Worked Solutions for ENGAA Papers by Topic

Section 1

Topic: Sequences and series

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ENGAA S1 2020 - Question 28

28 The sum of the first 20 terms of an arithmetic progression is 50.

The sum of the next 20 terms of the arithmetic progression is –50.

What is the sum of the first 100 terms of the arithmetic progression?

A –750

B –350

C –50

D $-\frac{159}{8}$

E $\frac{159}{8}$

F 50

G 350

H 750

ENGAA S1 2020 - Question 28 - Worked Solution

$$S_{20} = 50 = \frac{20}{2}(a + a + 19d)$$

$$S_{90-20} = -50 = \frac{40}{2}(a + a + 39d) - \frac{20}{2}(2a + 19d)$$

$$50 = 20a + 190d$$

$$-50 = 20a + 590d$$

$$d = -\frac{1}{4}$$

$$a = \frac{39}{8}$$

$$S_{100} = \frac{n}{2}(u_1 + u_{100})$$

$$= 50\left(\frac{39}{8} + \frac{39}{8} + 99 \times \left(-\frac{1}{4}\right)\right)$$

$$= 50\left(\frac{-60}{4}\right)$$

$$= -750$$

Answer is A

ENGAA S1 2020 - Question 32

32 *P* and *Q* are two different geometric progressions.

The 3rd term of each geometric progression is 4.

The 5th term of each geometric progression is 2.

What is the modulus of the difference between the sums to infinity of *P* and *Q*?

A 0

B 8

C $8\sqrt{2}$

D 16

E $16\sqrt{2}$

F 32

G $32\sqrt{2}$

ENGAA S1 2020 - Question 32 - Worked Solution

$$2 = u_1 \cdot r^4$$

$$4 = u_1 \cdot r^2$$

$$r^2 = \frac{1}{2}$$

$$r = \pm\sqrt{\frac{1}{2}}$$

$$S_{\infty+} - S_{\infty-} = \frac{8}{1-\sqrt{\frac{1}{2}}} - \frac{8}{1-\left(-\sqrt{\frac{1}{2}}\right)} = 16\sqrt{2}$$

Answer is E

ENGAA S1 2018 - Question 25

25 The first five terms of a sequence in order are:

2 17 42 77 122

The n^{th} term of this sequence is $pn^2 + q$ where p and q are integers.

What is the value of $\frac{p-q}{p+q}$?

A $\frac{1}{4}$

B $\frac{1}{2}$

C 1

D $\frac{23}{17}$

E $\frac{13}{7}$

F 2

G 4

H 14

ENGAA S1 2018 - Question 25 - Worked Solution

$$U_n = 5n^2 + cons^5$$

U_n	2	17	42	77	122
$S_n^{\,2}$	5	20	45	80	125
	-3	-3	-3	-3	-3

$$U_n = 5_n^{\,2} - 3$$
$$p = 5\,, q = -3$$
$$\frac{p-q}{p+q} = \frac{8}{2} = 4$$

Answer is G.

ENGAA S1 2018 - Question 37

37 In a particular arithmetic progression:

- the 13^{th} term is six times the 1^{st} term
- the 11^{th} term is 1 less than twice the 5^{th} term

What is the 3^{rd} term of the progression?

A -14.5

B -11

C $\frac{29}{19}$

D 3.5

E 11

F 14.5

ENGAA S1 2018 - Question 37 - Worked Solution

For arithmetic progression:

$$U_n = a + (n-1)d$$

$$U_{13} = a + 12d = 6U_1 = 6a$$

$$a + 12d = 6a$$

$$12d - 5a = 0 \quad ①$$

$$U_{11} = 2U_5 - 1$$

$$a + 10d = 2a + 8d - 1$$

$$2d - a = -1 \quad ②$$

① - 6②

$$-5a + 6a = 6$$

$$a = 6$$

$$2d - 6 = -1$$

$$d = \frac{5}{2}$$

$$U_3 = a + 2d$$

$$= 6 + 5$$

$$= 11$$

Answer is E.

ENGAA S1 2018 - Question 39

39 The first three terms of a geometric progression, whose terms are all greater than zero, are $(p-2)$, $(2p+2)$ and $(5p+14)$

What is the fifth term of the progression?

A 324

B 486

C 1250

D 1458

E 3888

ENGAA S1 2018 - Question 39 - Worked Solution

Geometric progression:

$$U_n = ar^{n-1}$$
$$U_1 = a = (p-2)$$
$$\frac{U_2}{U_1} = \frac{U_3}{U_2} = r$$
$$\frac{2p+2}{p-2} = \frac{5p+14}{2p+2}$$
$$(2p+2)^2 = (5p+14)(p-2)$$
$$4p^2 + 4 + 8p = 5p^2 + 14p - 10p - 28$$
$$p^2 - 4p - 32 = 0$$
$$p = 8 \; or \; p = -4$$
$$U_1 = p - 2$$
$$= 6 \;\; or \; -6 \quad (but \;\; U_n > 0)$$
$$p = 8$$
$$r = \frac{2p+12}{p-2}$$
$$r = 3$$
$$U_5 = ar^4$$
$$= (p-2) \times 3^4$$
$$= 6 \times 3^4$$
$$= 486$$

Answer is B.

ENGAA S1 2017 - Question 45

45 A geometric progression has first term equal to 1 and common ratio $\frac{1}{2}\sin 2x$

The sum to infinity of the series is $\frac{4}{3}$

Find the possible values of x in the range $\pi \le x \le 2\pi$

A $\frac{13}{12}\pi$, $\frac{17}{12}\pi$

B $\frac{7}{6}\pi$, $\frac{4}{3}\pi$

C $\frac{7}{6}\pi$, $\frac{11}{6}\pi$

D $\frac{5}{4}\pi$, $\frac{7}{4}\pi$

E there are no values of x in this range

ENGAA S1 2017 - Question 45 - Worked Solution

$$U_n = ar^{n-1}$$

$$a = 1\,, r = \frac{1}{2} sin\, sin\,(2x)$$

$$S_\infty = \frac{a}{1-r} = \frac{4}{3}$$

$$3 = 4(1-r)$$

$$\frac{1}{4} = r$$

$$\frac{1}{2} sin\, sin\,(2x) = \frac{1}{4}$$

$$sin\, sin\,(2x) = \frac{1}{2}$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13}{6}\pi, \frac{17}{6}\pi$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13}{12}\pi, \frac{17}{12}\pi$$

Answer is A

ENGAA S1 2017 - Question 47

47 The sequence of numbers $u_1, u_2, u_3, \dots, u_n, \dots$ is given by

$$u_1 = 2$$
$$u_{n+1} = pu_n + 3$$

where p is an integer.

The fourth term, u_4, is equal to -7

What is the value of $u_1 + u_2 + u_3 + u_4$?

A -10

B -2

C -1

D 8

E 26

ENGAA S1 2017 - Question 47 - Worked Solution

$$u_1 = 2$$
$$u_2 = 2p + 3$$
$$u_3 = p(2p + 3) + 3$$
$$= 2p^2 + 3p + 3$$
$$u_4 = p(2p^2 + 3p + 3) + 3$$
$$= 2p^3 + 3p^2 + 3p + 3$$
$$= -7$$
$$2p^3 + 3p^2 + 3p + 10 = 0$$

Find real solution by trying p =1 , p = 2 …

Clearly p = -2 is a solution:

$$u_2 = -1$$
$$u_3 = 5$$
$$u_1 + u_2 + u_3 + u_4 = 2 - 1 + 5 - 7$$
$$= -1$$

Answer is C

ENGAA S1 2016 - Question 33

33 The first term of a convergent geometric series is 8.

The fifth term is 2.

The sixth term is real and positive.

What is the sum to infinity of this series?

(The sum to infinity of a convergent geometric series is given by $\frac{a}{1-r}$, where a is the first term and r is the common ratio.)

A $8(1+\sqrt{2})$

B $8(1-\sqrt{2})$

C $8(2+\sqrt{2})$

D $8(2-\sqrt{2})$

E 16

F $\frac{8\sqrt[5]{4}}{\sqrt[5]{4}-1}$

G $\frac{8\sqrt[5]{4}}{\sqrt[5]{4}+1}$

ENGAA S1 2016 - Question 33 - Worked Solution

$$a = 8$$
$$ar^4 = 2$$
$$r > 0\,, r \in R$$
$$8r^4 = 2$$
$$r^4 = \frac{1}{4}$$
$$r = \left(\frac{1}{4}\right)^{\frac{1}{4}}$$
$$r = \frac{\sqrt{2}}{2}$$
$$S_\infty = \frac{a}{1-r}$$
$$S_\infty = 16 + 8\sqrt{2}$$
$$S_\infty = 8(2+\sqrt{2})$$

Answer is C