Worked Solutions for ENGAA Papers by Topic

Section 2

Topic: Waves

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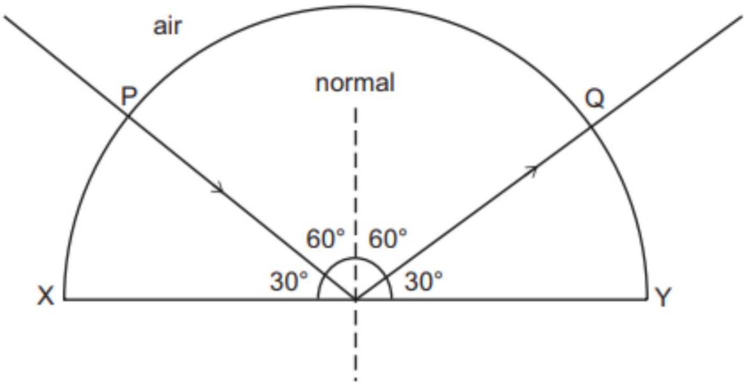
ENGAA S2 2020 - Question 3

3 A ray of light is directed into a semicircular transparent block, entering at P. The direction of the ray is adjusted until it strikes the centre of the flat face XY of the block at the critical angle and reflects to Q as shown.



The length of XY is L.

The speed of light in air is c.

What is the time taken by the light to travel from P to Q in the block?

A $\frac{L\sqrt{3}}{2c}$

B $\frac{L}{c}$

C $\frac{2L}{c\sqrt{3}}$

D $\frac{L\sqrt{3}}{c}$

E $\frac{2L}{c}$

F $\frac{4L}{c\sqrt{3}}$

ENGAA S2 2020 - Question 3 - Worked Solution

$$V = C\,.\,\sin\sin 60 = \frac{\sqrt{3}}{2}C$$

$$d = R + R = L$$

$$t = \frac{d}{s} = \frac{2L}{\sqrt{3}C}$$

Answer is C

ENGAA S2 2020 - Question 6

6 Diagram 1 shows the positions of nine equally spaced particles in a medium.

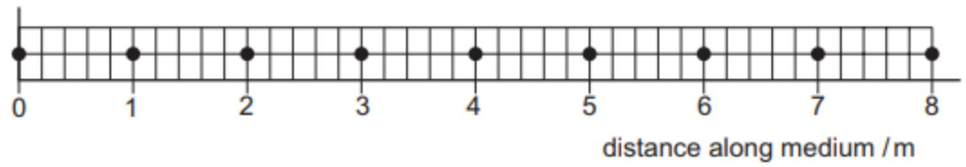


Diagram 1

Diagram 2 shows the positions of the same nine particles, at a particular time, while a longitudinal wave is travelling through the medium.

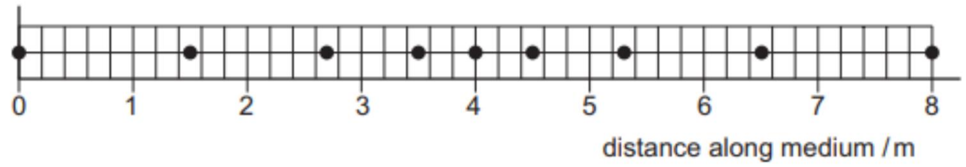


Diagram 2

What is the amplitude of the wave?

A 0.4 m

B 0.5 m

C 0.6 m

D 0.7 m

E 2.0 m

F 4.0 m

G 6.0 m

H 8.0 m

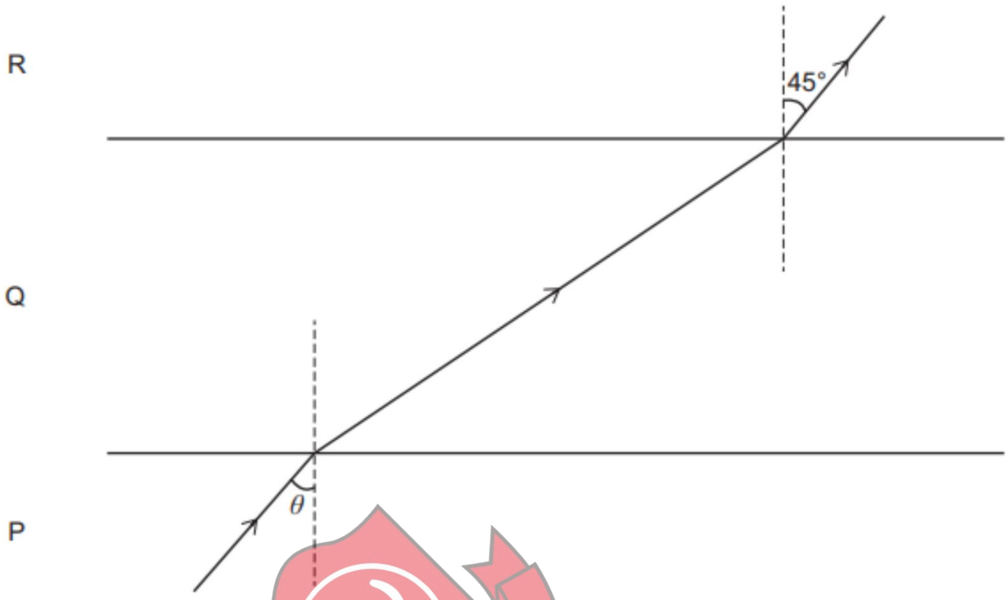
ENGAA S2 2020 - Question 6 - Worked Solution

Maximum displacement I 0.7m

Answer is D

ENGAA S2 2020 - Question 16

16 The diagram shows a ray of light passing through three mediums, P, Q and R. The boundaries between the three mediums are parallel.



[diagram not to scale]

The ratio of the speed of light in medium P to the speed of light in medium Q is $2 : \sqrt{5}$

The ratio of the speed of light in medium Q to the speed of light in medium R is $3 : \sqrt{6}$

What is the value of $\sin\theta$?

A $\frac{\sqrt{2}}{2}$

B $\frac{\sqrt{3}}{2}$

C $\frac{\sqrt{3}}{6}$

D $\frac{\sqrt{5}}{5}$

E $\frac{\sqrt{15}}{5}$

F $\frac{\sqrt{15}}{6}$

ENGAA S2 2020 - Question 16 - Worked Solution

$$6 \sin \sin \theta = 3 \sin \sin 45$$

$$\sqrt{5}\sin \sin \theta \ = \ 2 \sin \sin \theta$$

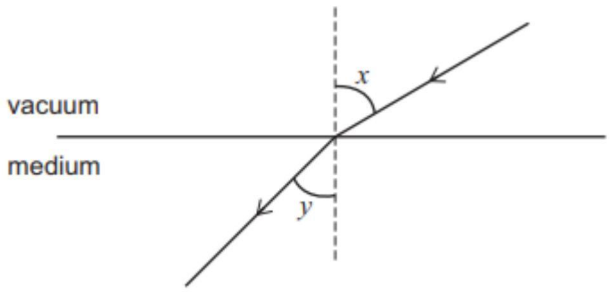
$$\sin \sin \theta \ \ = \frac{2}{\sqrt{5}} \cdot \frac{3}{\sqrt{6}} \ . \sin \sin 45$$

$$\sin sin\,\theta \ \ = \frac{\sqrt{15}}{5}$$

Answer is E

ENGAA S2 2019 - Question 1

1 The ray diagram shows light passing from a vacuum into a medium.



Two angles, x and y, are shown on the diagram.

When x is 60°, y is 45°.

When x is 45°, what is the value of $\sin y$?

A $\frac{1}{\sqrt{3}}$

B $\frac{2}{\sqrt{3}}$

C 1

D $\frac{\sqrt{3}}{2}$

E $\sqrt{3}$

ENGAA S2 2019 - Question 1 - Worked Solution

$$\sin \sin 45 = \frac{\sqrt{2}}{2}$$

$$\sin \sin 60 = \frac{\sqrt{3}}{2}$$

$$n \sin \sin x = \sin \sin y$$

$$n \,.\, \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{2}$$

$$n = \sqrt{\frac{2}{3}}$$

$$n.\sin \sin 45 = \sin \sin y$$

$$\sin \sin y = \frac{\sqrt{2}}{2} \,.\, \frac{\sqrt{2}}{3}$$

$$\sin \sin y = \frac{1}{\sqrt{3}}$$

Answer is A

ENGAA S2 2019 - Question 5

5 A stationary wave is set up in a medium in which the speed of the wave is $3.2\,\mathrm{m\,s^{-1}}$.

The stationary wave is formed by the superposition of two longitudinal waves, each of amplitude 1.5 cm, travelling in opposite directions.

The distance between adjacent nodes in the stationary wave is 4.0 cm.

What is the total distance travelled by a particle at an antinode during a time interval of 1.0 minute?

A 0 m

B 72 m

C 144 m

D 192 m

E 288 m

F 576 m

ENGAA S2 2019 - Question 5 - Worked Solution

$f = \frac{3.2ms}{0.08m} = \frac{v}{\lambda}$

$\mathrm{f} = 40\mathrm{Hz}$

in one minute : $40\mathrm{Hz} \times 60\mathrm{s} = 2400$ waves permin

Amplitude at antinode , $where$ opposite waves superpose:

$\mathrm{A} = 1.5\mathrm{cm} \times 2 = 3\mathrm{cm}$

If the particle experiences one full wave it does 4 amplitue distance

$\mathrm{d} = 4\mathrm{A}$

Hence:

$\mathrm{d}_0 = 3\mathrm{cm} \times 4 \times 2400\ \mathrm{min}^{-1}$

$= 28800\ \mathrm{cm}$

$= 288\ \mathrm{m}$

Answer is E

ENGAA S2 2019 - Question 6

6 A ray of light of single frequency f is travelling in a block of transparent material.

The ray strikes the boundary between the block and air at an angle θ **to the boundary**.

When $\theta = 65°$ the ray is just at the threshold of being totally internally reflected.

Which of the following is an expression for the wavelength of the light in the material?

(The speed of light in air is v_{air}.)

A $\dfrac{v_{\text{air}} \times \cos 65°}{f}$

B $\dfrac{v_{\text{air}} \times \sin 65°}{f}$

C $\dfrac{f}{v_{\text{air}} \times \cos 65°}$

D $\dfrac{f}{v_{\text{air}} \times \sin 65°}$

E $\dfrac{v_{\text{air}}}{f \times \cos 65°}$

F $\dfrac{v_{\text{air}}}{f \times \sin 65°}$

G $\dfrac{f \times \cos 65°}{v_{\text{air}}}$

H $\dfrac{f \times \sin 65°}{v_{\text{air}}}$

ENGAA S2 2019 - Question 6 - Worked Solution

$$\varphi = 90 - \theta$$

$$\frac{\text{sinsin } 90}{\text{sinsin } \varphi} = \frac{n_m}{n_{air}} = \frac{v_{air}}{v_{mat}} \quad \text{by defination of optiCal index}$$

$$v_{mat} = v_{air} - \sin \sin \varphi$$

$$\sin \sin \varphi = \sin \sin (90 - \theta) = \cos \text{Cos } \theta$$

$$v_{mat} = v_{air} \,.\, \cos \text{Cos } 65$$

$$\lambda_{mat} = \frac{v_{air} \cos\text{Cos } 65}{f}$$

Answer is A

ENGAA S2 2019 - Question 20

20 The critical angle for light incident on a boundary from medium X to air is 45°.

The critical angle for light of the same frequency incident on a boundary from medium Y to air is 60°.

There is a boundary between medium X and medium Y. Light of the same frequency travelling in one of these mediums is incident on this boundary.

In which direction of incidence is there a critical angle at this boundary, and within what range is this critical angle?

	direction of incidence	*critical angle*
A	X to Y	between 0° and 30°
B	X to Y	between 30° and 45°
C	X to Y	between 45° and 60°
D	X to Y	between 60° and 90°
E	Y to X	between 0° and 30°
F	Y to X	between 30° and 45°
G	Y to X	between 45° and 60°
H	Y to X	between 60° and 90°

ENGAA S2 2019 - Question 20 - Worked Solution

$$n_x = \frac{1}{\sin\sin 45^o}$$

$$n_y = \frac{1}{\sin\sin 60^o}$$

$$n_y < n_x$$

critical angel is from slower optical medium to higher so,

$$x \rightarrow y$$

$$n_x \sin\sin \theta_c = n_y \sin\sin 90^o$$

$$\sin sin\, \theta_c = \frac{n_y}{n_x} = \frac{\sin sin\, 45°}{\sin sin\, 60°} = \frac{\sqrt{2}/2}{\sqrt{3}/2} = \frac{\sqrt{2}}{\sqrt{3}} \cong 0.8$$

$$\sin sin\, 45 = \frac{\sqrt{2}}{2} \cong 0.7$$

$$\sin\sin 60 = \frac{\sqrt{3}}{2} \approx 0.87$$

$$\theta_c \text{ in between } 45^o \ \& \ 60^o$$

Answer is C

ENGAA Specimen S2 - Question 1

1 A wave of single frequency is travelling through a medium at a speed of $60\,cm\,s^{-1}$.

Each of the oscillating particles in the medium takes 0.20 seconds to move from its equilibrium position to its next maximum displacement.

What is the wavelength of the wave?

A 12 cm

B 24 cm

C 48 cm

D 75 cm

E 150 cm

F 300 cm

ENGAA Specimen S2 - Question 1 - Worked Solution

$$T = 4 \times 0.20S$$

$$T = 0.80S$$

$$f = \frac{1}{T} = 5Hz$$

$$v = f\lambda$$

$$\lambda = \frac{v}{f} = \frac{60}{5} = 12cm$$

Answer is A

ENGAA Specimen S2 - Question 11

11 A seismic wave causes the surface of the Earth to vibrate. The vibration at a building some distance from the epicentre of the earthquake has a period of 2.0 s.

A second building is 1.0 km farther from the epicentre. The vibration at the second building is $\pi/3$ radians out of phase with that at the first building.

What is the speed of the wave?

(Assume that the wavelength is greater than the separation of the buildings.)

A $\frac{1.5}{\pi}$ km s^{-1}

B $\frac{3.0}{\pi}$ km s^{-1}

C 1.5 km s^{-1}

D $\frac{6.0}{\pi}$ km s^{-1}

E 3.0 km s^{-1}

F $\frac{12.0}{\pi}$ km s^{-1}

G 6.0 km s^{-1}

H 12 km s^{-1}

ENGAA Specimen S2 - Question 11 - Worked Solution

$$1.0km = \frac{\pi/3}{2\pi}\lambda = \frac{\lambda}{6}$$

$$\lambda = 6.0\text{km}$$

$$f = \frac{1}{T} = \frac{1}{2.0} = 0.5Hz$$

$$v = f\lambda$$

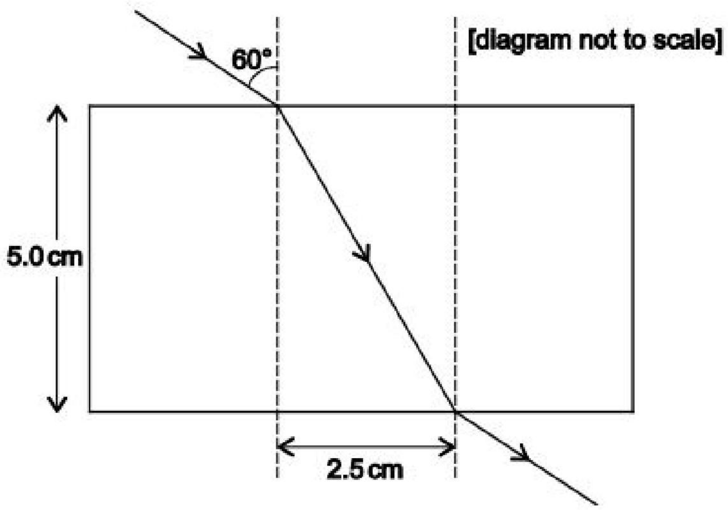
$$v = 0.5 \times 6$$

$$v = 3\text{kms}^{-1}$$

Answer is E

ENGAA Specimen S2 - Question 14

14 A ray of light in air strikes the surface of a rectangular transparent block at an angle of 60° to the normal. The ray passes through the block and exits from the far side as shown. The width of the block is 5.0 cm and the distance between the normal at the point of entry to the block and the normal at the point of exit from the block is 2.5 cm.



What is the refractive index of the block?

A $\frac{2}{\sqrt{15}}$

B $\frac{1}{\sqrt{3}}$

C $\frac{1}{\sqrt{2}}$

D $\frac{2}{\sqrt{5}}$

E $\frac{\sqrt{5}}{2}$

F $\sqrt{2}$

G $\sqrt{3}$

H $\frac{\sqrt{15}}{2}$

ENGAA Specimen S2 - Question 14 - Worked Solution

$$\tan tan\,\theta\ = \frac{2.5}{5} = \frac{1}{2}$$

$$\sin sin\,\theta\ = \frac{1}{\sqrt{5}}$$

$$n_1 \sin \sin\,\theta_1\ = n_2 \sin \sin\,\theta_2$$

$$\sin sin\,60\ = n\times\frac{1}{\sqrt{5}}$$

$$n\ = \sin sin\,60\times\sqrt{5} = \frac{\sqrt{15}}{2}$$

Answer is H

ENGAA S2 2018 - Question 6

6 Diagram 1 represents a stationary wave produced by sound in an open-ended tube of length 0.50 m containing a liquid. The speed of the wave in the liquid is $1000\,\text{m}\,\text{s}^{-1}$.

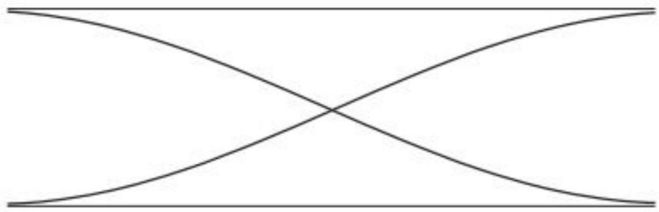


diagram 1

Diagram 2 is a displacement–time graph representing a progressive sound wave with the same frequency in the same liquid.

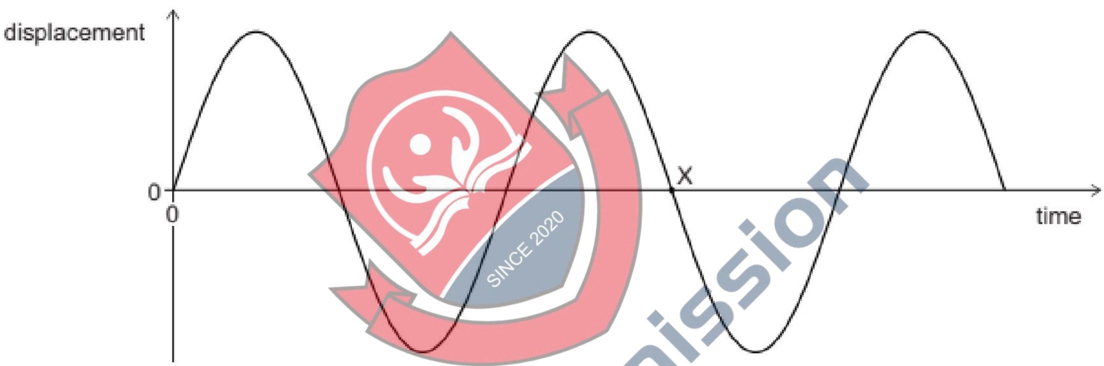


diagram 2

What is the value of the time at point X on the graph?

(End effects of the stationary wave can be ignored.)

A 2.5×10^{-4} s

B 3.75×10^{-4} s

C 5.0×10^{-4} s

D 7.5×10^{-4} s

E 1.0×10^{-3} s

F 1.5×10^{-3} s

ENGAA S2 2018 - Question 6 - Worked Solution

Shows half a cycle of the wave, so

$$\frac{\lambda}{2} = 0.5m$$

$$\lambda = 1\text{m}$$

Wave equation:

$$V = \lambda f$$

$$f = \frac{V}{\lambda} = \frac{1000}{1} = 1000Hz$$

$$T = \frac{1}{f} = 1\times10^{-3}$$

$$\text{X iS at} \quad 1.5T \rightarrow t = 1.5\times10^{-3}$$

$$= 1.5\times10^{-3}S$$

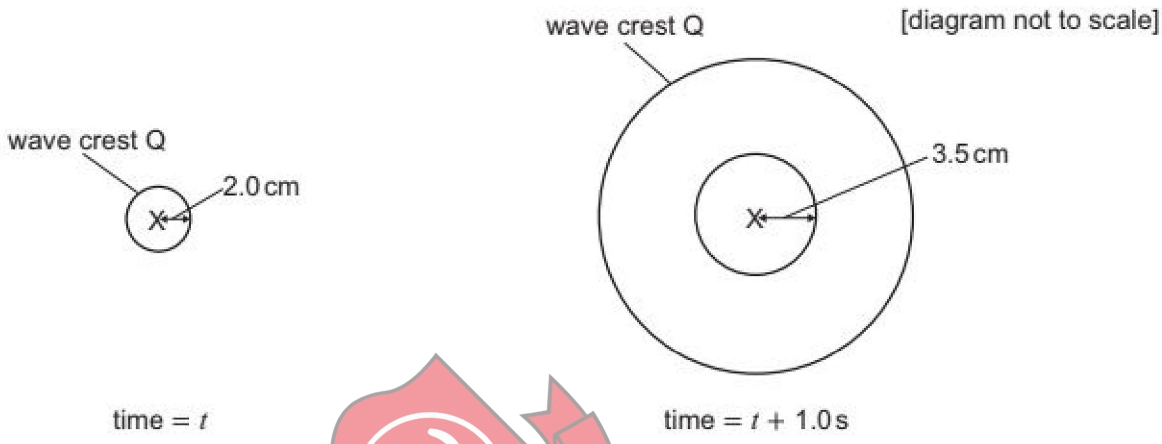
Answer is F.

ENGAA S2 2018 - Question 13

13 A stick at position X dips into water every 0.80 s, creating a circular wave which travels at a constant speed.

The diagrams show the wave crests at a time t and 1.0 s later. One of the wave crests, labelled Q, appears in both diagrams.

In each diagram, the distance from X to a wave crest is labelled.



What is the wavelength of the wave?

A 1.2 cm

B 1.5 cm

C 2.0 cm

D 3.5 cm

E 4.4 cm

F 6.0 cm

G 7.5 cm

H 14 cm

ENGAA S2 2018 - Question 13 - Worked Solution

Period of wave:

T = 0.8s

$$f = \frac{1}{T} = 1.25\ Hz$$

Speed of wave:

$$V = \lambda t$$

$$= 1.25\lambda$$

Also:

$$V = \frac{2}{t} = 1.25\lambda$$

$$t = \frac{8}{5\lambda} \quad ①$$

But also:

$$V = \frac{3.5+\lambda}{t+1} = \frac{2}{t} \quad ②$$

①→②

$$3.5 + \lambda = \frac{5\lambda}{8}\left(1 + \frac{8}{5\lambda}\right)\times 2$$

$$3.5 + \lambda = \frac{5\lambda}{4} + 2$$

$$1.5 = \frac{\lambda}{4}$$

$$\lambda = 6\ \text{cm}$$

Answer is F.

ENGAA S2 2017 - Question 4

a) The pair of slits is illuminated by laser light of wavelength $\lambda = 600\,\text{nm}$.

Which of the following statements are correct (where n is an integer)? **[2 marks]**

1 Points of maximum brightness on the screen occur where the distances r_1 and r_2 differ by nλ.

2 Points of maximum brightness on the screen occur where the distances r_1 and r_2 differ by $\left(n+\frac{1}{2}\right)\lambda$.

3 Points of minimum brightness on the screen occur where the distances r_1 and r_2 differ by $\left(n+\frac{1}{2}\right)\lambda$.

4 For a diffraction pattern to appear, the light from the two slits must be coherent.

5 The maxima are all of equal brightness.

A 1 and 4 only

B 1, 3 and 4 only

C 1, 3 and 5 only

D 1, 4 and 5 only

E 2 and 4 only

ENGAA S2 2017 - Question 4 - Worked Solution

a.

1) Constructive interference occurs when path difference is nλ, so TRUE
2) FALSE , as when path difference is $\left(n+\frac{1}{2}\right)\lambda$, waves arrive in anti-phase, so interference is destructive, see minima
3) TRUE, as shown above
4) TRUE, as there will be no diffraction pattern if waves are not coherent (have no fixed phase difference)
5) FALSE as maxima further away from source are less bright due to intensity inverse square law $\left[IIX(\text{distance})^{-2}\right]$

Answer is B

b.

b) A thin piece of transparent material, thickness 300 nm and in which the speed of light is half that in air, is now placed immediately behind one of the two slits.

Which one of the following statements is correct? **[3 marks]**

A The diffraction pattern is unchanged.

B The diffraction pattern disappears because the light from the two slits is no longer coherent.

C The diffraction pattern disappears because the light from the two slits is no longer in phase.

D The complete diffraction pattern shifts in the y direction.

E Each maximum is replaced by two because the material alters the wavelength of the light coming from it.

$S = 300\text{nm} = \lambda/2$

$v = c/2$

As $\lambda = 600\text{nm}$

- period of wave : $T = \lambda/c$
- Time to tranSverSe S in vaccum:

$$tv = \frac{s}{C} = \frac{\lambda}{2C} = \frac{1}{2}T$$

- Time to transverse s in material

$$tm = \frac{s}{C/2} = \frac{\lambda}{2C}\times 2 = T$$

- So light which passes through material will be half a cycle out of phase with the light that didn't
- However, the sources will still have a constant path difference
- So d is true as now light from one slit will be in anti-phase (half a cycle out of phase) with light from the slit. So maxima will now occur when $r_1 - r_2 = \left(n + \frac{1}{2}\right)\lambda$, as now at these points the waves will be in phase and so form maxima
- So pattern will shift in y-direction

Answer is D

c.

c) A radio transmitter transmits a signal at 600 MHz to a receiver 1 km away. In an attempt to double the strength of the signal at the receiver, a second antenna is added at the transmitter, 1 m away alongside the original one, and fed by the same signal. It is suggested that, instead of improving reception, diffraction effects might actually make reception much worse.

Which of the following statements is correct? **[3 marks]**

A Diffraction effects would not be a problem because light and radio are different types of wave.

B Diffraction effects would not be a problem because the waves are too low frequency to produce diffraction effects.

C Diffraction effects would not be a problem as the transmitting antennas are too far apart to produce diffraction effects.

D Diffraction effects will occur, but the maxima would be sufficiently close together that this would not be a problem.

E Diffraction effects could be a problem because the distance between the transmitting antennas is comparable to the wavelength.

- Wave length of the radio waves:

$$\lambda = \frac{c}{t} = \frac{3\times10^{8}}{600\times10^{6}}$$
$$= 0.5\text{m}$$

- λ is comparable to gap between sources (d = m) so diffraction occurs
- So D and E are true
- Diffraction creating formula

$$d \sin\sin\theta = n\lambda$$
$$\sin\sin\theta \approx \tan\tan\theta \approx \frac{x}{D},\ d = 1\text{m}$$
$$x = Dn\lambda$$

- So spacing between maxima is:

$$\Delta x = D\lambda$$
$$= 0.5\times1\text{km}$$
$$= 500\text{m}$$

- So spacing between maxima is large
- So D is wrong and E is true

Answer is E

ENGAA S2 2016 - Question 3

3 The speed of light in vacuum and air can be taken to be $c = 3.0 \times 10^5\,\text{km s}^{-1}$. The refractive index n of a material is the ratio of the speed of light c in vacuum to the speed of light in the material.

a) A lighthouse emits a beam of light. How far does this beam of light travel in 1.0 ns? **[1 mark]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A 0.30 mm

B 300 m

C 0.30 m

D 3.0×10^{-12} m

ENGAA S2 2016 - Question 3 - Worked Solution

a)

$$d = vt = 3.0\times10^{8}\times1.0\times10^{-9} = 0.3\text{m}$$

Answer is C

b)

b) The propagation time T is the time taken for a pulse of light to travel directly along an optical fibre. A straight optical fibre has a length of 9 km. Its refractive index is 1.5. What is T for this fibre? **[1 mark]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A 20 ms

B 20 μs

C 30 ms

D 30 μs

E 45 ms

F 45 μs

$$v^1 = \frac{c}{n} = \frac{3.0\times10^5}{1.5} = 2.0\times10^5 kms^{-1}$$

$$T = \frac{d}{v} = \frac{9}{2\times10^5} = 45\mu s$$

Answer is F

c)

c) An engineer has used a refractive index of $n = 1.5$ to estimate the nominal propagation time T_{nom} for an optical fibre. The actual refractive index of the fibre depends on the wavelength of the light. For red and blue light the refractive indices obey the inequality $n_{red} < n_{blue} < 1.5$. If T_{red} and T_{blue} are the propagation times for red and blue light, respectively, which of the following inequalities is correct? **[2 marks]**

SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.

A $T_{red} < T_{blue} < T_{nom}$

B $T_{blue} < T_{red} < T_{nom}$

C $T_{red} < T_{nom} < T_{blue}$

D $T_{blue} < T_{nom} < T_{red}$

E $T_{nom} < T_{red} < T_{blue}$

F $T_{nom} < T_{blue} < T_{red}$

$$T \propto \frac{1}{v} \propto \frac{1}{1/n} \propto n$$

$$n_{red} < n_{blue} < 1.5$$

$$T_{red} < T_{blue} < T_{nom}$$

Answer is A