Mathematics BMAT Model Answers (2009-2016)

M2 – Gradients and Graphs

2016 Q4 – the correct answer is A: An equation of a parallel line has the **same** gradient as the line PQ. Gradient: y_2-y_1/ x_2-x_1 => y_Q-y_P/ x_Q-x_P → 9-3/6-(-3) = 6/9 = 2/3 Hence, the gradient is 2/3. We are given no information on the coordinates of the parallel line, hence we cannot find the y intercept. Hence, the answer is y=2x/3 – 3

2011 Q16 – the correct answer is E: Algebraically, for graphs **not** to intersect, there is no **real** solution. To demonstrate this, you can try to solve for x for answers A-E.

A) 1 and 2: $3x-2=x^2$

$x^2-3x+2=0$

$(x-1)(x-2)=0$ Hence, equations 1 and 2 intersect at x=1, x=2

B) 1 and 3: $3x-2=1-x^2$

$x^2+3x-3=0$

This equation **cannot** be solved by the sum and product method, hence we use **completing the square.**

$(x+3/2)^2-3=0$ (Divide 3 (the coefficient of x) by 2, and arrange in the form $(x+b/2)^2+c=0$, where $ax^2+bx+c=0$))

$(x+3/2)^2-9/4-3=0$ (Square the divided x coefficient, and subtract it from the **entire** equation ($(3/2)^2 = 9/4$; $(b/2)^2=b^2/4$ –> $(x+b/2)^2+c-b^2/4=0$

$(x+3/2)^2-21/4=0$ → $(x+3/2)^2 = 21/4$

$x+3/2= \pm\sqrt{21}/2$ (Take the square root of the equation. Remember that the number on the right hand side is ±)

Rearrange: $x=-3/2\pm\sqrt{21}/2$ → Hence, equations 1 and 3 intersect at $x=-3/2+\sqrt{21}/2$, $x=-3/2-\sqrt{21}/2$

C) 2 and 3: $x^2=1-x^2$

$2x^2=1$ → $x^2=1/2$ → $x=\pm1/\sqrt{2}$ Hence, equations 2 and 3 intersect at $x=1/\sqrt{2}$, $x=-1/\sqrt{2}$

D) 2 and 4: $x^2=x+6$

$x^2-x-6=0$

$(x-3)(x+2)=0$ Hence, equations 2 and 4 intersect at x=3, x=2

E) 3 and 4: $1-x^2=x+6$

$x^2+x+5=0$

This equation **cannot** be solved by the sum and product method, hence we use **completing the square.**

$(x+1/2)^2+5=0$

$(x+1/2)^2-1/4+5=0$

$(x+1/2)^2+19/4=0$

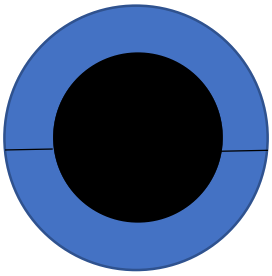
$(x+1/2)^2=-19/4$ → **A square root of a negative number will give an imaginary number, not an integer. Hence, no real solution is observed. Hence, lines 3 and 4 do not intersect.**

M3 – Means

2014 Q16 – the correct answer is D: Class 1 contains **twice** as many students as Class 2 - if there are n students in Class 2, then there are 2n students in Class 1. The total of all of the scores in Class 1 must be 61 × 2n = 122n. The total of all of the scores in Class 2 must be 63n. If there are m students in Class 3, then the total of all the scores in Class 3 must be 70m. The total score of all of the students must be 65 × (2n + n + m), so 195n + 65m = 122n + 63n + 70m. This simplifies to 10n = 5m, so m = 2n. The number of students in Class 3 is 2n, which is the same as the number of students in Class 1.

M4 – Area

2016 Q12 – the correct answer is D: To find the shaded area, the formula: $A = \pi r^2$ is needed. We are given the diameter of the **artery** as 1.6cm, with the extension from the **artery wall** being 1mm. Be careful of units here – convert **all** into **mm**, as the answers are in **mm.** Hence, the diameter of the artery is 16mm. In order to get the radius of the artery wall**, 2mm must be subtracted** from the artery diameter – 2mm as t=1mm, which extends in both directions from the diameter.

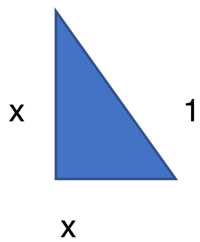
1mm  1mm

Hence, 16-2=14. Radius=Diameter/2, hence r=7. A= $\pi(7)^2$=49 π

2013 Q20 – the correct answer is D:
This is a very involved question, as it required the successive use of the Pythagorean Theorem.
With the smallest cube, all sides are of length 1cm.

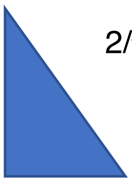
The vertexes of the cube are at the midpoint of the side of the second largest cube. The length of the side of the cube, and 2 sides of the larger cube – which are bounded by 2 midpoints – form a triangle. There are 4 triangles on the second largest cube. As the lengths of the 2 sides bounded by the midpoints are **equal,** let each length = x. The hypotenuse of the triangle = 1.

$x^2+x^2=1^2 \rightarrow 2x^2=1 \rightarrow x^2=1/2$ Hence, x=1/√2
Area of 4 triangles: 4 (1/2 b x h) = 4 (1/2 x $(1/\sqrt{2})^2$) = 1



The length of all sides of the second largest cube = 2(1/√2) = 2/√2
This becomes the length of the hypotenuse of the next triangle.

Using the above method,

x  $2/\sqrt{2}$ $x^2+x^2=(2/\sqrt{2})^2 \rightarrow 2x^2=2 \rightarrow x^2=1$

Area of 4 triangles: $4\ (1/2\ b \times h) = 4\ (1/2 \times 1^2) = 2$

x

The length of all sides of the largest cube = 2(1) = 2
The surface area of the entire shape:
Largest Cube: 5 sides – 5 x (area of each square) = $5\ (2^2) = 20$
4 triangles – 2
Second Largest Cube: 4 sides – 4 x (area of each square) = $4\ (2/\sqrt{2})^2 = 8$
4 triangles – 1
Smallest Cube: 5 sides - 5 x (area of each square) = $5\ (1^2) = 5$
Total Surface Area = 20 + 2 + 8 + 1 + 5 = $36cm^2$

2012 Q4 – the correct answer is A:
Remember to answer in terms of **diameter** – although radius is required for the area, express r as, r = d/2.
Subtract the 2^{nd} largest circle from the largest circle:
Largest Area: $\pi(4d/2)^2 = 4\pi d^2$
2^{nd} Largest Area: $\pi(3d/2)^2 = 9/4\pi d^2$
Largest A – 2^{nd} Largest A => $4\pi d^2 - 9/4\pi d^2 = 7/4\pi d^2$

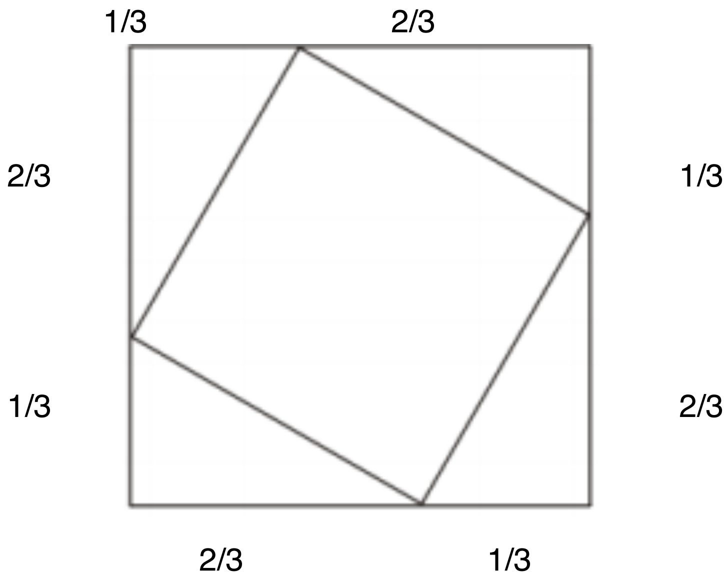
Subtract the smallest circle from the 3^{rd} largest circle:
3^{rd} Largest Area: $\pi(2d/2)^2 = \pi d^2$
Smallest Area: $\pi(d/2)^2 = 1/4\pi d^2$
3^{rd} Largest A – Smallest Area => $\pi d^2 - 1/4\pi d^2 = 3/4\pi d^2$

Hence, adding these areas together: $3/4\pi d^2 + 7/4\pi d^2 = 10/4\pi d^2 \Rightarrow 5/2\pi d^2$

2011 Q20 – the correct answer is B:
The area of the triangle given is ½bh = ½ x 3 x 1 = 3/2
As all 3 triangles are **similar,** their angles all have the same values. Assuming the top of the triangle is the largest, we can calculate their hypotenuses. Let's call each triangle (beginning from the bottom) A, B and C respectively. The hypotenuse of the triangle A becomes the base of triangle B.
$3^2+1^2=10$; Base of B = $\sqrt{10}$. As similar triangles, the height of B **must** follow the ratio of height : base from A. 1:3 → Height B: $\sqrt{10}$. $\sqrt{10}/3$ is the scale factor, hence the height of B: 1 x $\sqrt{10}/3 = \sqrt{10}/3$
Hypotenuse of C:
$(\sqrt{10})^2 + (\sqrt{10}/3)^2 = 10 + 10/9 = 100/9$; Base of C = 10/3. As similar triangles, the height of B **must** follow the ratio of height : base from A. 1:3 → Height C: 10/3
10/9 is the scale factor, hence the height of C: 1 x 10/9 = 10/9
Area of the largest triangle = ½ x 10/3 x 10/9 = ½ x 100/27 = 50/27

<u>2010 Q24</u> – the correct answer is C:
This question uses **repeated Pythagorean theorem.** It can quickly be solved by using a geometric progression (this solution will be also given, however, it will be perhaps more appreciated by A Level Mathematics students. Geometric Progression, Arithmetic Progression, or any knowledge of series beyond GCSE Mathematics is **not** required by the BMAT specification)

The side of the second square, forms a hypotenuse of a triangle bounded by its vertexes, and the sides of the larger square. The longer side, until the vertex of the smaller square is 2/3 of the way down. Whereas, the shorter side, until the vertex of the bigger square is 1/3 of the way down (approaching the diagram on the left), and opposite for the right hand side.



The hypotenuse can be calculated via Pythagoras: $\sqrt{((2/3)^2+(1/3)^2)} = \sqrt{5}/3$

As the process is repeated, the 2nd square is in the same arrangement as above, however, the lengths expressed are: $1/3\text{x}(\sqrt{5}/3)$ and $2/3\text{x}(\sqrt{5}/3)$.

Hence, the length of the side of the 3rd square:

$\sqrt{(1/3\text{x}(\sqrt{5}/3))^2+(2/3\text{x}(\sqrt{5}/3))^2} = \sqrt{(5/81 + 20/81)} = \sqrt{(25/81)}$

As the process is repeated, the 3rd square is in the same arrangement as above, however, the lengths expressed are: $1/3\text{x}(\sqrt{(25/81)})$ and $2/3\text{x}(\sqrt{(25/81)})$.

Hence, the length of the side of the 4th square:

$\sqrt{(1/3\text{x}\sqrt{(25/81)})^2+(2/3\text{x}\sqrt{(25/81)})^2} = \sqrt{(25/729 + 100/729)} = \sqrt{(125/729)}$

The length of the side of the 4th square can be squared, to give the area.
Hence, $(\sqrt{(125/729)})^2 = 125/729$

(Geometric Progression
Let the side of the first square = a, hence its area = a^2. The common ratio, r, between each term = 5/9, as the ratio between the side of length a and the second square = 5/9, which was deduced from Pythagoras. To find the area of the 4th square, the formula for a GP => $a_n = a_1 r^{n-1}$ $a_1=a^2$, n=4, r=5/9
Hence, Area of 4th square = $a^2 \text{ x } (5/9)^{4-1} = a \text{ x } (5/9)^3 = 125/729a^2$

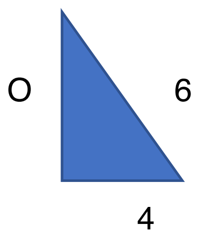
M5 – Trigonometry

2016 Q16 – the correct answer is C:

tan θ= Opposite/Adjacent = 4/3
To find the area, we must divide the shape into a right angled triangle PQX (where X is the vertex of this triangle on the line PS), and a rectangle QRSX.
The area of the triangle = ½ bh; The opposite to θ is the height (4) and the adjacent to θ is the base (3). However, the actual length of PX is 11-5=6. Remember tan is a **ratio** of sides, hence the ratio of O:A is 4:3. The actual length is O:6, hence O = 8 as the factor is 2.
Area of triangle = ½ x 6 x 8 = 24
Area of rectangle = 8 x 5 = 40
Area of quadrilateral = area of triangle + area of rectangle = 24 + 40 = 64

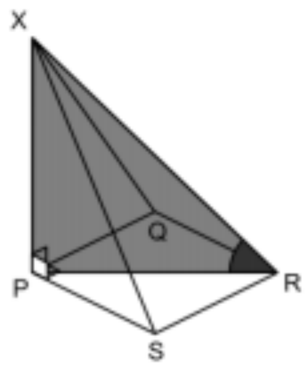
2015 Q8 – the correct answer is E:

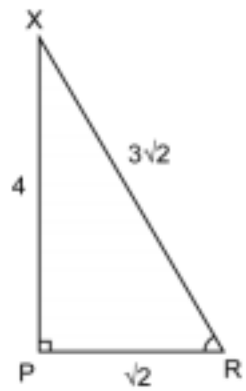
As PQR is an **isosceles triangle**, we can split the shape into 2 right angled triangles. PQ=PR=6cm is the hypotenuse of both triangles, and QR is split into the base of 4cm **each.** As Tan θ= Opposite/Adjacent, we already have the adjacent length of 4cm. As we have the hypotenuse too, we can use the Pythagorean Theorem: $\sqrt{(6^2-4^2)}=\sqrt{20} = 2\sqrt{5}$
Tan θ= Opposite/Adjacent → tan θ = $2\sqrt{5}/4 = \sqrt{5}/2$



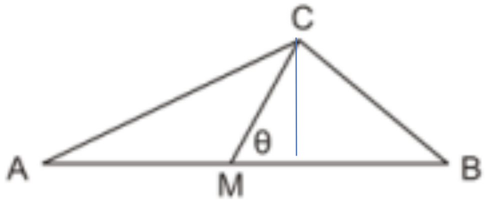
2014 Q20 – the correct answer is A:

The angle required is XRP. Using the Pythagorean Theorem PR = √2, since PR is a diagonal of square PQRS. Then triangle XPR is a right angled triangle as X is vertically above P, and by Pythagoras XR = √18 = 3√2
Hence, $\cos(XRP) = PR/XR = \sqrt{2}/(3\sqrt{2}) = 1/3$





<u>2012 Q16</u> – the correct answer is C:



To find tan θ, a vertical line must be drawn from C to the line AB, as an opposite and adjacent side, with respect to a right angled triangle, is needed. This line also maps out the triangle CAD (where D is the vertex of the triangle of angle A, and of the triangle of angle θ). This line also gives the triangle CBD, corresponding to the angle at B. tan A = CD/AD = 1/6 Hence, CD (according to A) = 1, AD=6 (according to A)
Tan B = CD/BD = 2/3 Hence, CD = 2, BD = 3 - the tan of A is given at a scale factor of 2. Multiplying the tan A by 2 to standardise both fractions – tan A = 2/12. Hence, CD = 2, AD = 12. As M is the midpoint of AB, AB = AD + DB = 12 + 3 = 15.
AM = 15/2. For tan θ = CD/MD
Hence, MD = 12-7.5 = 4.5
Tan θ = 2/4.5 = 4/9

<u>M6 – Algebra</u>

<u>2016 Q20</u> – the correct answer is C: f needs to be made the subject.

Firstly, add $\frac{a}{b} = \frac{c}{d} + \frac{e}{f} = \frac{fc+ed}{fd}$ here, we have cross multiplied fc and ed, as the lowest common multiple (LCM) of the denominator is fd.
Then, multiply **both sides** by **fd** in order to factorise later:

$$\frac{afd}{b} = fc + ed$$

$$\frac{afd}{b} - fc = ed$$

$$f(\frac{ad}{b} - c) = ed$$ Factorise, by making f the subject

$$f = \frac{ed \quad \times b}{\frac{da}{b} - c \times b}$$ Multiply numerator and denominator by b

$$f = \frac{bde}{ad - bc}$$

<u>2015 Q12</u> – the correct answer is B: $a<c<b$

Comparing a and c first: $a<c$: $\frac{3}{5+X} < \frac{3+X}{5+X}$ This statement is true - as the denominators are the same, only the numerators need to be compared. $3<3+X$ is **always true**, as $X>0$ and the addition of 3 with a **whole number greater than 0** is **always greater than 3.**

Comparing b and c: $\frac{3+X}{5+X} < \frac{3+X}{5}$ This statement is also true – as the numerators are the same, only the denominators need to be compared. However, in this case, the smaller the denominator, the **greater** the value of the fraction (quotient rule: For the same numerator, a larger denominator will give a smaller value). 5+X is **always greater** than 5, as $X>0$. Hence, the $c<b$ is true. As $c<b$ is true, $a<b$ is **also true**.

<u>2015 Q20</u> – the correct answer is D: Let x = **total score.**

Mean Score = Total Score/Number of pupils → m = $\frac{x}{n}$ → mn= x

Inclusion of another pupil's test score: m-2 = $\frac{x+n}{n+1}$

For every x substitute mn= x

m-2 = $\frac{mn+n}{n+1}$ → (multiply both sides by **(n+1)** to get rid of denominator)

(m-2)$(n+1)$=mn+n

Expand brackets on Left Hand Side: mn – 2n + m – 2 = mn + n

As we want to express in terms of n, rearrange equation with all 'n's on one side:

3n=m-2

Divide both sides by 3: $\boldsymbol{n} = \frac{\boldsymbol{m-2}}{\boldsymbol{3}}$

<u>2015 Q24</u> – the correct answer is D:

$$\frac{2x+3}{2x-3} + \frac{2x-3}{2x+3} - 2$$

Lowest common multiple of all 3 fractions is: $(2x-3)(2x+3)$

Hence, re write the equation, using cross multiplication:

$$\frac{(2x+3)(2x+3)+(2x-3)(2x-3)}{(2x-3)(2x+3)} - \frac{2(2x-3)(2x+3)}{(2x-3)(2x+3)}$$

Simplify the numerators:

$(2x+3)^2 = 4x^2+12x+9$

$(2x-3)^2 = 4x^2-12x+9$

Hence, $(2x+3)^2 + (2x-3)^2 = 8x^2+18$

$2(2x-3)(2x+3) = 8x^2 - 18$
$8x^2 + 18 - (8x^2 - 18) = 8x^2 + 18 - 8x^2 + 18 = 36$
Hence, $\frac{36}{(2x-3)(2x+3)}$

<u>2014 Q4</u> – the correct answer is B:
Factorise **both** numerator and denominator.
Numerator: x is the common factor, hence: $x^2 - 4x = x(x-4)$
Denominator: Via **difference of two squares,** $x^2 - 16 = (x+4)(x-4)$
Cancel **both** $(x-4)$, as they appear in **both** numerator and denominator:

$$\frac{x(x-4)}{(x+4)(x-4)} = \frac{x}{(x+4)}$$

<u>2014 Q8</u> – the correct answer is B:
Express 4 and 8 as **powers of 2: $4 = 2^2$, $8 = 2^3$**
Hence: $4^p \times 8^q = (2^2)^p \times (2^3)^q = 2^{2p} \times 2^{3q} = 2^{2p+3q}$
Hence, **$n = 2p + 3q$**

<u>2013 Q4</u> – the correct answer is A: This question is a simple calculation. However, beware of your substitution and powers! First substitute any terms involving coefficients: y –> $7y = 7(2 \times 10^6) = 14 \times 10^6$; $2y = 2(2 \times 10^6) = 4 \times 10^6$
(x+7y)/(x-2y) => $x + 7y = 4.6 \times 10^7 + 14 \times 10^6$ **: convert all in terms of 10^6 as this is the lowest power.** Hence, $4.6 \times 10^7 = 46 \times 10^6$
Hence, $(46 \times 10^6) + (14 \times 10^6) = 60 \times 10^6$
x – 2y => $(46 \times 10^6) - (4 \times 10^6) = 42 \times 10^6$
Hence, $60 \times 10^6/42 \times 10^6 = 10/7$ (10^6 cancels out completely)

<u>2013 Q8</u> – the correct answer is F: In order to simplify this expression, approach it in steps. First, recognise that $(1-16x^2)$ is a **difference of 2 squares** which can be split up into: (1-4x)(1+4x). Although the denominator is (4x-1), the minus sign can be applied to the (1-4x) bracket, giving (4x-1). Hence, the (4x-1) **cancels out** in **both** the numerator **and** denominator. The expression is now: 4 **+** $(x^2(4x+1))/(2x^3)$
The x^2 and x^3 can cancel out to obtain x in the **denominator** (in the rule of powers, $x^{2-3} = x^{-1} = 1/x$). The expression is now: 4 + (4x+1)/(2x) = 4 + 2 + 1/(2x) = 6 + 1/(2x)
((4x/2x) + (1/2x) = (4x+1)/(2x) = 4 + 2 + 1/(2x))

<u>2013 Q16</u> – the correct answer is D: We can express the proportionalities algebraically, by adding constants **a** (for expression 1) and **b** (for expression 2) to show equality: $x = \mathbf{a}z^2$; $y = \mathbf{b}/(z^3)$
Solve for k: $\mathbf{a} = x/(z^2)$; $\mathbf{b} = yz^3$
To equate the two formulae, the powers of z need to be the same. As $z^2 \times z^3 = z^5$, raise the x expression by a power of 3, and the y expression by a power of 2.
$z^5 = x^3/a^3$, $z^5 = b^2/y^2$ Hence, $x^3/a^3 = b^2/y^2$
$x^3 = a^3\, b^2/\, y^2$; Redefine a **new** constant of proportionality $\mathbf{k} = a^3\, b^2$

$x^3 = k/y^2$
Hence, the cube of x is **inversely proportional** to the square of y

2012 Q8 – the correct answer is E: Firstly, subtract 5 from **both sides**.
$G – 5 = \sqrt{(7(9-R)^2+9)}$
Square both sides: $(G – 5)^2 = 7(9-R)^2+9$
Subtract 9 on both sides: $(G – 5)^2 – 9 = 7(9-R)^2$ ➔
Divide by 7 on both sides: $((G – 5)^2 – 9) / 7 = (9-R)^2$
Square root both sides: $\sqrt{(((G – 5)^2 – 9) / 7)} = 9-R$
Add R to both sides and subtract $\sqrt{(((G – 5)^2 – 9) / 7)}$ on both sides:
$R = 9 - \sqrt{(((G – 5)^2 – 9) / 7)}$

2011 Q4 – the correct answer is C: Firstly, deal with the powers. Powers inside brackets, are **always** multiplied by the power **outside** the bracket. Hence,
$(3x^{-1/3})^3=27x^{-1}$
$27x^{-1} \times 3x = 81$

2011 Q12 – the correct answer is D: We do not have enough information on w, hence we can ignore it. As $x^2>y^2$, it can be inferred that x>y. However, we must check if this is the **only** true inequality, with the next 2 inequalities: $y^2<z^2$ - we do not know whether x is greater/less than z. Inequality 3; x>z, hence it can be deduced that $x^2>z^2$. Hence, x>y>z. As the only statement of x>y exists, which is correct, it is selected.

2010 Q4 – the correct answer is C: The big container is full to begin with. That counts as '1' unit of water. When the water is poured, you stop pouring until both containers have the same amount of water – the 1 unit is **halved** (0.5 + 0.5 = 1) to get 0.5 in **each** container. The big container has a capacity of 1 but only has 0.5 units of water in it, which is half of its capacity. Hence, p = 0.5. The smaller container now contains 0.5 units. Because it's smaller than the big container, it must be more than half full. If the volume of water was half the capacity of the small container, you would times by 0.5. However, the we know that the volume of water is more than half capacity. Therefore, we need to times capacity by more than 0.5 to find volume of water. Therefore, q must be bigger than 0.5. i.e q>0.5 Overall we now know that p=0.5 and q>0.5.
p and q **do not** stand for the volume of water - they stand for what you times capacity by to find the volume of water. To find capacity of water in the big container:
Volume = capacity x p
Volume = 1 x 0.5
Volume = 0.5 .

Hence, p=0.5 and q>0.5.

2010 Q12 – the correct answer is C: Total time for both groups/total people = mean
First Group: T (total time) / 20 = 54. Hence, T = 54 x 20 = 1080

Second Group: Total Time = P x T
Total People: P + 20
Hence, 1080 + PT = 56(20 + P)
1080 + PT = 1120 + 56P
PT – 56P = 1120 – 1080
Factorise P: P (T-56) = 40
Hence, P = 40/(T-56)

2010 Q16 – the correct answer is C: This question involves ratios, similar triangles and algebra. As BC and DE are **parallel**, similar triangles and isosceles triangles can be assumed. Set up ratios between the smaller triangle ABC and the larger triangle ADE. ABC → AB:BC ➔ 4 : x
ADE → AD:DE ➔ x : x+3
As the multiplying factor between these ratios must be equal, find out the multiplying factors and equalise them:

(multiply by x/4)	4 : x	(multiply by x+3/x)
	x : x+3	

Hence, x/4 = x+3/x
Rearrange and simplify: $x^2 = 4(x+3)$ → $x^2 - 4x-12 = 0$
Solve for x: (x-6)(x+2) where x=6, x= -2. As a length can **never** be negative, x=6
To find x+3, substitute: 6 + 3 = 9

2010 Q20 – the correct answer is A: Total Surface Area = Volume
Total Surface area = $2\pi r^2 + 2\pi rh$ ($2\pi r^2$ as there are 2 circles; $2\pi rh$ as it's the diameter (2r) multiplied by the height (h) and pi for the curved surface area.
Volume = $\pi r^2 h$
Hence, $2\pi r^2 + 2\pi rh = \pi r^2 h$ → (cancelling pi and rearranging) → $2r^2=r^2h-2rh$
Factorise r and h on RHS: $2r^2=rh(r-2)$
Divide by r(r-2) on both sides, and obtain: 2r/(r-2)

2009 Q12 – the correct answer is C: This question may appear to be confusing, however, this is just a substitution and calculation question. Firstly, identify that y X (mathematical binary operation) x = y^x/x
Let mathematical binary operation = !
Hence, wherever that expression is seen, **replace** it with y^x/x, where x and y are defined.
For (2!3)!2 ➔ Firstly, y = 2, x=3.
Hence, $2^3/3 = 8/3$
Then, y = 8/3, x=2
Hence, $(8/3)^2/2 = 64/9/2 = 32/9$

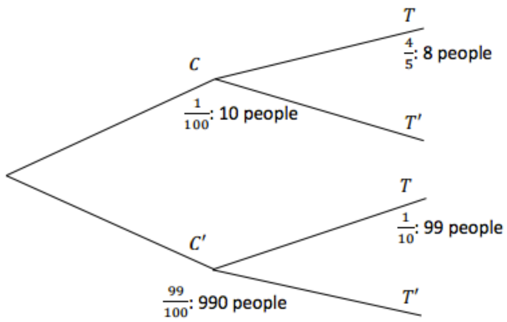
2009 Q24 – the correct answer is A:
Add 10 to both sides: $y + 10 = 5((x/2)-3)^2$
Divide by 5 on both sides: $(y+10)/5 = ((x/2)-3)^2$
Square root both sides (remembering that there are **positive** and **negative** square roots): $\pm\sqrt{((y+10)/5)} = ((x/2)-3)$
Add 3 to both sides: $3 \pm \sqrt{((y+10)/5)} = x/2$
Multiply by 2 on both sides: $6 \pm 2(\sqrt{((y+10)/5)}) = x$

M7 – Probabilities

2016 Q24 – the correct answer is D: Defining: P(AB') = 0.45, P(BA') = 0.09 P(A'B')=0.43, P(AB)=0.03.
Rewrite probabilities as equations:
A(1-B) = 0.45 → A-AB=0.45
B(1-A) = 0.09 → B-AB=0.09
(1-A)(1-B) = 0.43 → AB – (A+B)+1=0.43
AB = 0.03
Hence, substituting AB = 0.03, A-0.03 = 0.45 , hence A = 0.48
B – 0.03 = 0.09 ; B = 0.12
To check: 0.03 – (0.12 + 0.48) + 1 = 1.03 – 0.60 = 0.43
Given the presence of Type B antigens, the probability off **both** types need to be found. This is, the probability of both/given
Hence, 0.03/0.12 = ¼

2015 Q4 – the correct answer is B: The probability of getting a black in the first selection, is 3/8. As the ball is **not** replaced, the sample size: 7. Hence, the probability of getting a black ball in the second selection is 2/7. The probability of getting 2 black balls, BB, is 3/8 X 2/7 = 6/56 = 3/28.

2014 Q24 – the correct answer is E:



Hence, 8 + 99 = 107 people test **positive**, of which 8 have the condition, so the probability that someone has the condition, given that they test positive, is 8/107. This is only about 7.5%, so the test is quite poor on this population: over 90% of the positive test results are false positives. Tests for rare conditions which are applied indiscriminately will often suffer from this problem, and can lead to costly treatments (physically, financially and emotionally) for many perfectly healthy people.

2012 Q20 – the correct answer is B: The smallest probability to win is if you split the 4 red balls so that two are in each bag, and then 1 bag will consist of 2 yellow and 2 red, and the other bag will consist of 1 yellow, 1 blue and 2 red.
There are three ways that you can win
Pick bag 1 **and** then get 2 yellow
Pick bag 1 **and** then get 2 red
Pick bag 2 **and** then get 2 red
P(bag 1 **and** 2 yellow) = P(Bag 1) X P(first yellow) X P(2nd = yellow)

P(Bag 1) = 1/2,
P(first = yellow) = 2/4, i.e. two yellows in a bag that contains 4 balls
Once this is removed, only 3 balls remain in the bag and only 1 is yellow. Given that the first was yellow,
P(second is also yellow) = 1/3
P(bag1 **and** 2 yellows) = 1/2 X 2/4 X 1/3 = 2/24 = 1/12
P(Bag 1 **and** 2 reds) = 1/2 X 2/4 X 1/3 = 1/12
P(Bag 2 **and** 2 reds) = 1/2 X 2/4 X 1/3 = 1/12
Hence, the probability of winning = 1/12 + 1/12 + 1/12 = 3/12 = 1/4

<u>2011 Q24</u> – the correct answer is B: To get a total of 12, both die can only display 6 (as 6+6=12). The probability on the unbiased die: 1/6.
As 1/18 = P(6,unbiased) x P(6,biased)
P(6, biased) = 1/18 / (1/6) = 1/3
Hence, P(1-5, biased) = 1-1/3 = 2/3
As 1-5 all have **equal probabilities,** (2/3)/5 = 2/15
To get a 2, only possibility is 1+1
P(1,unbiased) = 1/6, P(1,biased) = 2/15
Hence, 1/6 X 2/15 = 2/90 = 1/45

<u>2009 Q4</u> – the correct answer is C: The total number of balls: x+y+z
The probability of red: x/(x+y+z) , blue: y/(x+y+z) , yellow: z/(x+y+z)
Red first: x/(x+y+z) X blue second: y/(x+y+z) = $xy/(x+y+z)^2$

<u>M8 – Ratios</u>

<u>2015 Q16</u> – the correct answer is E: Multiply the ratio by the **lowest common multiple**, to eliminate denominators. The **lowest common multiple** of 1,3 and 5 is 15. Hence, 1 : 2/3 : 4/5 → 15: 10: 12
Compare this with A:B:3000
The multiplying factor is, 3000/12 = 250
10 X 250 = 2500
15 X 250 = 3750
Hence, 3750: 2500: 3000
Adding them up: 3750 + 2500 + 3000 = 9250

<u>2012 Q24</u> – the correct answer is A:
Units of wood with price x/unit => cost ax
b units of metal with price 3x/unit => cost 3bx

But $ax = k_1 X D$, where D = diameter
and $3bx = k_2 X D^2$

Total cost: $x(a + 3b) = k_1D + k_2D^2 = D(k_1 + k_2D)$,
Double diameter = triple to cost $3x(a + 3b) = k_1(2D) + k_2(2D)^2 = 2D(k_1 + k_2 2D)$

Divide both equations: $3 = 2(k_1 + k_2 2D)/ (k_1 + k_2D)$
$3k_1 + 3k_2D = 2k_1 + 4k_2D$

$k_1 = k_2D$
$k_2/k_1 = 1/D$

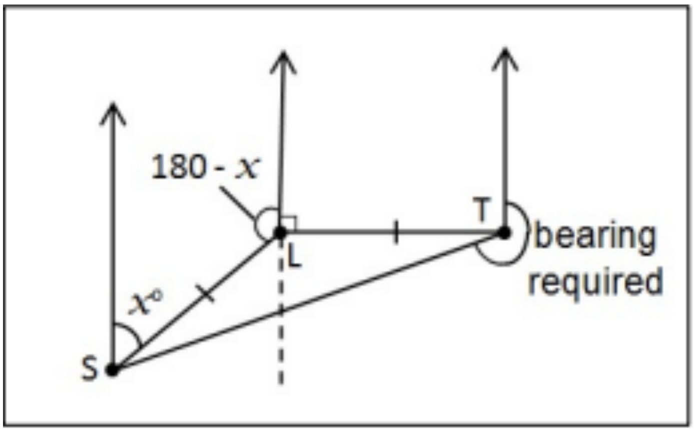
From above, $ax = k_1D$
$3bx = k_2D^2$
Divide the equations
$3b/a = k_2/k_1D$
$b/a = 1/3\ (k_2/k_1D) = 1/3\ (1/(D \times D)) = 1/3$

Part of the metal: b/(a+b) = (b/a) / (1 + b/a) = (1/3) / (1 + 1/3) = 1/4 = 25%

The proportion of metal is 25%

M9 – Geometry

2014 Q12 – the correct answer is F: Extend north line down at L and recall that alternate angles between parallel lines are equal, so angle SLT = 90 + x
Because the **interior angles** of any triangle **add up to 180**°, LTS + LST = 180 – (90 + x)
LTS + LST = 90 − x.
Triangle SLT is isosceles so both angles LTS and LST = (90−x)/2
Bearing S **from** T = 270 – ((90−x)/2) = 270 − 45 + x/2 = 225 + x/2



2011 Q8 – the correct answer is D: When it is 9:00pm, the angle between the hour and minute hands is 90 degrees. There are 3 equal divisions between 9 and 12. Hence, the angle between each hour is 90/3 = 30. However, the hour hand is ¾ of the way between 9 and 10, whereas the minute hand is **exactly** at 9. As there is a 30 degree angle between 9 and 10, ¾ x 30 = 22.5

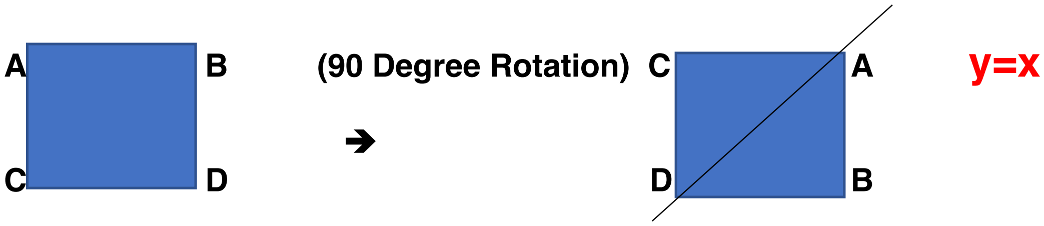
2009 Q8 – the correct answer is B: Unit length is defined as a length of 1. To find the diagonal of the base, use the Pythagorean Formula: $\sqrt{(1^2+1^2)} = \sqrt{2}$
The length of the midpoint of the diagonal: $\sqrt{2}/2$
Repeat usage of Pythagorean Formula: (the vertical height is 1, the base length to midpoint is $\sqrt{2}$, and the length in question is the hypotenuse of this triangle)

Hence, √((√2/2)^2+1^2) = √(1/2 + 1) = √(3/2)

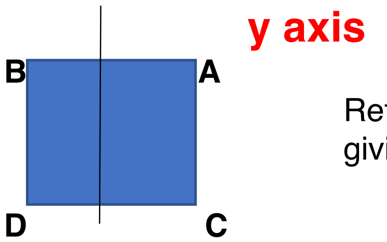
2009 Q20 – the correct answer is D: The question says that, the cylinder has the **same length as the internal diameter** of the sphere. Hence, h (length) = 2r for the cylinder
To find the fraction: Volume of sphere/Volume of cylinder
$\frac{4}{3}\pi r^3/(\pi r^2 h) = \frac{4}{3}\pi r^3/(\pi r^2(2r))$ = 2/3

M10 – Transformations

2013 Q12 – the correct answer is B: Rotating the square clockwise through 90 degrees about the origin does **not** change the coordinates, as it is a square. Neither will the reflection in the line y=x affect the square; the coordinates will be **interchanged**, but the square will remain unmoved.
Beware, the question asks for the **orientation** of the shape. Hence, diagrams displaying coordinate movement will be useful
Let A (-1,1), B (1,1), C (-1,-1), D(1,-1)



(Reflection in y = x – points **A** & **D** = **INVARIANT** as they lie on the line. Only **C** and **B** are swapped diagonally)



Reflection in y axis will swap A and B, and D and C, giving the **original shape**

M11 – Calculations

2012 Q12 – the correct answer is E:
Here, for the first power, there is a lot of cancelling – it can be re-written as:
((2 X 10^5)/ (25 X 10^{-6}))$^{1/3}$ = (2/25 X 10^{11})$^{1/3}$ = (8 X 10^{10})$^{1/3}$ = 2000
For the second power, convert it all to the **smallest power** – 10^2 ➔ 40X10^2 – 4X10^2 = 36 X 10^2

$\sqrt{(36 \times 10^2)} = 6 \times 10 = 60$
Hence, 2000-60=1940