

# Worked Solutions for ENGAA Papers by Topic

## Section 2

### Topic: Electricity

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### ENGAA S2 2020 - Question 2

- 2 A single strand of wire has a radius of  $2.0 \times 10^{-4}$  m and length 15 m. The resistivity of the material from which the wire is made is  $4.8 \times 10^{-7} \Omega \text{ m}$ .

Twelve strands of this wire are connected in parallel to make a cable.

What is the resistance of the cable?

- A  $\frac{\pi}{2160} \Omega$
- B  $\frac{\pi}{180} \Omega$
- C  $\frac{\pi}{15} \Omega$
- D  $\frac{15}{\pi} \Omega$
- E  $\frac{180}{\pi} \Omega$
- F  $\frac{2160}{\pi} \Omega$

### ENGAA S2 2020 - Question 2 - Worked Solution

$$A = \pi r^2 = \pi \cdot 4 \times 10^{-8}$$
$$R = \frac{\rho \cdot l}{A} = \frac{15}{\pi}$$

Answer is D

**ENGAA S2 2020 - Question 11**

**11** Three identical resistors can be combined in four different arrangements.

One of the arrangements has a resistance of  $18\Omega$ .

A different arrangement has a resistance of  $8.0\Omega$ .

What are the resistances of the other two arrangements?

(All three resistors contribute to the total resistance in all arrangements.)

**A**  $2.0\Omega$  and  $4.0\Omega$

**B**  $2.0\Omega$  and  $9.0\Omega$

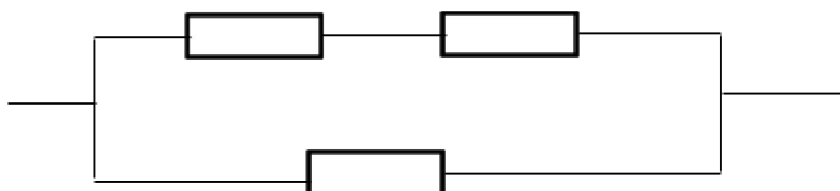
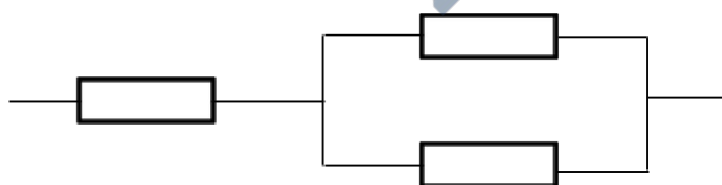
**C**  $4.0\Omega$  and  $12\Omega$

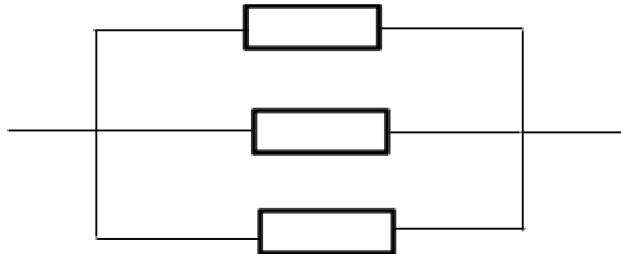
**D**  $4.0\Omega$  and  $36\Omega$

**E**  $36\Omega$  and  $162\Omega$

**F**  $81\Omega$  and  $162\Omega$

**ENGAA S2 2020 - Question 11 - Worked Solution**





Let each resistor have resistance  $R$

$$R_0 = 3R$$

$$R_o = \frac{3}{2}R$$

$$R_o = \frac{3}{2}R$$

$$R_o = \frac{1}{3}R$$

If  $R = 6$ ,  $R_o \in \{2\Omega, 4\Omega, 9\Omega, 18\Omega\}$

$R = 12$ ,  $R_o \in \{4\Omega, 8\Omega, 18\Omega, 36\Omega\}$

thus  $R = 12\Omega$

Answer is D

### ENGAA S2 2020 - Question 12

- 12 A  $4.0\text{ k}\Omega$  fixed resistor is connected in series with a light dependent resistor (LDR) across a  $100\text{ V}$  dc power supply.

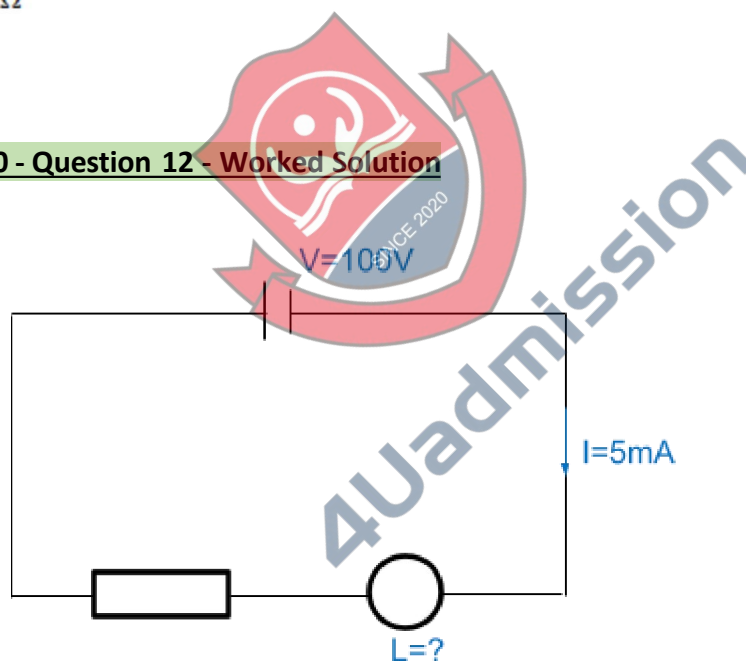
The current in the LDR is  $5.0\text{ mA}$ .

The intensity of light falling on the LDR now decreases and the voltage across the fixed resistor changes by  $50\%$ .

What is the change in the resistance of the LDR as a result of the change in intensity?

- A  $8.0\text{ k}\Omega$
- B  $12\text{ k}\Omega$
- C  $16\text{ k}\Omega$
- D  $20\text{ k}\Omega$
- E  $32\text{ k}\Omega$
- F  $36\text{ k}\Omega$

### ENGAA S2 2020 - Question 12 - Worked Solution



$$V_{R_0} = IR = 20\text{V}$$

$$VR' = 10\text{V}$$

$$VR' = 100\text{V} \cdot \frac{4\text{ k}\Omega}{L+4\text{ k}\Omega} = 10\text{V}$$

$$L = 20000\Omega$$

Answer is D

### ENGAA S2 2020 - Question 20

- 20 A cell has emf  $E$  and internal resistance  $r$  that varies with current  $I$  according to:

$$r = kI^2$$

where  $k$  is a constant.

A variable resistor is connected to the terminals of the cell. The resistance of the variable resistor is adjusted.

Which expression gives the resistance of the variable resistor, in terms of  $k$  and  $E$ , that causes maximum power dissipation in it?

A  $3\left(\frac{kE^2}{2}\right)^{\frac{1}{3}}$

B  $3\left(\frac{kE^2}{4}\right)^{\frac{1}{3}}$

C  $3\left(\frac{kE^2}{9}\right)^{\frac{1}{3}}$

D  $3\left(\frac{kE^2}{16}\right)^{\frac{1}{3}}$

E  $(2kE^2)^{\frac{1}{3}}$

F  $(4kE^2)^{\frac{1}{3}}$

G  $(9kE^2)^{\frac{1}{3}}$

H  $(16kE^2)^{\frac{1}{3}}$



### ENGAA S2 2020 - Question 20 - Worked Solution

By ohm's law

$$E = I(R + r)$$

$$r = KI^2$$

$$:IR = E - KI^3$$

$$P = I^2 R = EI - KI^4$$

$$\frac{dP}{dI} = E - 4KI^3$$

$$\frac{dP}{dI} = 0 \text{ when } E = 4kI^3$$

OR

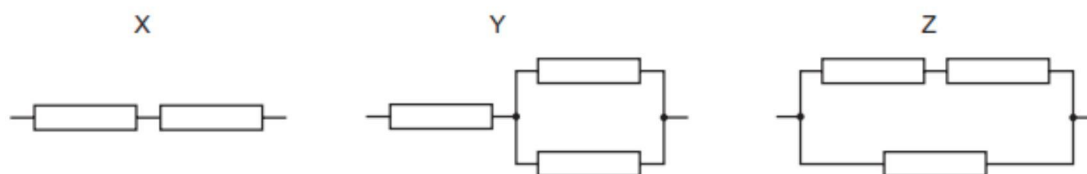
$$I = \left(\frac{E}{4k}\right)^{\frac{1}{3}}$$

$$\begin{aligned} IR &= E + KI^3 \\ &= IR + \frac{E}{4} \end{aligned}$$



## ENGAA S2 2019 - Question 2

- 2 Identical resistors are used to produce three different arrangements X, Y and Z.



Each arrangement is connected, in turn, across the same battery which has a negligible internal resistance.

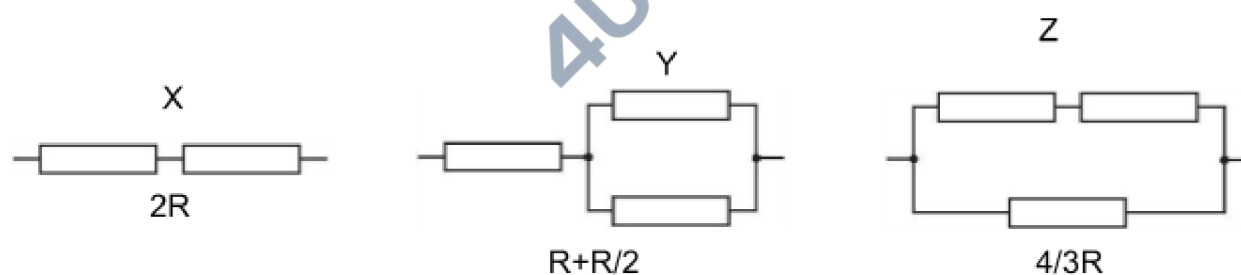
The total power developed in each of the arrangements is determined.

What is the order of the arrangements when placed in order of **increasing** power?

- A X, Y, Z
- B X, Z, Y
- C Y, X, Z
- D Y, Z, X
- E Z, X, Y
- F Z, Y, X



## ENGAA S2 2019 - Question 2 - Worked Solution



$$P = IR$$

$$V = IR$$

$$\therefore P = \frac{V^2}{R}$$

We want to rank them  
lowest power  $\rightarrow$  highest power



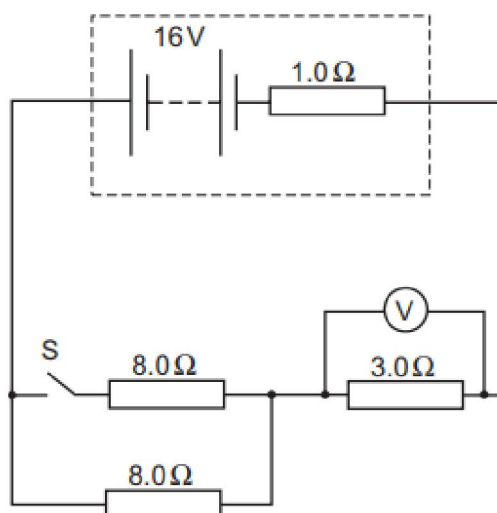
Thus:  
highest R  $\rightarrow$  lowest R  
X, Y, Z

Answer is A



# ENGAA S2 2019 - Question 4

- 4 The battery in the circuit shown has an emf of 16 V and an internal resistance of  $1.0\ \Omega$ .



Which line in the table gives the voltmeter readings when switch S is in its open and closed states?

	voltmeter reading / V	
	when S is open	when S is closed
A	$\frac{4.0}{3.0}$	2.0
B	4.0	6.0
C	4.0	2.4
D	6.0	2.4
E	6.0	4.0
F	$\frac{48}{11}$	$\frac{48}{19}$
G	$\frac{48}{11}$	$\frac{48}{7.0}$
H	$\frac{128}{11}$	$\frac{64}{7.0}$

## ENGAA S2 2019 - Question 4 - Worked Solution

$$V_{measured} = \frac{3}{R_0} \cdot 16V$$

$$V_{open} = \frac{13}{12} \cdot 16 = 4V$$

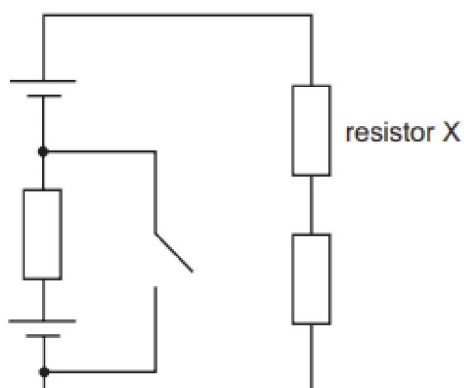
$$V_{closed} = \frac{3}{8} \cdot 16 = 6V$$

Answer is B



### ENGAA S2 2019 - Question 13

- 13 The circuit below contains three identical resistors, and two identical cells. When the switch is open, the power dissipated by resistor X is  $P$ .



What is the power dissipated by resistor X after the switch is closed?

- A  $\frac{P}{4}$
- B  $\frac{9P}{16}$
- C  $\frac{3P}{4}$
- D  $P$
- E  $\frac{16P}{9}$
- F  $\frac{9P}{4}$



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### ENGAA S2 2019 - Question 13 - Worked Solution

Switch	$\sum v$	$\sum R$	I	P
Open	2V	3R	$\frac{2}{3} V/R$	$\frac{4}{9} V^2/R$
Closed	V	2R	$\frac{1}{2} V/R$	$\frac{1}{4} V^2/R$

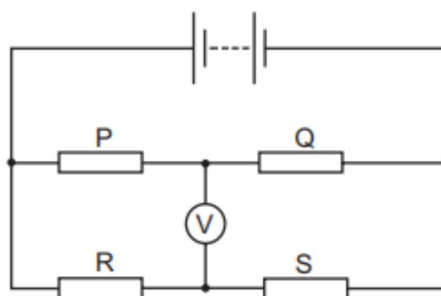
$$P' = \frac{9}{16} P$$

Answer is B

**ENGAA S2 2019 - Question 15**

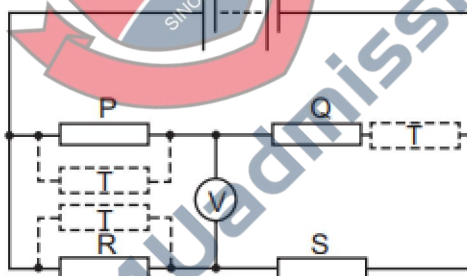
- 15** Four resistors, P, Q, R and S, are connected to a battery with negligible internal resistance, as shown in the diagram.

P and S each have resistance  $x$ . Q and R each have resistance  $2x$ .



A fifth resistor, T, which has resistance  $x$ , is to be added to the circuit in one of the following listed positions, as shown in the diagram:

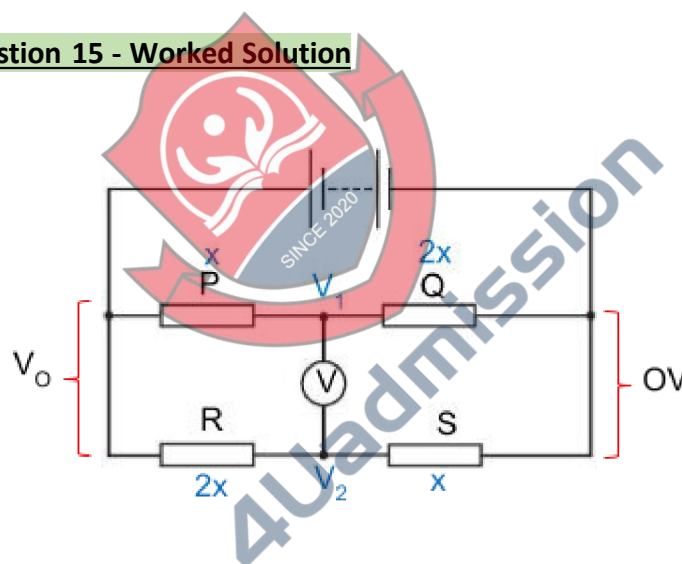
- 1** in parallel with P
- 2** in series with Q
- 3** in parallel with R



Which of the positions for resistor **T** causes an increase in the magnitude of the voltmeter reading?

- A none of them
- B 1 only
- C 2 only
- D 3 only
- E 1 and 2 only
- F 1 and 3 only
- G 2 and 3 only
- H 1, 2 and 3

**ENGAA S2 2019 - Question 15 - Worked Solution**



To maximize Voltmeter reading, increase difference between  $V_1$  &  $V_2$

- 1 in parallel with P  $\rightarrow$  lowers  $R_p$ , lowers  $V_1$
- 2 in series with Q  $\rightarrow$  increases  $R_q$ , lower  $V_1$
- 3 in parallel with R  $\rightarrow$  makes  $V_2$  closer to  $V_1$

Answer is E

### ENGAA Specimen S2 - Question 6

- 6 A wire of length 4.0 m with a uniform cross-sectional area of  $0.020 \text{ mm}^2$  is connected in series with a  $1.0 \text{ k}\Omega$  resistor.

There is a pd of 1.2 V across this arrangement and a voltmeter connected across the  $1.0 \text{ k}\Omega$  resistor reads 1.0 V.

Under these conditions, what is the resistivity of the material from which the wire is made?

- A  $1.0 \times 10^{-6} \Omega \text{ m}$
- B  $1.1 \times 10^{-5} \Omega \text{ m}$
- C  $1.0 \times 10^{-3} \Omega \text{ m}$
- D  $1.1 \times 10^{-2} \Omega \text{ m}$
- E  $8.0 \times 10^{-2} \Omega \text{ m}$
- F  $8.0 \times 10^2 \Omega \text{ m}$
- G  $4.0 \times 10^7 \Omega \text{ m}$
- H  $4.0 \times 10^{10} \Omega \text{ m}$

### ENGAA Specimen S2 - Question 6 - Worked Solution

Since the wire is in series with the resistor, the sum of the p. d. s is 1.2v  
The p. d across the wire = 0.2v

for resistors in series  $\frac{V_1}{V_2} = \frac{R_1}{R_2}$

$$\frac{0.2}{1.0} = \frac{R}{1.0 \times 10^3}$$

$$R = 200 \Omega$$

$$R = \frac{\rho l}{A}$$

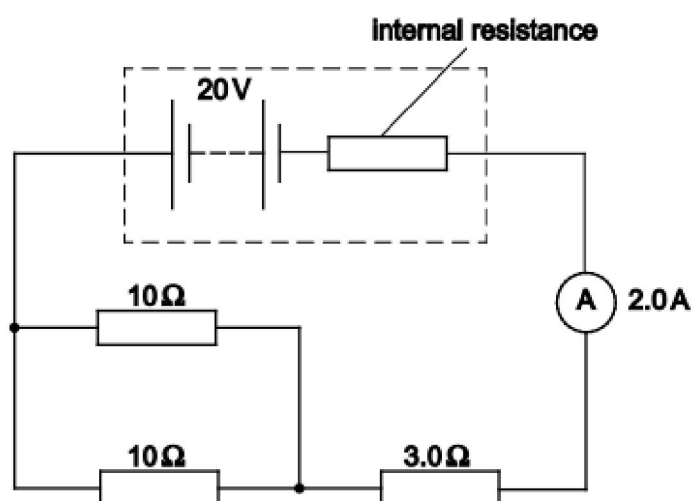
$$\frac{P \times 4.0}{0.02 \times 10^{-6}} = 200$$

$$P = 1 \times 10^{-6} \Omega \text{ m}$$

Answer is A

### ENGAA Specimen S2 - Question 8

- 8 The circuit shown includes a battery with an emf of 20V and an ideal ammeter. The reading on the ammeter is 2.0A.



What is the internal resistance of the battery?

- A 2.0Ω
- B 4.0Ω
- C 7.0Ω
- D 10Ω
- E 18Ω

### ENGAA Specimen S2 - Question 8 - Worked Solution

For resistors in parallel:

$$\frac{1}{R_e} = \sum_i \frac{1}{R_i}$$
$$\frac{1}{R_e} = \frac{1}{10} + \frac{1}{10}$$
$$R_e = 5\Omega$$

For resistors in series :  $R_T = \sum_i R_i$

$$R_T = 5 + 3 + r$$

$$V = IR$$

$$20 = 2(8 + r)$$

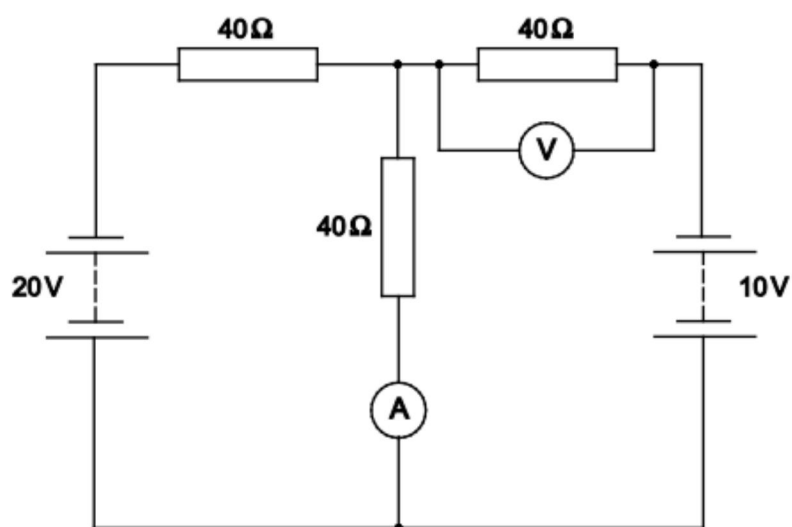
$$r = 2\Omega$$

Answer is A



### ENGAA Specimen S2 - Question 18

- 18 An electric circuit contains two different power supplies with negligible internal resistance, three identical resistors, an ideal ammeter and an ideal voltmeter.



What are the readings on the ammeter and the voltmeter?

What are the readings on the ammeter and the voltmeter?

	ammeter reading	voltmeter reading
A	0.25 A	0 V
B	0.50 A	0 V
C	0.75 A	0 V
D	0.25 A	10 V
E	0.50 A	10 V
F	0.75 A	10 V

### ENGAA Specimen S2 - Question 18 - Worked Solution

Using Kirchoff's voltage law

$$\text{loop ① : } I_1 \times 40 + I_3 \times 40 = 20 \text{ ----- ①}$$

$$\text{loop ② : } I_2 \times 40 + I_3 \times 40 = 10 \text{ ----- ②}$$

Using Kirchoff's current law:

$$I_1 + I_2 = I_3 \text{ ----- ③}$$

$$\text{①} + \text{②}$$

$$(I_1 + I_2) \times 40 + 2I_3 \times 40 = 30 \text{ ----- ④}$$

Sub ③ into ④

$$120I_3 = 30$$

$$I_3 = \frac{3}{12} = 0.25a$$

$$40I_3 + 40I_2 = 10$$

$$10 + 40I_2 = 10$$

$$I_2 = 0$$

the voltmeter reading = 0v

Answer is A



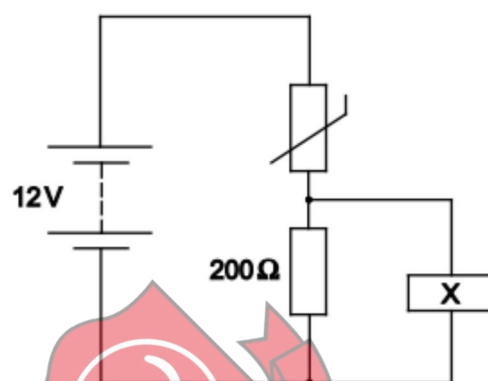
### ENGAA Specimen S2 - Question 20

- 20 In the following circuit, the thermistor has a resistance  $R$  at temperature  $T$  °C given by the equation

$$R = R_0 b^{-\mu T}$$

where  $R_0$  is the resistance at 0 °C, and  $\mu$  is a positive constant and  $b > 1$ .

X is a component with very high resistance that emits light when the pd across it exceeds 2.0 V.



What is the full range of temperatures of the thermistor for which component X emits light?

- A greater than  $(1/\mu)(\log_b R_0 - \log_b 1000)$
- B greater than  $(1/\mu)(\log_b 1000 - \log_b R_0)$
- C greater than  $(1/\mu)(\log_b R_0 - \log_b 1200)$
- D greater than  $(1/\mu)(\log_b 1200 - \log_b R_0)$
- E less than  $(1/\mu)(\log_b R_0 - \log_b 1000)$
- F less than  $(1/\mu)(\log_b 1000 - \log_b R_0)$
- G less than  $(1/\mu)(\log_b R_0 - \log_b 1200)$
- H less than  $(1/\mu)(\log_b 1200 - \log_b R_0)$

### ENGAA Specimen S2 - Question 20 - Worked Solution

Potential divider circuit can be used as the resistance of X is very high.

$$V_{out} = V_{in} \left( \frac{R_2}{R_1 + R_2} \right)$$
$$V_x = 12 \times \frac{200}{200 + R_0 b^{-\mu T}}$$

$$\frac{2400}{200 + R_o b^{-\mu T}} > 2.0$$

$$1200 > 200 + R_o b^{-\mu T}$$

$$R_o b^{-\mu T} < 1000$$

$$b^{-\mu T} < \frac{1000}{R_o}$$

$$-\mu T \log \log b < \log \log 1000 - \log \log R_o$$

$$\mu T \log \log b > \log \log R_o - \log \log 1000$$

$$T > \frac{\log \log b}{\mu} (\log \log R_o - \log \log 1000)$$

Make b the base of the logs such that  $b = 1$

$$T > \frac{1}{\mu} (\log \log R_o - \log \log 1000)$$

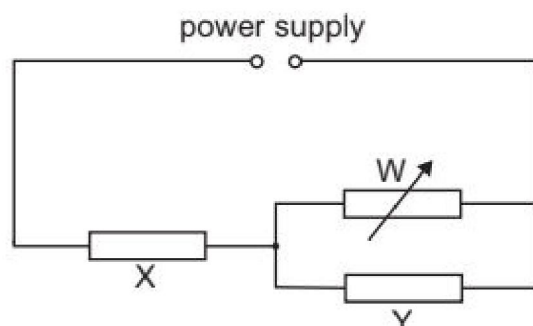
Answer is A



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### ENGAA S2 2018 - Question 3

- 3 A circuit contains two fixed resistors, X and Y, and a variable resistor W. The power supply has no internal resistance.



The resistance of W increases.

What happens to the power dissipated in X and in Y?

	<i>power dissipated in X</i>	<i>power dissipated in Y</i>
<b>A</b>	decreases	decreases
<b>B</b>	decreases	stays constant
<b>C</b>	decreases	increases
<b>D</b>	increases	decreases
<b>E</b>	increases	stays constant
<b>F</b>	increases	increases

### ENGAA S2 2018 - Question 3 - Worked Solution

Total resistance for a parallel combination:

$$R_T = \left( \frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}$$

So as W increases, total resistance of combination of W and Y increases.

P.d across then increases (potential divider → potential is split in the same ratio as the resistance).

So P.d across X decreases.

$P = \frac{V^2}{R}$ , so power dissipation across X decreases

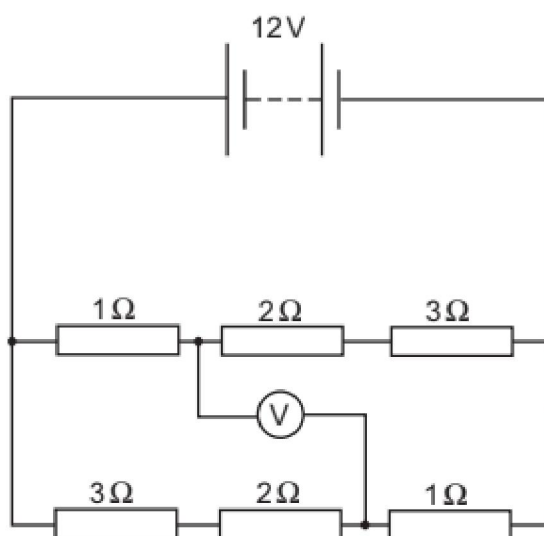
As the potential across Y increases, power dissipation in Y increases, by same argument as above.

Answer is C.



**ENGAA S2 2018 - Question 9**

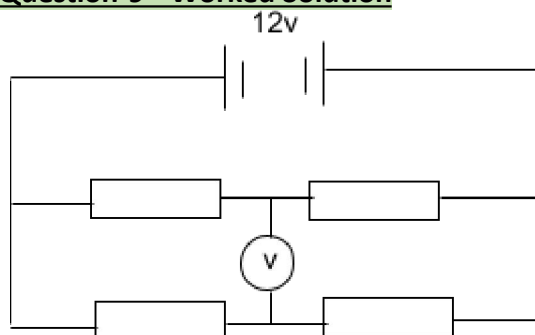
- 9 The circuit shown in the diagram contains six resistors and an ideal digital voltmeter.



What is the reading on the voltmeter?

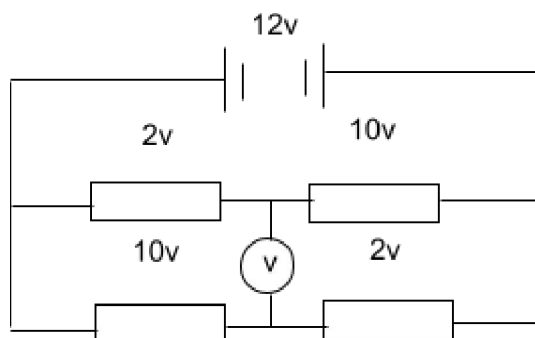
- A 0V
- B 2V
- C 4V
- D 6V
- E 8V
- F 10V
- G 12V

**ENGAA S2 2018 - Question 9 - Worked Solution**



Redraw circuit:

Potential is split in ratio of resistance:



So, reading on voltmeter is  $10\text{v} - 2\text{v} = 8\text{v}$

Answer is E.

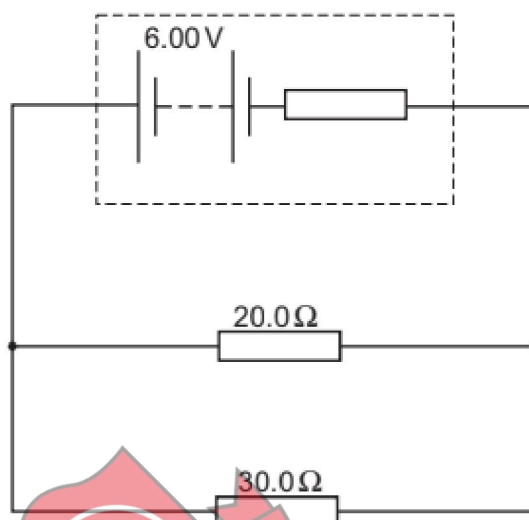




### ENGAA S2 2018 - Question 12

- 12 A circuit contains a battery with internal resistance and two resistors, connected as shown in the diagram.

The emf of the battery is 6.00 V. The pd across the 20.0  $\Omega$  resistor is 4.80 V.



What is the internal resistance of the battery?

- A 0.0800  $\Omega$
- B 0.333  $\Omega$
- C 0.480  $\Omega$
- D 3.00  $\Omega$
- E 12.0  $\Omega$
- F 12.5  $\Omega$
- G 15.0  $\Omega$

### ENGAA S2 2018 - Question 12 - Worked Solution

Parallel circuit so:

$$4.8 = 6 - V_r$$

$$V_r = 1.2 \quad (\text{p.d lost in internal resistance})$$

$$I_1 = \frac{V_r}{R} = \frac{4.8}{20} = 0.24 \text{ A}$$

$$I_2 = \frac{V_r}{R} = \frac{4.8}{30} = 0.16 \text{ A}$$

$$I = 0.24 + 0.16$$

$$I = 0.4A$$

$$V_r = I_r$$

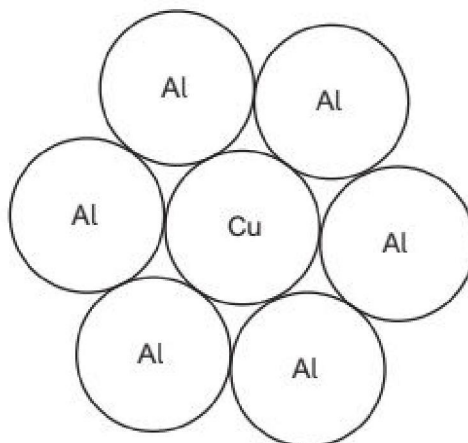
$$r = \frac{V_r}{I} = \frac{1.2}{0.4} = 3\Omega$$

Answer is D.



**ENGAA S2 2018 - Question 16**

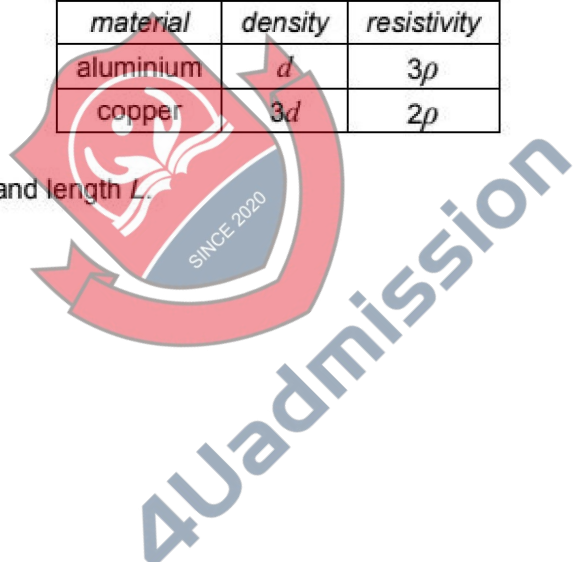
- 16** A power cable consists of a cylindrical copper (Cu) wire surrounded by six cylindrical aluminium (Al) wires. All the wires are of the same cross-sectional area as shown:



The table gives the densities and resistivities of aluminium and copper.

<i>material</i>	<i>density</i>	<i>resistivity</i>
aluminium	$d$	$3\rho$
copper	$3d$	$2\rho$

The cable has mass  $M$  and length  $L$ .



Which expression gives the resistance between the two ends of the cable?

A  $\frac{18\rho dL^2}{5M}$

B  $\frac{21\rho dL^2}{M}$

C  $\frac{81\rho dL^2}{5M}$

D  $\frac{180\rho dL^2}{M}$

E  $\frac{12\rho dL^2}{5M}$

F  $\frac{28\rho dL^2}{3M}$

G  $\frac{36\rho dL^2}{5M}$

H  $\frac{80\rho dL^2}{M}$

ENGAA S2 2018 - Question 16 - Worked Solution

$$P = \frac{RA}{L} \quad (P = \text{respectively})$$

$$d = \frac{m}{V} \quad (d = \text{density})$$

$$V = A \times L$$

$$d = \frac{m}{AL} \rightarrow m = ALd$$

Now for whole cable:

$$M = 6 \times M_A + M_C$$

$$= 6 \times ALd + AL(3d)$$

$$= 9ALd$$

$$A = \frac{M}{9Ld}$$

$$R_C = \frac{(2\rho) \times L}{A}$$

$$= \frac{2\rho L}{M} \times 9Ld$$

$$= \frac{18\rho dL^2}{M}$$

$$R_a = \frac{(3\rho)L}{A}$$

$$= \frac{3\rho L}{M} \times 9Ld$$

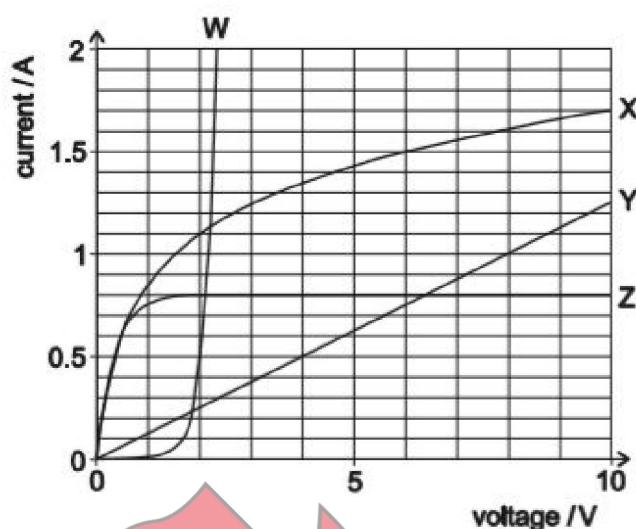
$$\begin{aligned}
 &= \frac{27\rho dL^2}{M} \\
 R_T &= \left( \frac{6}{R_a} + \frac{1}{R_c} \right)^{-1} \\
 &= \left( \frac{6M}{27\rho dL^2} + \frac{M}{18\rho dL^2} \right)^{-1} \\
 &= \frac{18\rho dL^2}{5M}
 \end{aligned}$$

Answer is A.



## ENGAA S2 2017 - Question 2

- 2 The graph shows the current against voltage characteristics of four different electronic devices W, X, Y and Z. One of the devices is an  $8\Omega$  resistor and one is a filament lamp rated  $9\text{ W}$  at  $6\text{ V}$ . You may assume that the filament lamp does not 'blow' in the context of this question.



- a) Which of the devices is the resistor?

- A device W
- B device X
- C device Y
- D device Z

## ENGAA S2 2017 - Question 2 - Worked Solution

a)

- Resistor follows Ohm's law with constant  $R$   
 $V = IR$
- So  $V \propto I$ , and so resistor has a straight line  $I - V$  graph, through the origin
- So device V is a resistor

Answer is C

b)

b) Which of the devices is the filament lamp?

- A device W
- B device X
- C device Y
- D device Z

- As the current through a lamp increases, the lamp heats up, and so its resistance increases.
- So as the instantaneous resistance of the lamp is  $R = V/I$ , the gradient of its  $I - V$  graph will decrease.
- This means x is the filament lamp.

Answer is B

c)

c) The filament lamp and the resistor are connected in parallel to a 6.0V power supply with negligible internal resistance.

Approximately what current is drawn from the supply?

[3 marks]

- A 0.75A
- B 1.5A
- C 1.83A
- D 2.25A
- E 2.42A

Draw circuit and label currents

- Currents are parallel so voltage across resistor and lamp is 6V
- From question, at 6V, power dissipated by lamp is  $P = 9W$
- $P = IV = 6I_2 = 9$

$$I_2 = \frac{3}{2} A$$

$$V = I_1 R$$

$$6 = I_1 \times 8$$

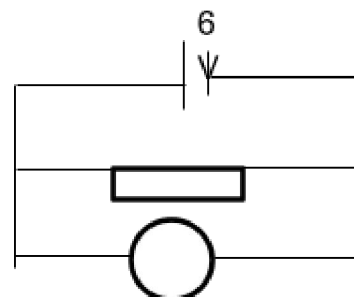
$$I_1 = \frac{3}{4} A$$

$$I = I_1 + I_2$$

$$I = \frac{3}{4} \times \frac{3}{2}$$

$$I = 2.25A$$

- For resistor



Answer is D

d)

- d) The previous circuit is disconnected, and then devices W and Y are connected in series to the same 6.0V power supply.

Which one of the following statements about the new circuit must be correct?

[1 mark]

- A Devices W and Y dissipate equal power.
- B Devices W and Y have equal voltages across them.
- C Equal currents flow through devices W and Y.
- D The power supply delivers more power than it would if device W or device Y were connected alone.
- E The power supply delivers more power than it would if devices W and Y were connected in parallel.

- C is correct as for a series circuit I is constant throughout.

Why other statements are false

- a.  $P = IV$ , I is the same for W and Y, but V is not as they don't have equal resistance.
- b. Again W and Y don't have equal resistance, so as p.d are shared in ratio of resistance, p.d across W and Y is not equal.
- c.
- d.  $P = IV = \frac{V^2}{R}$ , and if W and Y were connected alone, power supply would deliver more power.
- e. If connected in parallel, total R is lower, so similarly to d, power supply would deliver more power.

Answer is C

e)

- e) In the new circuit, approximately what power is dissipated by device W?

[3 marks]

- A 0.5W
- B 1.0W
- C 1.5W
- D 2.0W
- E 2.5W

- I is the same in W and Y
- Require  $V_W + V_Y = 6V$
- So need to use I – V graphs to find such an I such that this holds
- At  $I = 0.5A$



$$V_Y + V_W = 4 + 2 = 6V$$

$$P_W = I V_W$$

$$= 0.5 \times 2$$

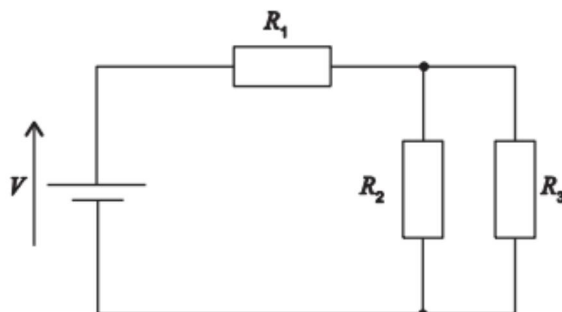
$$= 1.0W$$

Answer is C



### ENGAA S2 2016 - Question 2

- 2 The figure below shows a network of three non-zero resistances  $R_1$ ,  $R_2$ ,  $R_3$  connected to a voltage source  $V$  with zero internal resistance.



- a) Which of the following statements must be correct?

[1 mark]

**NO WORKING NEEDS TO BE GIVEN FOR THIS PART OF THIS QUESTION.**

- A The currents through resistances  $R_1$  and  $R_2$  are the same.
- B The currents through resistances  $R_1$  and  $R_3$  are the same.
- C The currents through resistances  $R_2$  and  $R_3$  are the same.
- D The voltages across resistances  $R_1$  and  $R_2$  are the same.
- E The voltages across resistances  $R_1$  and  $R_3$  are the same.
- F The voltages across resistances  $R_2$  and  $R_3$  are the same.

### ENGAA S2 2016 - Question 2 - Worked Solution

- a) by Kirchoff's voltage law

Answer is F

b)

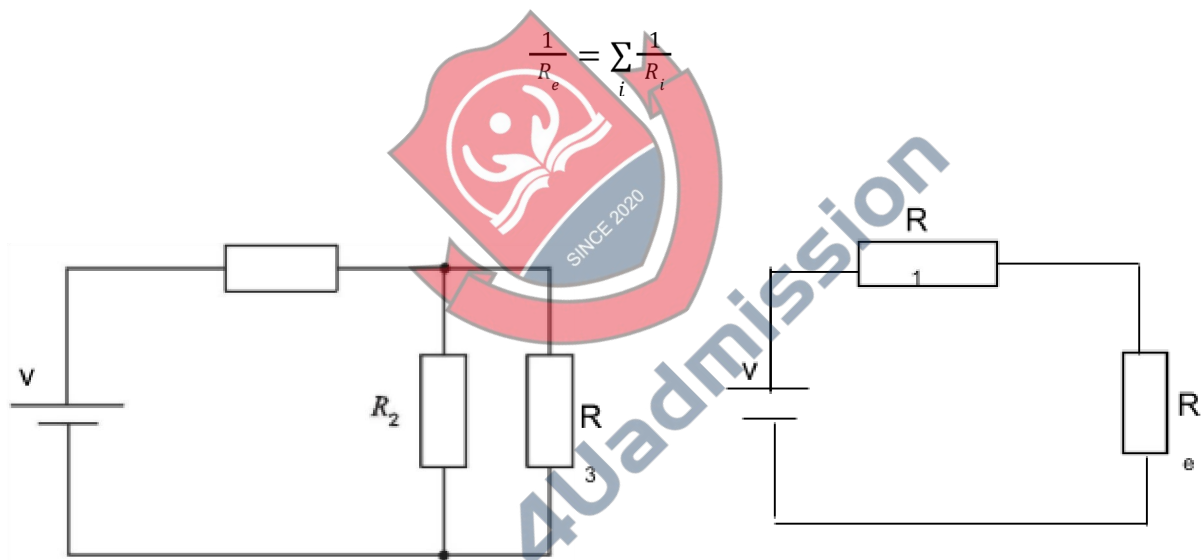
b) Which of the following expressions gives the current through the voltage source? [2 marks]

[2 marks]

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

- A  $\frac{V(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$
- B  $\frac{V(R_1 R_2 + R_1 R_3 + R_2 R_3)}{R_2 + R_3}$
- C  $\frac{V R_2 R_3}{R_2 + R_3 + R_1 R_2 R_3}$
- D  $\frac{V}{R_1 + R_2 + R_3}$
- E  $\frac{V(R_1 R_2 + R_1 R_3 + R_2 R_3)}{R_1 R_2 R_3}$

For resistance in parallel:



$$\frac{1}{R_e} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_e = \frac{R_2 R_3}{R_2 + R_3}$$

For resistance in series:

$$R_T = \sum_i R_i$$

The total resistance of the circuit

$$R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$\text{The current in the circuit} = \frac{V}{R_T}$$

The current is the same everywhere in a series circuit

$$I = \frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} = \frac{V(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Answer is A

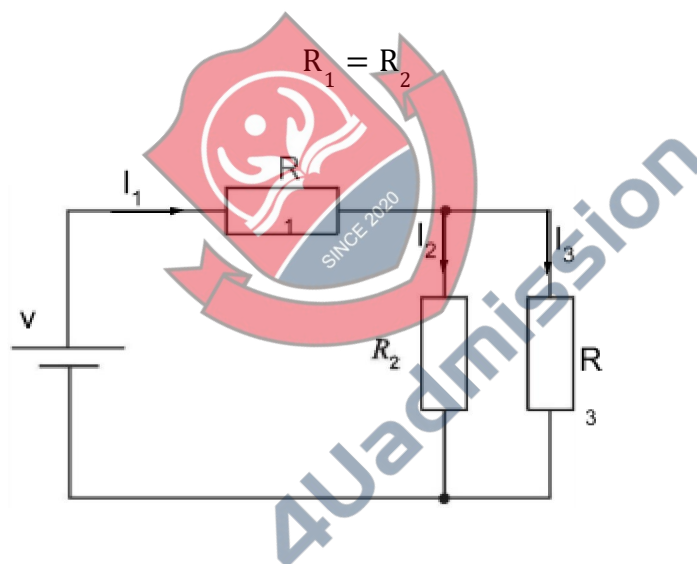


c)

- c)  $R_1$  and  $R_2$  are now fixed such that  $R_1 = R_2$ . Which of the following expressions gives the power  $P$  that is dissipated by resistance  $R_3$ ? [3 marks]

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

- A  $\frac{V^2 R_3^3}{(R_1 + 2R_3)^2}$   
 B  $\frac{V^2}{2R_1 + R_3}$   
 C  $\frac{V^2 (R_1 + 2R_3)^2}{R_3^3}$   
 D  $\frac{V^2 R_3}{(R_1 + 2R_3)^2}$   
 E  $\frac{V^2 (R_1 + R_3)^2}{(R_1 + 2R_3)^2 R_3}$



$$I_1 = I_2 + I_3 \quad \text{by kirchoff's current law}$$

$$I_1 R_1 + I_3 R_3 = V \quad \text{by kirchoff's voltage law}$$

$$I_1 = \frac{V(R_1 + R_3)}{R_1(R_1 + 2R_3)}$$

$$I_1 R_1 = \frac{V(R_1 + R_3)}{(R_1 + 2R_3)}$$

$$I_3 = \frac{V}{R_3} \left( 1 - \frac{R_1 + R_3}{R_1 + 2R_3} \right)$$

$$I_3 = \frac{V}{R_3} \left( \frac{R_1 + 2R_3 - R_1 - R_3}{R_1 + 2R_3} \right)$$

$$I_3 = \frac{V}{R_1 + 2R_3}$$
$$P = I_3^2 R_3 = \frac{V^2 R_3}{(R_1 + 2R_3)^2}$$

Answer is D



d)

- d) For the case where  $R_1 = R_2$ , which of the following values of  $R_3$  maximises its power dissipation? You may find it helpful to use the fact that any value of  $R_3$  that maximises  $P$  also minimises  $1/P$ . [3 marks]

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A  $R_3 = \frac{1}{4}R_1^2$

B  $R_3 = \frac{1}{2}R_1$

C  $R_3 = \exp\left(-\frac{4}{R_1^2}\right)$

D  $R_3 = \frac{1}{\sqrt{2}}R_1$

E  $R_3 = \frac{1}{4}R_1$

F Either  $R_3 = \frac{1}{2}R_1$  or  $R_3 = -\frac{1}{2}R_1$  would result in maximum power dissipation in  $R_3$ .

To find  $P_{max}$ , set  $\frac{dP}{dR_3}$  to 0

$$\frac{dP}{dR_3} = \frac{1}{(R_1 + 2R_3)^4} \left[ (R_1 + 2R_3)^2 - R_3 \times 4(R_1 + 2R_3) \right]$$

$$\frac{dP}{dR_3} = \frac{R_1 + 2R_3 - 4R_3}{(R_1 + 2R_3)^3}$$

$$= \frac{R_1 - 2R_3}{(R_1 + 2R_3)^3} = 0$$

$$R_1 = 2R_3$$

$$R_3 = \frac{R_1}{2}$$

Answer is B