# Worked Solutions for ENGAA Papers by Topic

## Section 2

## Topic: Mechanics

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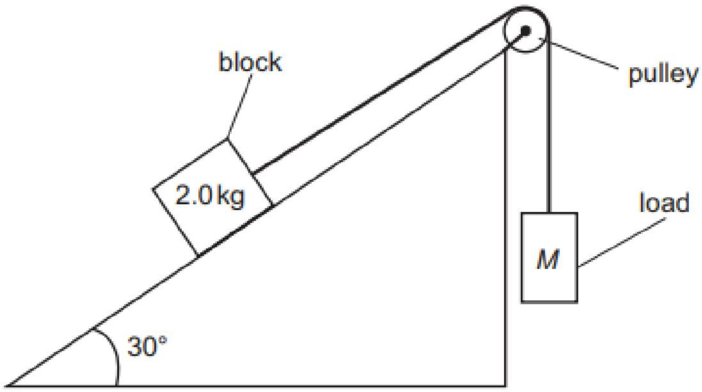
**ENGAA S2 2019 - Question 3**

**3** A block of mass 2.0 kg is on a plane which is inclined to the horizontal at an angle of 30°.

The block is attached to a load of mass $M$ by a light, inextensible string which passes over a smooth pulley.




The load moves downwards at a constant speed.

A constant friction force of 5.0 N acts on the block while it moves.

What is the value of $M$?

(gravitational field strength = $10\,\text{N kg}^{-1}$; assume that air resistance is negligible)

**A** 0.50 kg

**B** 1.0 kg

**C** 1.5 kg

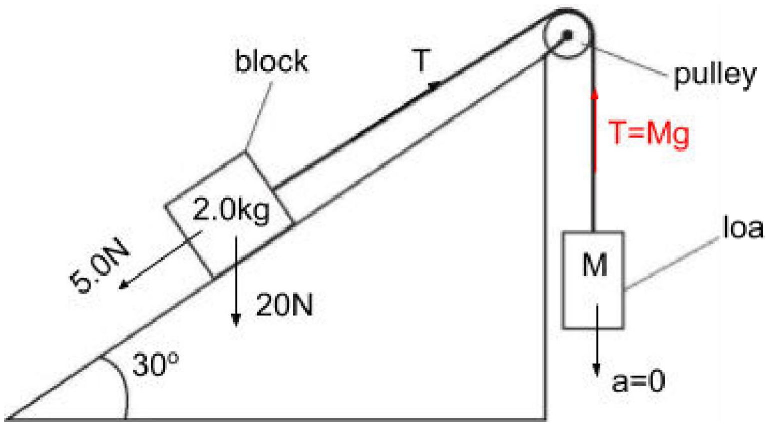
**D** 2.5 kg

**E** 4.0 kg

**F** 6.0 kg



**ENGAA S2 2019 - Question 3 - Worked Solution**




wait of $the$ block acting down slope:

$$w' = mg\ \sin\sin\ \theta$$

$$:T = mg\ \sin\sin\ \theta + 5N$$

$$Mg = 10N .\ \sin\sin\ 30\ o + 5$$

$$M = \frac{15}{10} = 1.5kg$$

Answer is C

**ENGAA S2 2019 - Question 10**

**10** A non-uniform beam PQ of length 5.0 m and weight $X$ rests on a pivot placed 3.0 m from end P. It is kept in equilibrium in a horizontal position by an upward force of magnitude $0.60X$ acting at end P.

The normal contact force at the pivot is 800 N.

What is the weight of the beam and how far is the centre of gravity of the beam from the pivot?

| | *weight of beam* / N | *distance from pivot* / m |
|---|---|---|
| **A** | 500 | 0.50 |
| **B** | 500 | 1.8 |
| **C** | 500 | 3.0 |
| **D** | 2000 | 0.50 |
| **E** | 2000 | 1.8 |
| **F** | 2000 | 3.0 |

**ENGAA S2 2019 - Question 7 - Worked Solution**

Let d be the distance to the Center of Mass:

Swimming torques about pivot.

$$d \cdot X = 3 \cdot (0.6X)$$

$$d = 1.8\text{m from pivot}$$

$$0.6X + 800 = X$$

$$X = 2000\text{N}$$

Answer is E

## ENGAA S2 2019 - Question 14

**14** A car of mass $m$ is pulling a caravan of mass $M$.

The caravan is connected to the car by a metal bar of length $l$ and cross-sectional area $A$.

The Young modulus of the metal from which the bar is made is $E$.

The car and caravan have a constant forward acceleration $a$ and there are total resistive forces $D_1$ acting on the car and $D_2$ acting on the caravan.

What is the extension of the bar?

(Assume that the bar obeys Hooke's law and that the cross-sectional area of the bar remains unchanged.)

**A** $\dfrac{Mal}{EA}$

**B** $\dfrac{Ma}{EA}$

**C** $\dfrac{(Ma + D_2)l}{EA}$

**D** $\dfrac{Ma + D_2}{EA}$

**E** $\dfrac{(Ma + ma + D_1 + D_2)l}{EA}$

**F** $\dfrac{Ma + ma + D_1 + D_2}{EA}$

## ENGAA S2 2019 – Question 14 - Worked Solution

By Newton's 2nd law on Caravan

$$T - D_2 = Ma$$

By definition of Young's Modulus

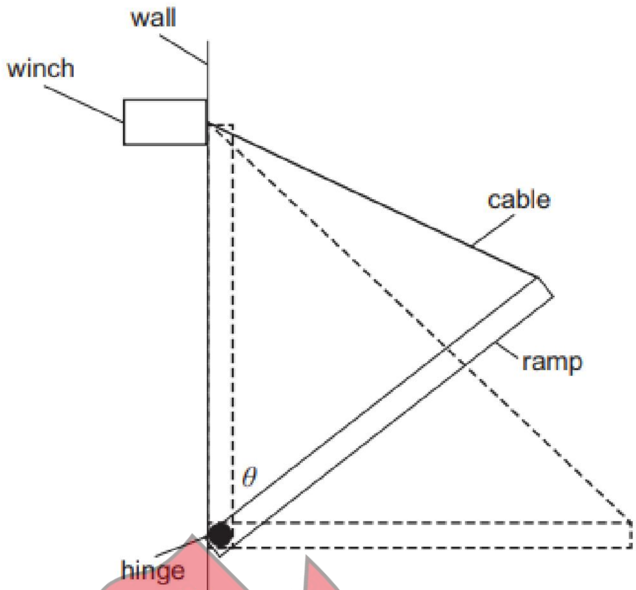
$$E = \frac{T \times l}{A \times x} \rightarrow \frac{EAx}{l} - D_2 = Ma$$

$$x = \frac{(Ma + D_2)l}{EA}$$

Answer is C

**ENGAA S2 2019 - Question 18**

**18** A drawbridge system consists of a uniform ramp, of weight $W$, that is smoothly hinged at its lower end. The upper end is connected by a light, inextensible cable to a winch that is fixed to the wall in the position shown in the diagram.




The ramp is lowered slowly, at constant speed, from its closed (vertical) position ($\theta = 0°$) to its open (horizontal) position ($\theta = 90°$).

What is the maximum tension in the cable during this process?

(double-angle identities: $\sin 2\theta = 2\sin\theta\cos\theta$; $\cos 2\theta = \cos^2\theta - \sin^2\theta$)

**A** $\dfrac{W}{2}$

**B** $\dfrac{W}{\sqrt{2}}$

**C** $\dfrac{\sqrt{3}W}{2}$

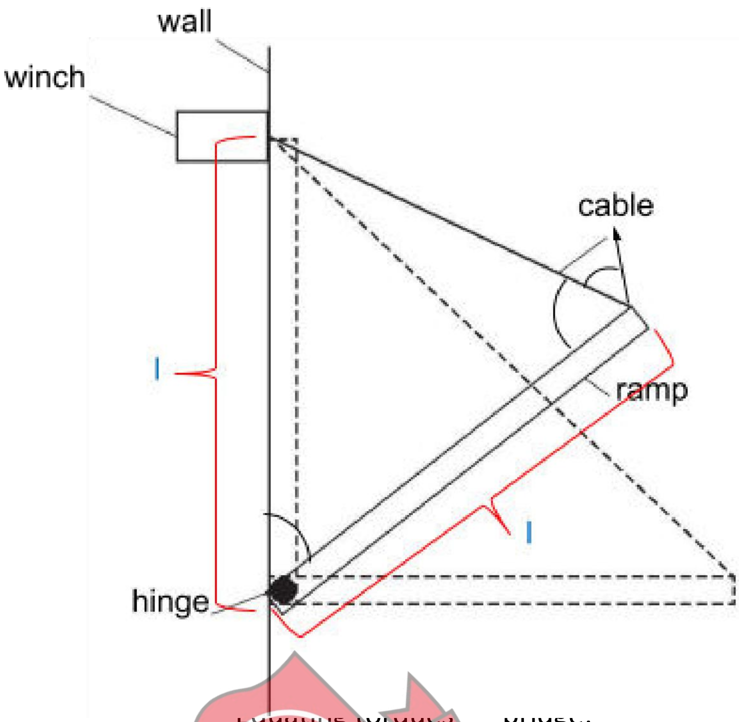
**D** $W$

**E** $\dfrac{2W}{\sqrt{3}}$

**F** $\sqrt{2}W$

**G** $2W$

**ENGAA S2 2019 - Question 18 - Worked Solution**




$$w \times l \sin \sin \theta \ = T \ \cos Cos \frac{\theta}{2} \times l$$

$$T \ = \ w \sin\sin \frac{\theta}{2}$$

Max value of T is at $\theta \ = \ 90o$

$$\therefore T \ = \frac{w}{\sqrt{2}}$$

under limits : $\theta \in [0 \ , \ 90]$

Answer is B

## ENGAA S2 2019 - Question 19

19 A particle of mass $m$ has kinetic energy $E$ when it collides with a stationary particle of mass $M$. The two particles coalesce.

Which of the following expressions gives the total kinetic energy transferred to other forms of energy in the collision?

**A** $0$

**B** $\dfrac{ME}{(M+m)}$

**C** $\dfrac{mE}{(M+m)}$

**D** $\dfrac{(M+m)E}{m}$

**E** $\dfrac{(M+m)E}{M}$

**F** $\dfrac{mME}{(M+m)^2}$

**G** $E$

## ENGAA S2 2019 - Question 19 - Worked Solution

KE of small particle:

$$E = \frac{1}{2}mv^2$$

$$u = \sqrt{\frac{2E}{m}}$$

Conservation of momentum

$$mu = (m + M)v$$

$$v = \frac{m}{m+M}u$$

Initial KE — Final KE

$$E_{lost} = \frac{1}{2}mv^2 - \frac{1}{2}(m + M)\left(\frac{m}{m+M}\right)^2$$

$$= E_0 - \frac{1}{2}\frac{m^2u^2}{m+M}$$

$$= E_0\left(1 - \frac{m}{m+M}\right)$$

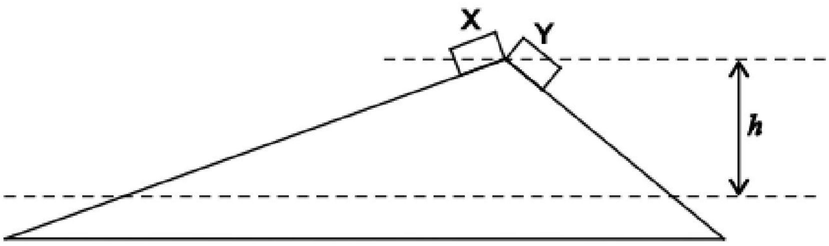
$$= E_0\frac{M}{m+M}$$

Answer is B

**ENGAA Specimen S2 - Question 2**

2 Two identical wooden blocks X and Y are released from rest from the top of two frictionless slopes. Block Y is on a steeper slope than block X.

The times taken for each block to fall through the same vertical distance $h$ are $t_X$ and $t_Y$ respectively, and at these times the blocks have speeds $v_X$ and $v_Y$.




Which of the following options gives the relationships between times and speeds for blocks X and Y?

**A** $t_X = t_Y$ and $v_X = v_Y$

**B** $t_X > t_Y$ and $v_X < v_Y$

**C** $t_X = t_Y$ and $v_X > v_Y$

**D** $t_X > t_Y$ and $v_X = v_Y$

**E** $t_X > t_Y$ and $v_X > v_Y$

**F** $t_X = t_Y$ and $v_X < v_Y$



**ENGAA Specimen S2 - Question 2 - Worked Solution**

The speed will be the same after a given height change because gravitational potential energy I turned into kinetic energy and there is no friction.
Block X takes longer as it moves a further distance

$$t_x > t_y \ , \ v_x = v_y$$

Answer is D

**ENGAA Specimen S2 - Question 3**

3 A space probe is travelling through the vacuum of deep space in a straight line at constant speed; its engine is switched off and there are no gravitational forces acting on it.

Fuel in the probe explodes and the probe splits into two sections. One section continues on at a speed greater than the speed of the probe before the explosion whilst the other section travels on more slowly than before.

Which statement is correct?

**A** Both the total kinetic energy and the total momentum after the explosion are the same as before.

**B** The total energy after the explosion is the same as before but the total kinetic energy has decreased.

**C** The total energy after the explosion is the same as before but the total momentum has increased.

**D** The total kinetic energy after the explosion is the same as before but the total momentum has decreased.

**E** The total kinetic energy after the explosion is the same as before but the total momentum has increased.

**F** The total momentum after the explosion is the same as before but the total kinetic energy has decreased.

**G** The total momentum after the explosion is the same as before but the total kinetic energy has increased.

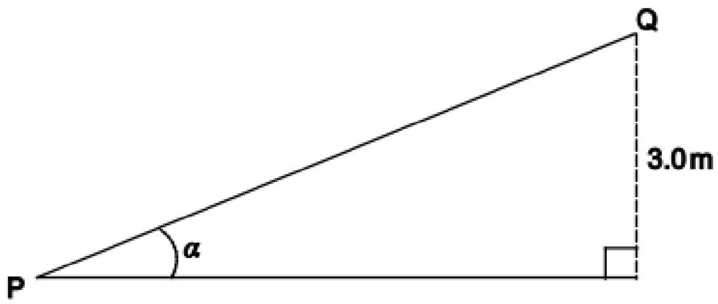
**ENGAA Specimen S2 - Question 3 - Worked Solution**

Energy from the explosion is transferred into kinetic energy as kinetic energy increases.
Momentum is conserved.
Answer is G

## ENGAA Specimen S2 - Question 4

**4** PQ is a rough plane which is inclined to the horizontal at an angle $\alpha$, where $\tan\alpha = \frac{3}{4}$

The point Q is a vertical distance of 3.0 m above the horizontal level of P.




A small object of mass 5.0 kg is projected from P along a line of greatest slope of the plane towards Q.

The object loses 210 J of kinetic energy as it travels from P to Q.

What is the friction force between the plane and the object as it travels from P to Q?

(gravitational field strength = 10 N kg$^{-1}$; air resistance can be ignored)

**A** 12 N

**B** 15 N

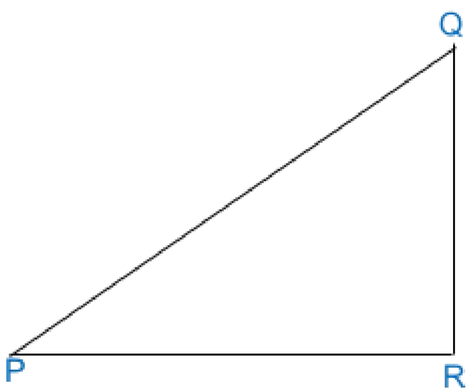
**C** 39 N

**D** 42 N

**E** 72 N



## ENGAA Specimen S2 - Question 4 - Worked Solution




$\tan \tan \alpha \ = \frac{3}{4}$

$QR \ = \ 3m$

$PR \ = \ 4m$

$PQ \ = \ 5m$

$energy \ = \ force \times distance$

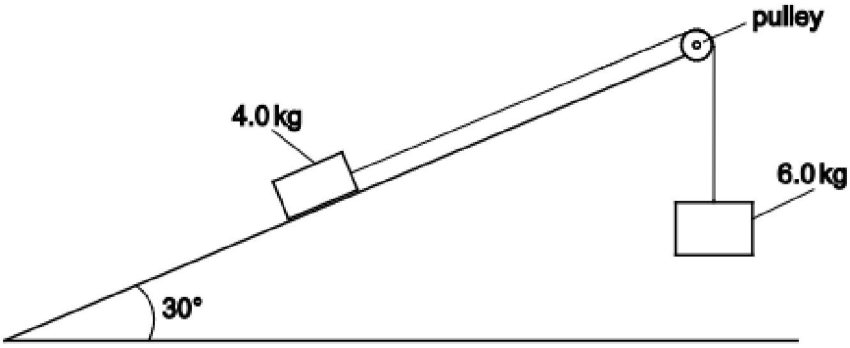
$210 \ = F_R \times 5$

$F_K \ = \ 42N$

Answer is D

**ENGAA Specimen S2 - Question 7**

7 A block of mass 4.0 kg is on a rough plane which is inclined at 30° to the horizontal. The block is attached to one end of a light inextensible string, the other end of which is attached to a block of mass 6.0 kg. The rope passes over a frictionless pulley of negligible mass at the top of the plane, and the 6.0 kg mass hangs vertically, as shown in the diagram.




The friction force acting on the 4.0 kg block is 15.0 N.

What is the acceleration of the 4.0 kg block?

**A** $0.50\,\text{m}\,\text{s}^{-2}$

**B** $2.5\,\text{m}\,\text{s}^{-2}$

**C** $4.5\,\text{m}\,\text{s}^{-2}$

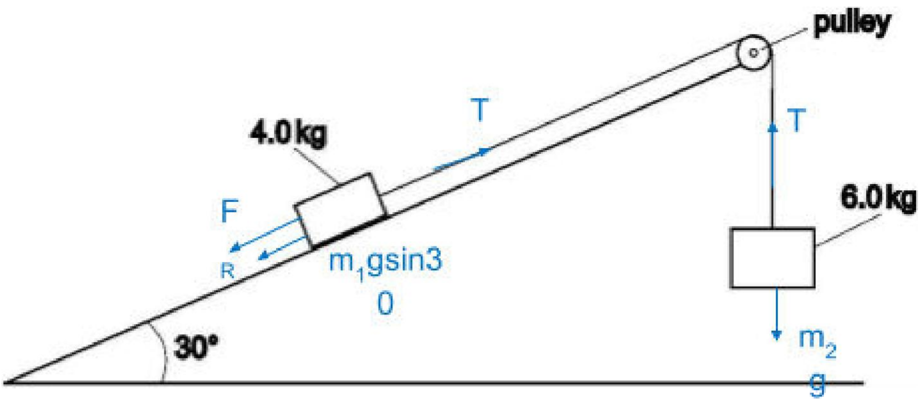
**D** $5.5\,\text{m}\,\text{s}^{-2}$

**E** $6.25\,\text{m}\,\text{s}^{-2}$

**F** $8.75\,\text{m}\,\text{s}^{-2}$



**ENGAA Specimen S2 - Question 7 - Worked Solution**




4. 0kg block:

$$T - F_R - m_1 g \,\text{sinsin}\, \theta = m_1 a$$

$$T - 15 - 4\times 10 \cup \frac{1}{2} = 4a$$

$$T - 35 = 4a \quad \text{———} \quad ①$$

6. 0kg block:

$$m_2 g - T = m_2 a$$

$$6\times 10 - T = 6a$$

$$60 - T = 6a \quad \text{———} \quad ②$$

$$① + ②$$

$$25 = 10a$$

$$a = 2.5$$

sub into ①

$$T - 35 = 10$$

$$T = 45\text{N}$$

Answer is B

**ENGAA Specimen S2 - Question 9**

9 The main part of a telescope is made from three uniform cylindrical tubes each 20 cm long, joined end to end.

The tubes have masses 0.40 kg, 0.60 kg and 1.0 kg respectively.

The telescope rests horizontally on a single pivot and is in equilibrium.

How far from the lighter end of the telescope is the pivot?

**A** 30 cm

**B** 34 cm

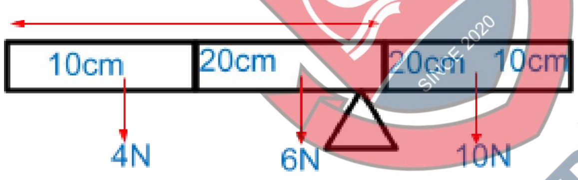
**C** 36 cm

**D** 40 cm

**E** 45 cm

**ENGAA Specimen S2 - Question 9 - Worked Solution**

Start with a diagram




Take moments about P

$4(x - 10) + 6(x - 30) = 10(50 - x)$

$4x - 40 + 6x - 60 = 500 - 10x$

$20x = 600$

$x = 30\text{cm}$

Answer is A

**ENGAA Specimen S2 - Question 12**

12 An object of mass $m$ moving through air experiences an air resistance (drag) force $F$ given by

$$F = kv^n$$

where $k$ and $n$ are positive constants.

The object is released from rest from a great height and falls vertically. No horizontal forces act on the object.

When it is travelling at a speed of $v_0$, its acceleration is 50% of the acceleration of free fall.

What is the terminal speed of the object?

**A** $2^n v_0$

**B** $2^{(1/n)} v_0$

**C** $2v_0$

**D** $2^{-n} v_0$

**E** $2^{-1} v_0$

**F** $2^{-(1/n)} v_0$

**ENGAA Specimen S2 - Question 12 - Worked Solution**

In freefall $mg = ma$ , $a = g$

at speed $v_o$, $mg - kv_o^n = \frac{mg}{2}$

$$\frac{mg}{2} = kv_o^n$$

At terminal speed $v_t$, $mg - kv_t^n = 0$

$$mg = kv_t^n = 2kv_o^n$$

$$v_t^n = 2v_o^n$$

$$v_t = 2^{\frac{1}{n}} v_o$$

Answer is B

## ENGAA Specimen S2 - Question 15

**15** Two smooth spheres of masses 3 kg and 1 kg are moving towards each other along a straight line. Their speeds are 2 m s$^{-1}$ and 6 m s$^{-1}$ respectively. The spheres collide and separate.

As a result of the collision their total kinetic energy decreases by 25%.

What is the speed of the 1 kg sphere after the collision?

**A** 3 m s$^{-1}$

**B** $3\sqrt{3}$ m s$^{-1}$

**C** $4\sqrt{3}$ m s$^{-1}$

**D** 4.5 m s$^{-1}$

**E** 6 m s$^{-1}$

## ENGAA Specimen S2 - Question 15 - Worked Solution

conservation of p : $1\times6 - 3\times2 = 3v_2 - 1v_1$

$0 = 3v_1 - v_1$

$v_1 = 3v_2$

Kinetic energy

$\frac{1}{2}\times1\times v_1^{\ 2} + \frac{1}{2}\times3\times v_2^{\ 2} = 0.75\left(\frac{1}{2}\times1\times6^2 + \frac{1}{2}\times3\times2^2\right)$

$\frac{1}{2}\times(3v_2)^2 + \frac{3}{2}v_2^{\ 2} = 18$

$6v_2^{\ 2} = 18$

$v_2 = \sqrt{3}$

$v_1 = 3\sqrt{3}\text{ms}^{-1}$

Answer is B

## ENGAA Specimen S2 - Question 19

**19** Water at the top of a waterfall has zero vertical velocity. The water falls 45 m vertically onto a flat horizontal rock of area $2.0\,m^2$.

Each second, 40 kg of water hits the rock.

When the water hits the rock it flows away horizontally.

At any instant the average depth of water on the rock surface is 0.050 m.

What is the total average pressure on the rock due to water?

(gravitational field strength = $10\,N\,kg^{-1}$; density of water = $1000\,kg\,m^{-3}$; assume air resistance is negligible)

**A** 400 Pa

**B** 500 Pa

**C** 600 Pa

**D** 1000 Pa

**E** 1100 Pa

**F** 1200 Pa

**G** 1700 Pa

**H** 2200 Pa

## ENGAA Specimen S2 - Question 19 - Worked Solution

When the water hits the rock it loses all vertical velocity instantly.
40kg of water hits the rock every second
The velocity of the water can be found using conservation of energy:

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{900} = 30ms^{-1}$$

The momentum of water transferred to the rock every second

$$= 40\times30 = \frac{1200Ns}{s} = 1200N$$

$$F = \frac{dp}{dt}$$

The force on the rock is 1200N from water hitting it

$$P_1 = 1200\div2 = 600Pa$$

There is also pressure from water sitting on the rock

$$P_2 = \rho gh$$

$$= 1000\times10\times0.050$$

$= 500\text{Pa}$

$P_T = P_1 + P_2 = 1100\text{Pa}$

Answer is E

**ENGAA S2 2018 - Question 4**

**4** The diagram shows two bar magnets, X and Y, held at rest a short distance apart from one another on a smooth horizontal surface. They are aligned as shown in the diagram and both are released at the same time. They move towards each other, collide and coalesce. Just before they collide X has a velocity $v$ to the right.


The mass of X is double the mass of Y.

Which row in the table gives the magnitude and direction of the velocity of the two magnets after the collision?

| | *magnitude of velocity* | *direction of velocity* |
|---|---|---|
| **A** | 0 | not applicable |
| **B** | $\frac{1}{3}v$ | to the left |
| **C** | $\frac{2}{3}v$ | to the left |
| **D** | $\frac{4}{3}v$ | to the left |
| **E** | $\frac{1}{3}v$ | to the right |
| **F** | $\frac{2}{3}v$ | to the right |
| **G** | $\frac{4}{3}v$ | to the right |



**ENGAA S2 2018 - Question 4 - Worked Solution**

Let Y has mass m, so X has mass 2m

Equal but opposite force acts on both magnets, so momentum is conserved, as no external forces act.

Conserve momentum:

$3mv_{final} = 2m\times0 + m\times0$

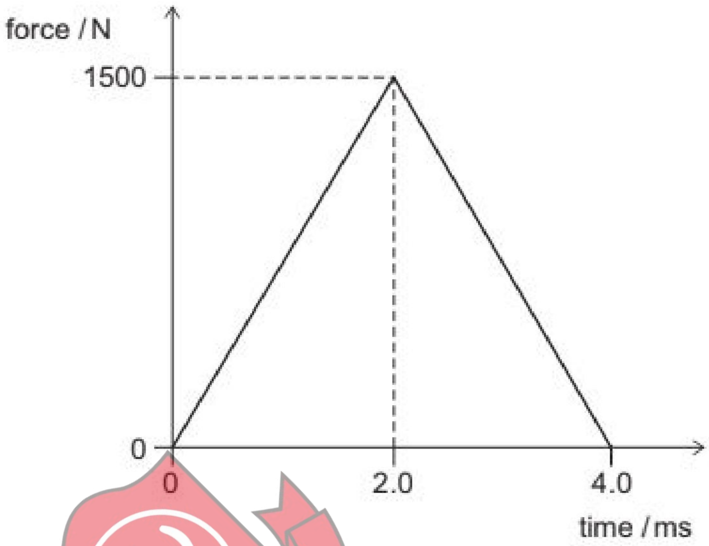
$V_{final} = 0$

Answer is A.

**ENGAA S2 2018 - Question 7**

**7** A tennis ball travelling at a speed of $30\,\text{m}\,\text{s}^{-1}$ hits a racket elastically with a kinetic energy of 27 J.

The racket applies a variable force to the tennis ball for a time of 4.0 ms as shown.




The ball moves away in the opposite direction to its initial motion.

With what speed does the ball leave the racket?

(Assume that resistive forces are negligible.)

**A** $20\,\text{m}\,\text{s}^{-1}$

**B** $28\,\text{m}\,\text{s}^{-1}$

**C** $32\,\text{m}\,\text{s}^{-1}$

**D** $50\,\text{m}\,\text{s}^{-1}$

**E** $70\,\text{m}\,\text{s}^{-1}$

**F** $80\,\text{m}\,\text{s}^{-1}$

**G** $100\,\text{m}\,\text{s}^{-1}$

**H** $130\,\text{m}\,\text{s}^{-1}$

**ENGAA S2 2018 - Question 7 - Worked Solution**

Find mass of ball:

$$\frac{1}{2}mv^2 = 27J$$

$$m(30)^2 = 54$$

$$m = 0.6kg$$

$$F = \frac{dP}{dE}$$

$$\Delta p = \int Fdt = \text{area under graph}$$

$$M(-V - U) = -\frac{1}{2}\times 4\times 10^{-3}\times 1500$$

$$0.06(-V - 30) = -3$$

$$V = -20ms^{-1}$$

Answer is A.

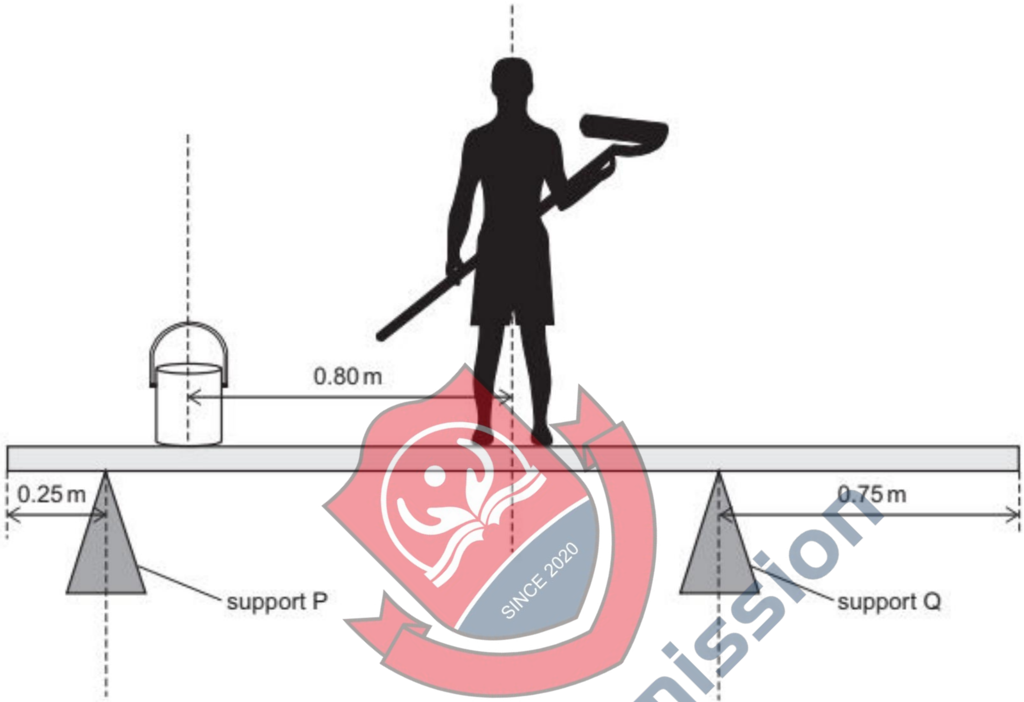
**ENGAA S2 2018 - Question 8**

8 A painter of mass 74 kg stands on a uniform wooden plank of length 2.5 m and of mass 24 kg.

The painter stands at the middle of the plank. The plank rests on two supports.

Support P is 0.25 m from one end of the plank and support Q is 0.75 m from the other end.

A pot of paint of mass 5.0 kg is 0.80 m from the centre of mass of the painter.




What is the force exerted by support Q on the plank?

(gravitational field strength = 10 N kg$^{-1}$)

**A** 50 N

**B** 51.5 N

**C** 66 N

**D** 177 N

**E** 500 N

**F** 515 N

**G** 660 N

**H** 1770 N

**ENGAA S2 2018 - Question 8 - Worked Solution**

Label forces

Plank is uniform so centre of mass is in the middle

Resolve moments about P:

$$[5g\times 0.2] + [24g\times(0.2 + 0.8)] + [74g\times(0.2 + 0.8)] = FQ\times(2.5 - 0.75 - 0.25)$$

$$99g = 1.5FQ$$

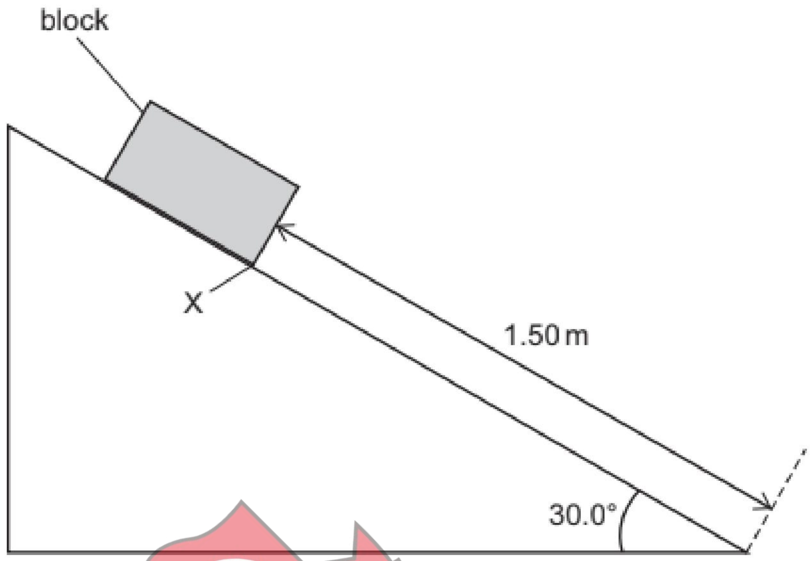
$$FQ = \frac{99\times 10}{1.5}$$

$$= 660\text{N}$$

Answer is G.

## ENGAA S2 2018 - Question 11

**11** A block of mass 3.60 kg is held stationary on a rough slope inclined at 30.0° to the horizontal. The edge X of the block is 1.50 m from the bottom of the slope.




The block is released and it accelerates uniformly down the slope. When X reaches the bottom of the slope, the speed of the block is 2.00 m s$^{-1}$.

What is the average rate at which work is done against resistive forces?

(gravitational field strength = 10 N kg$^{-1}$)

**A** 4.8 W

**B** 13.2 W

**C** 15.6 W

**D** 18.0 W

**E** 26.4 W

## ENGAA S2 2018 - Question 11 - Worked Solution

By conservation of energy:

$$\Delta EP = \Delta E_k + W_R$$

$$-\frac{1}{2} \times (3.6) \times 2^2 + mgh = W_R$$

$$-7.2 + (3.6) \times 10 \times 1.5 \sin\sin(30) = W_R$$

$$W_R = 19.85$$

Time taken to slide down:

$$S = \frac{1}{2} \times 2 \times t$$

$$t = 1.5s$$

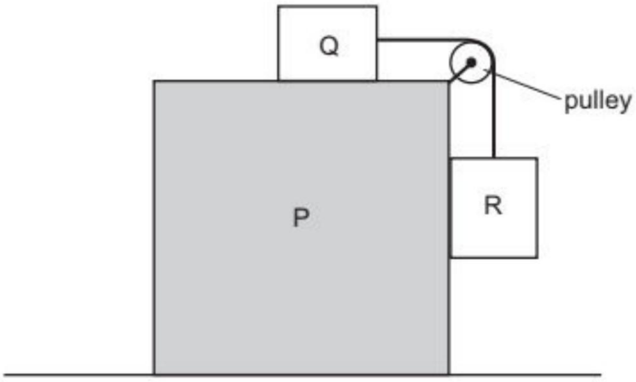
$$P_R = \frac{W_R}{t} = \frac{19.8}{1.5} = 13.2W$$

Answer is B.

**ENGAA S2 2018 - Question 14**

**14** A block P has a smaller block Q resting on its top surface.

Q is connected to a hanging block, R, by a light, inextensible string. The string passes over a smooth pulley which is connected to block P, as shown in the diagram.




The masses of blocks P, Q and R are $m_P$, $m_Q$ and $m_R$ respectively.

The surfaces of the three blocks are smooth.

P is accelerated horizontally to the right by an external force. While this is happening, Q and R do not move relative to P.

What is the acceleration of P?

(gravitational field strength = $g$)

**A** $\dfrac{m_Q g}{m_R}$

**B** $\dfrac{m_R g}{m_Q}$

**C** $\dfrac{m_R g}{m_R + m_Q}$

**D** $\dfrac{m_Q g}{(m_P + m_Q + m_R)}$

**E** $\dfrac{m_R g}{(m_P + m_Q + m_R)}$

**F** $\dfrac{(m_Q + m_R)g}{(m_P + m_Q + m_R)}$



**ENGAA S2 2018 - Question 14 - Worked Solution**

R doesn’t move w . r . t P, but P only moves horizontally, so R doesn’t move vertically in lab frame:

$$T = M_R g$$

$$M_Q a_Q = T$$

$$a_Q = \frac{M_R g}{M_Q}$$

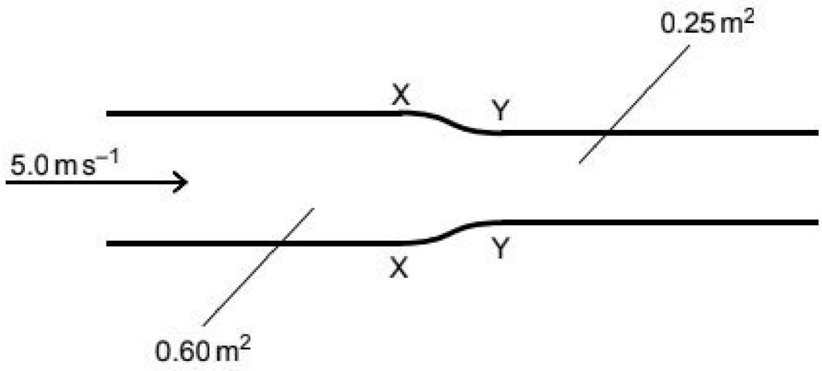
$a_Q = a_P$ as a doesn’t move relative to P.

$$a_P = \frac{M_R g}{M_Q}$$

Answer is B.

## ENGAA S2 2018 - Question 17

**17** Oil of density 800 kg m$^{-3}$ is pumped through a pipe of circular cross-sectional area 0.60 m$^2$ at a speed of 5.0 m s$^{-1}$. Between X and Y, the cross-sectional area of the pipe decreases to 0.25 m$^2$.




What is the resultant force exerted on the oil as it passes from X to Y?

**A** 0 N

**B** 7000 N

**C** 12 000 N

**D** 16 800 N

**E** 19 000 N

**F** 24 000 N

**G** 40 800 N

**H** 143 000 N



## ENGAA S2 2018 - Question 17 - Worked Solution

A soil is incompressible, flow rate at X = flow rate at Y
(Otherwise oil would build up between X and Y)

$$A_X V_X = A_Y V_Y$$

$$V_Y = \frac{A_X V_X}{A_Y}$$

$$= \frac{0.6 \times 5}{0.25}$$

$$= 12 \text{ ms}^{-1}$$

$$\Delta V = V_X - V_Y$$

$$= 7\ ms^{-1}$$

In a timest volume flows through X

$$V = \frac{M}{\rho}$$

$$\Delta t = \frac{V}{A_X V_X} = \frac{M}{\rho} \times \frac{1}{0.6\times 5} = \frac{M}{3\rho}$$

Then:

$$F = \frac{\Delta P}{\Delta t}$$

$$= M\Delta V \times \frac{3\rho}{M}$$

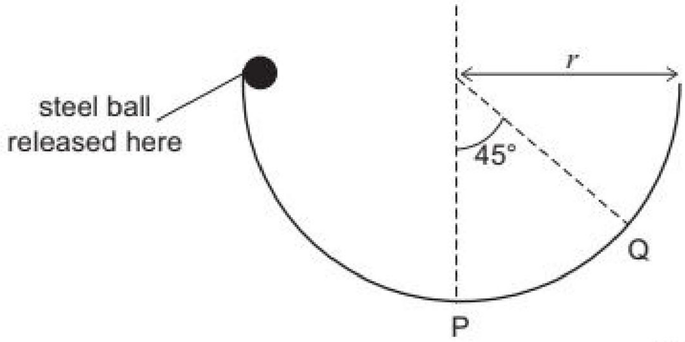
$$= 3\rho \times 7$$

$$= 21\rho = 21\times 800 = 16800\ N$$

Answer is D.

ENGAA S2 2018 - Question 18

**18** A small steel ball of mass $m$ is released from the top of a semi-circular ramp of radius $r$ as shown in the diagram:




[diagram not to scale]

After being released, the ball moves around the semi-circle to the lowest point at position P and then rises to a maximum height on the other side at position Q before falling down again.

Assume that the friction force acting on the ball has a constant magnitude whilst the ball is moving.

What is the kinetic energy of the ball as it first passes position P?

(gravitational field strength = $g$)

**A** $mgr(\sqrt{2}-1)$

**B** $mgr\left(1-\frac{\sqrt{2}}{3}\right)$

**C** $mgr\left(1-\frac{\sqrt{2}}{4}\right)$

**D** $\frac{2mgr}{3}$

**E** $\frac{3mgr}{4}$

**F** $mgr\left(\frac{1+\sqrt{2}}{3}\right)$

**G** $mgr$

**H** $mgr\left(2-\frac{\sqrt{2}}{2}\right)$



ENGAA S2 2018 - Question 18 - Worked Solution

F = friction
Distance moved:

$d_Q = r\theta$

$= \frac{3\pi}{4}r$

Initially:

E = 0

At Q:

$E_Q = mgr\cos Cos 45_\circ$

By conservation of energy:

$E - E_Q = FdQ$

$$mgr \cos cos\,(45) = \frac{3\pi}{4}rF$$

$$F = \frac{2\sqrt{2}}{3\pi}mg$$

At P:

$E_p = E_k - mgr$

$E - E_p = F\times dp$

$$mgr - E_k = \frac{\pi}{2}r\times F$$

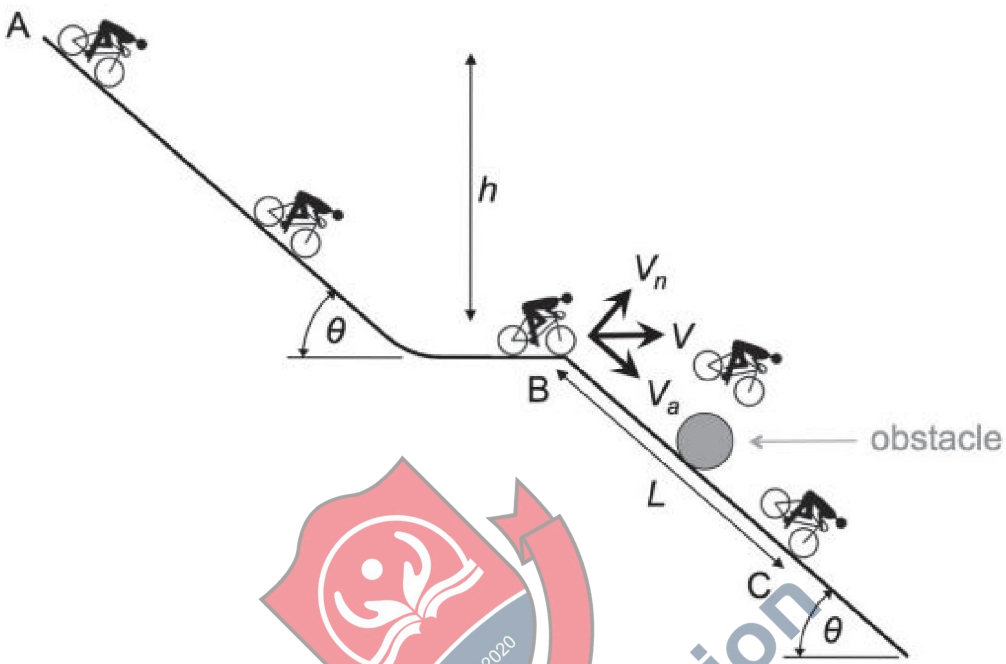
$$E_k = mgr - \frac{\pi}{2}r\times\frac{2\sqrt{2}}{3\pi}mg$$

$$= mgr\left[1 - \frac{\sqrt{2}}{3}\right]$$

Answer is B.

**ENGAA S2 2016 - Question 4**

**4** A stunt cyclist is preparing a new trick. The track on which he will perform the trick is shown schematically in the figure below. As shown, most of the track is sloped at an angle $\theta$ to the horizontal.




The cyclist starts riding from rest at A. In riding down the slope from A to B he transfers an amount of energy $E$ from his muscles to provide kinetic energy to the rider-bicycle system and descends through a vertical distance $h$. The cyclist leaves the track at B, travelling horizontally initially. He lands on the track at C, a distance $L$ down the slope.

Assume that the rider-bicycle system can be modeled as a point mass of mass $M$, that frictional forces and air resistance can be neglected, and that the gravitational field strength is $g$.

**a)** What is $V_a$, the component of the velocity of the rider-bicycle system along (parallel to) the slope, immediately after the cyclist has left the track at B? **[2 marks]**

**SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.**

**A** $\sqrt{\left[\frac{E}{M} + 2gh\right]} \cos\theta$

**B** $\sqrt{\frac{2}{M}[E + Mgh]} \sin\theta$

**C** $\sqrt{\frac{2}{M}[E + Mgh]} \frac{1}{\cos\theta}$

**D** $\sqrt{\left[\frac{E}{M} + 2gh\right]} \sin\theta$

**E** $\sqrt{2\left[\frac{E}{M} + gh\right]} \cos\theta$

**ENGAA S2 2016 - Question 4 - Worked Solution**

a) At B the cyclist has transferred an energy E and dropped a height h , therefore:

$$\frac{1}{2}MV^2 = Mgh + E$$

$$V^2 = 2gh + \frac{2E}{M}$$

$$V = \sqrt{2gh + \frac{2E}{M}}$$

At B all the V is horizontal due to the shape of the track.

$$:V_a = V \cos Cos\ \theta$$

$$V_a = \sqrt{2\left(\frac{E}{M} + gh\right)} \cos cos\ \theta$$

Answer is E

b)

**b)** The cyclist leaves the track at B at time $t = 0$ with an initial speed $V$. By considering motion parallel and/or perpendicular to the slope, or otherwise, find an expression for the time taken to land at C. **[2 marks]**

**SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.**

**A** $2\frac{V}{g}$

**B** $2\frac{V}{g}\tan\theta$

**C** $\frac{V}{g}\sin\theta$

**D** $\frac{V}{g}\tan\theta$

**E** $2\frac{V}{g}\sin\theta$

The horizontal speed is constant at V

The horizontal distance from B to C = L cos Cos θ

$$time = \frac{L coscos\ \theta}{V} \quad ------ \ (1)$$

Consider motion perpendicular to the slope

$$s = 0$$

$$u = V_n = V \sin \sin \theta$$

$$v = ?$$

$$a = -g \cos \text{Cos}\ \theta$$

$$t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$0 = Vt \sin \sin \theta - \frac{1}{2} g \cos \text{Cos}\ \theta\, t^2$$

$$t = \frac{2V sinsin\ \theta}{g coscos\ \theta} = \frac{2V tantan\ \theta}{g} \quad ------ \ (2)$$

Answer is B

c)

**c)** How far along the slope will the cyclist land, i.e. what is the value of $L$? **[3 marks]**

**SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.**

**A** $2\frac{V^2}{g}\sin\theta$

**B** $\frac{V^2}{2g}$

**C** $2\frac{V^2}{g}\frac{\sin\theta}{\cos^2\theta}$

**D** $2\frac{V^2}{g}\frac{\cos\theta}{\sin^2\theta}$

**E** $2\frac{V^2}{g}[\sin\theta + \tan^2\theta]$

From ① & ②

$$\frac{2V\tan tan\,\theta}{g} = \frac{L\cos cos\,\theta}{V}$$

$$L = \frac{2V^2}{g}\frac{\sin sin\,\theta}{cos^2\theta}$$

Answer is C

d)

**d)** As part of the trick, the cyclist wants to clear an obstacle placed on the slope between B and C. To give the cyclist the greatest chance of clearing the obstacle it should be placed at the point at which the cyclist's perpendicular distance from the track is greatest. At what distance from B should the obstacle be placed? **[3 marks]**

**SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.**

**A** $\frac{1}{2}L$

**B** $\frac{V^2 \sin^2\theta}{2g\cos\theta}$

**C** $\frac{1}{2}h$

**D** $\frac{V^2}{g}\sin\theta\left[1 + \frac{1}{2}\tan^2\theta\right]$

**E** $\frac{V^2}{2g}$

Distance perpendicular to the slope

$$S_n = Vt\sin sin\,\theta - \frac{1}{2}gt^2\cos cos\,\theta$$

$$to\ find\ S_{max},\ set\ \frac{dS_n}{dt}\ to\ 0$$

$$\frac{dS_n}{dt} = V\sin sin\,\theta - gt\cos cos\,\theta = 0$$

$$\text{gt cos Cos}\ \theta = \text{V sin sin}\ \theta$$

$$t = \frac{V}{g}\tan tan\,\theta$$

Distance parallel to the slope:

$$S_a = ut + \frac{1}{2}at^2$$

$$S = Vt\cos cos\,\theta + \frac{1}{2}gt^2\sin sin\,\theta$$

$$t = \frac{V}{g}\tan tan\,\theta$$

$$S_a = \frac{V^2}{g}\text{tan tan}\ \theta\ \text{cos Cos}\ \theta + \frac{1}{2}\text{g sin sin}\ \theta\ \frac{V^2 tan^2\theta}{g^2}$$

$$S_a = \frac{V^2 \sin sin\,\theta}{g} + \frac{1}{2}\frac{V^2}{g}\sin sin\,\theta\ tan^2\theta$$

$$S_a = \frac{V^2}{g}\sin sin\,\theta\left(1 + \frac{1}{2}tan^2\theta\right)$$

Answer is D