosed by this cylinder?



[diagram not to scale]

A $\frac{5}{2\pi}$

B $\frac{25}{4\pi}$

C $\frac{125}{2\pi}$

D 62.5π

E $\frac{125}{\pi}$

F 250π

ENGAA S1 2016 - Question 19 - Worked Solution

$$\text{circumference of cylinder} = 5\text{m}$$

$$2\pi r = 5$$

$$r = \frac{5}{2\pi}$$

$$v = \pi r^2 h$$

$$\text{h} = 10\text{m}$$

$$v = \left(\frac{5}{2\pi}\right)^2 \times \pi \times 10$$

$$v = \frac{250}{4\pi}$$

$$v = \frac{125}{2\pi}$$

Answer is C

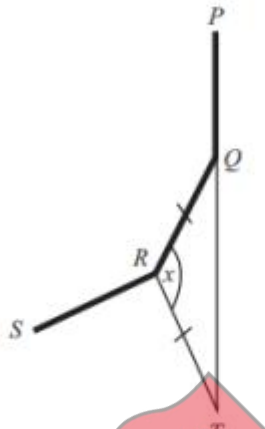
ENGAA S1 2016 - Question 27

27 In the diagram below, $PQRS$ is part of a regular polygon.

The polygon has n sides.

The side PQ is extended to T such that PQT is a straight line.

The length of RQ is the same as the length of RT.



[diagram not to scale]

Find an equation for n in terms of x, where x is the size of angle $\angle QRT$ in degrees.

A $n = \dfrac{180}{x - 90}$

B $n = \dfrac{180 - x}{720}$

C $n = \dfrac{360 - x}{90}$

D $n = \dfrac{360}{180 - x}$

E $n = \dfrac{720}{180 - x}$

F $n = \dfrac{720}{360 - x}$

G $n = \dfrac{360}{360 - x}$

ENGAA S1 2016 - Question 27 - Worked Solution

$\angle RQT = \angle QTR = y$

$2y + x = 180$

$\angle RQp + \angle RQT = 180$

$$\angle RQP = \frac{(n-2)(180)}{n} = 180\left(1 - \frac{2}{n}\right)$$

$$<RQT = 180 - <RQp$$

$$= 180 - 180\left(1 - \frac{2}{n}\right)$$

$$= \frac{2}{n} \times 180^{\circ}$$

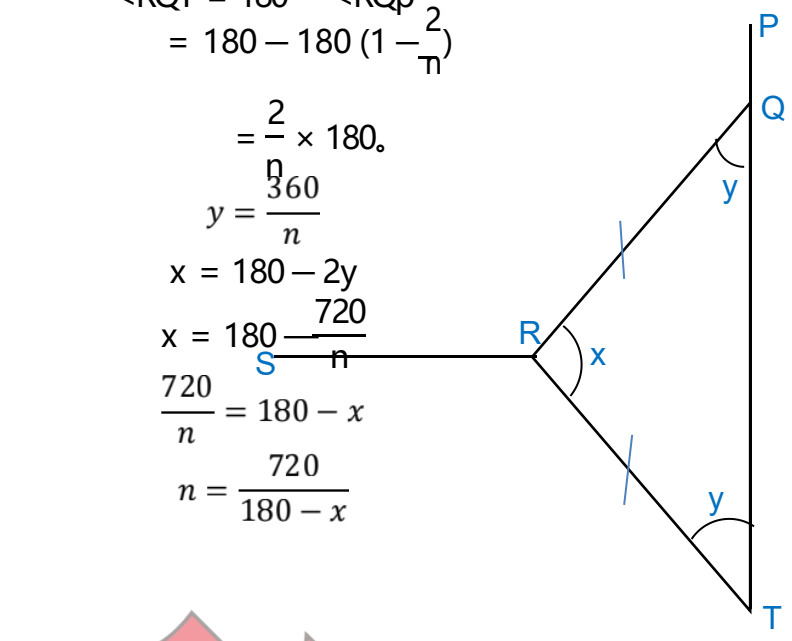
$$y = \frac{360}{n}$$

$$x = 180 - 2y$$

$$x = 180 - \frac{720}{n}$$

$$\frac{720}{n} = 180 - x$$

$$n = \frac{720}{180 - x}$$



Answer is E

ENGAA S1 2016 - Question 31

31 A square PQRS is drawn above the x-axis with the side PQ on the x-axis.

P is the point $(-5, 0)$ and Q is the point $(1, 0)$.

A circle is drawn inside the square with diameter equal in length to the side of the square.

Which one of the following is an equation of the circle?

A $x^2 + y^2 - 4x + 6y + 4 = 0$

B $x^2 + y^2 - 4x + 6y + 9 = 0$

C $x^2 + y^2 + 4x - 6y + 4 = 0$

D $x^2 + y^2 + 4x - 6y + 9 = 0$

E $x^2 + y^2 - 6x - 4y + 9 = 0$

F $x^2 + y^2 - 6x + 4y + 4 = 0$

G $x^2 + y^2 + 6x - 4y + 4 = 0$

H $x^2 + y^2 + 6x + 4y + 9 = 0$

ENGAA S1 2016 - Question 31 - Worked Solution

The square has a side length 6.

If the circle is fully inside the square and its diameter is equal to the side length of the square then its center must be at the center of the square.

The center of the square is either at (- 2 , 3) or (- 2 , - 3)

the circle has equation : $(x + 2)^2 + (y - 3)^2 = 3^2$, or $(x + 2)^2 + (y + 3)^2 = 3^2$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 9$$

$$x^2 + 4x + 4 + y^2 + 6y + 9 = 9$$

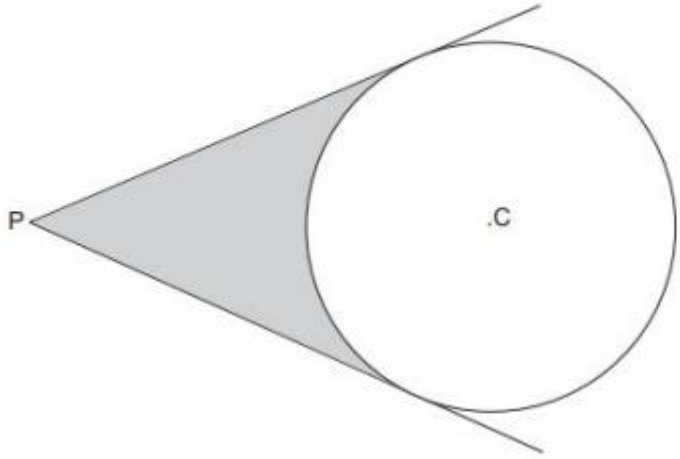
Only C is either of the possible equations

Answer is C

ENGAA S1 2016 - Question 35

35 Tangents are drawn from a point P to a circle of radius 10 cm.

The centre of the circle is C and the distance PC is 20 cm.



[diagram not to scale]

Which one of the following is an expression for the shaded area in square centimetres?

A $\frac{100}{3}(3\sqrt{3}-\pi)$

B $\frac{100}{3}(3\sqrt{5}-\pi)$

C $\frac{50}{3}(6\sqrt{3}-\pi)$

D $\frac{50}{3}(6\sqrt{5}-\pi)$

E $\frac{50}{3}(\sqrt{3}-2\pi)$

F $\frac{50}{3}(2\pi-\sqrt{3})$

ENGAA S1 2016 - Question 35 - Worked Solution

$$AP = 10\text{cm} \quad AC = 10\text{cm}$$

$$PB = PD = x$$

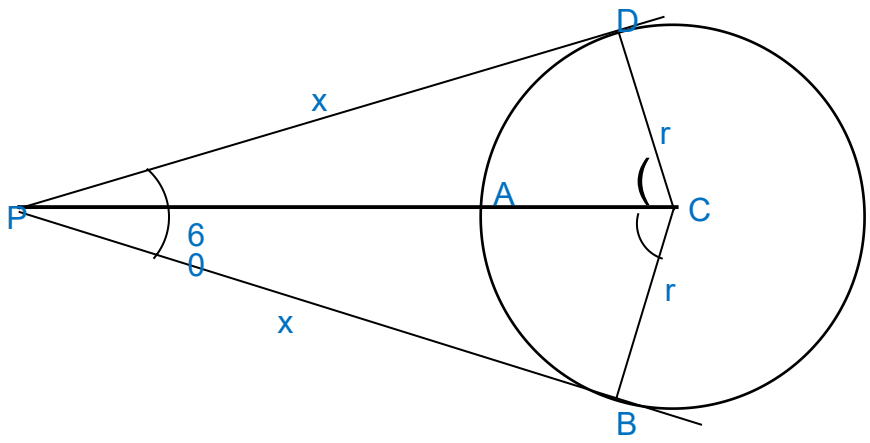
$$x^2 + r^2 = 20^2$$

$$x^2 = 300$$

$$x = 10\sqrt{3}\text{cm}$$

$$\cos\cos\frac{\theta}{2} = \frac{1}{2}$$

$$\frac{\theta}{2} = \frac{\pi}{3}$$



$$\theta = \frac{2\pi}{3}$$

The area of the shaded area is equal to the area of the triangle with vertices PBD subtract the area of the segment of the circle formed by ABD, denoted by V. That is equal to the sector of the circle formed by BCD subtract the area of the triangle with vertices BCD, shown as W.

$$A_{PBD} = \frac{1}{2}x^2 \, sin \, sin \, 60°$$

$$= \frac{1}{2} \times 300 \times \frac{\sqrt{3}}{2}$$

$$= 75\sqrt{3}$$

$$A_{sector} = \pi r^2 \times \frac{2\pi/3}{2\pi} = \frac{\pi r^2}{3} = \frac{100\pi}{3}$$

$$A_W = \frac{1}{2}r^2 \, sin \, sin \frac{2\pi}{3} = \frac{1}{2} \times 100 \times \frac{\sqrt{3}}{2} = 25\sqrt{3}$$

$$\mathrm{A_v} = 100\frac{\pi}{3} - 25\sqrt{3}$$

$$A_{shaded} = 75\sqrt{3} - \left(100\frac{\pi}{3} - 25\sqrt{3}\right)$$

$$= 100\sqrt{3} - 100\frac{\pi}{3}$$

$$= \frac{100}{3}\left(3\sqrt{3} - \pi\right)$$

Answer is A