STEP Past Papers by Topic

STEP Topic - Collision

STEP I 1987 Question 12 (Mechanics)

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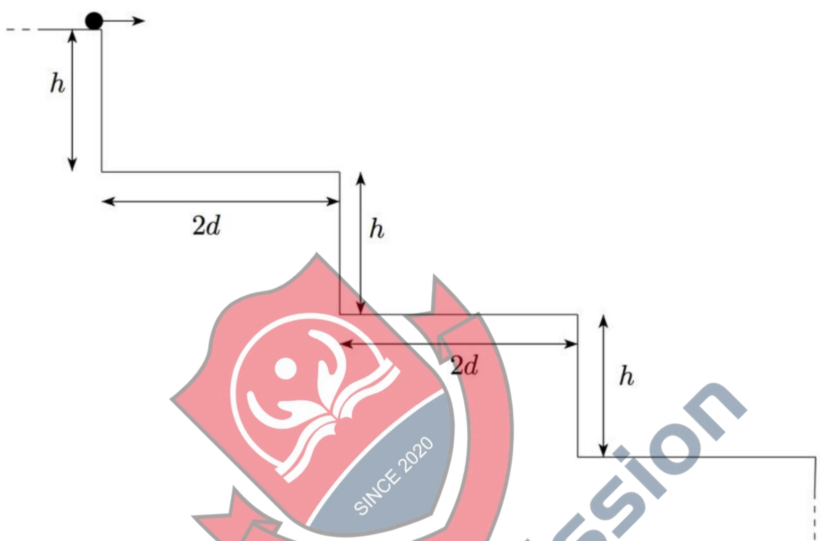

A particle is placed at the edge of the top step of a flight of steps. Each step is of width $2d$ and height h. The particle is kicked horizontally perpendicular to the edge of the top step. On its first and second bounces it lands exactly in the middle of each of the first and second steps from the top. Find the coefficient of restitution between the particle and the steps.

Determine whether it is possible for the particle to continue bouncing down the steps, hitting the middle of each successive step.

STEP II 1987 Question 13 (Mechanics)

13 Ice snooker is played on a rectangular horizontal table, of length L and width B, on which a small disc (the *puck*) slides without friction. The table is bounded by smooth vertical walls (the *cushions*) and the coefficient of restitution between the puck and any cushion is e. If the puck is hit so that it bounces off two adjacent cushions, show that its final path (after two bounces) is parallel to its original path.

The puck rests against the cushion at a point which divides the side of length L in the ratio $z : 1$. Show that it is possible, whatever z, to hit the puck so that it bounces off the three other cushions in succession clockwise and returns to the spot at which it started.

By considering these paths as z varies, explain briefly why there are two different ways in which, starting at any point away from the cushions, it is possible to perform a shot in which the puck bounces off all four cushions in succession clockwise and returns to its starting point.

STEP III 1988 Question 12 (Mechanics)

12 A smooth billiard ball moving on a smooth horizontal table strikes another identical ball which is at rest. The coefficient of restitution between the balls is $e(<1)$. Show that after the collision the angle between the velocities of the balls is less than $\frac{1}{2}\pi$.

Show also that the maximum angle of deflection of the first ball is

$$\sin^{-1}\left(\frac{1+e}{3-e}\right).$$

STEP I Specimen Question 12 (Mechanics)

12 A game consists of a player sliding a small uniform disc A, initially at a point O, across horizontal ice. The disc collides with an identical disc B at rest at a distance d away from O. After the collision, B comes to rest at a target C distant $2d$ away from O. All motion is along the line OC and the discs may be treated as point masses. Show that the speed of B immediately after the collision is $\sqrt{2\mu g d}$, where μ is the coefficient of friction between the discs and the ice. Deduce that A is started with speed U given by

$$U = \frac{\sqrt{2\mu g d}}{1+e}(5+2e+e^2)^{\frac{1}{2}},$$

where e is the coefficient of restitution between the discs.

STEP III 1991 Question 13 (Mechanics)

13 A smooth particle P_1 is projected from a point O on the horizontal floor of a room with has a horizontal ceiling at a height h above the floor. The speed of projection is $\sqrt{8gh}$ and the direction of projection makes an acute angle α with the horizontal. The particle strikes the ceiling and rebounds, the impact being perfectly elastic. Show that for this to happen α must be at least $\frac{1}{6}\pi$ and that the range on the floor is then

$$8h\cos\alpha\left(2\sin\alpha - \sqrt{4\sin^2\alpha - 1}\right).$$

Another particle P_2 is projected from O with the same velocity as P_1 but its impact with the ceiling is perfectly inelastic. Find the difference D between the ranges of P_1 and P_2 on the floor and show that, as α varies, D has a maximum value when $\alpha = \frac{1}{4}\pi$.

STEP III 1989 Question 11 (Mechanics)

11 A smooth uniform sphere, with centre A, radius $2a$ and mass $3m$, is suspended from a fixed point O by means of a light inextensible string, of length $3a$, attached to its surface at C. A second smooth unifom sphere, with centre B, radius $3a$ and mass $25m$, is held with its surface touching O and with OB horizontal. The second sphere is released from rest, falls and strikes the first sphere. The coefficient of restitution between the spheres is $3/4$. Find the speed U of A immediately after the impact in terms of the speed V of B immediately before impact.

The same system is now set up with a light rigid rod replacing the string and rigidly attached to the sphere so that OCA is a straight line. The rod can turn freely about O. The sphere with centre B is dropped as before. Show that the speed of A immediately after impact is $125U/127$.

STEP II 1989 Question 11 (Mechanics)

11 A lift of mass M and its counterweight of mass M are connected by a light inextensible cable which passes over a light frictionless pulley. The lift is constrained to move vertically between smooth guides. The distance between the floor and the ceiling of the lift is h. Initially, the lift is at rest, and the distance between the top of the lift and the pulley is greater than h. A small tile of mass m becomes detached from the ceiling of the lift. Show that the time taken for it to fall to the floor is

$$t = \sqrt{\frac{2\,(M-m)\,h}{Mg}}.$$

The collision between the tile and the lift floor is perfectly inelastic. Show that the lift is reduced to rest by the collision, and that the loss of energy of the system is mgh.

STEP II 1990 Question 12 (Mechanics)

12 A straight staircase consists of N smooth horizontal stairs each of height h. A particle slides over the top stair at speed U, with velocity perpendicular to the edge of the stair, and then falls down the staircase, bouncing once on every stair. The coefficient of restitution between the particle and each stair is e, where $e < 1$. Show that the horizontal distance d_n travelled between the nth and $(n+1)$th bounces is given by

$$d_n = U\left(\frac{2h}{g}\right)^{\frac{1}{2}}(e\alpha_n + \alpha_{n+1}),$$

where $\alpha_n = \left(\dfrac{1-e^{2n}}{1-e^2}\right)^{\frac{1}{2}}$.

If N is very large, show that U must satisfy

$$U = \left(\frac{L^2 g}{2h}\right)^{\frac{1}{2}}\left(\frac{1-e}{1+e}\right)^{\frac{1}{2}},$$

where L is the horizontal distance between the edges of successive stairs.

STEP III 1992 Question 11 (Mechanics)

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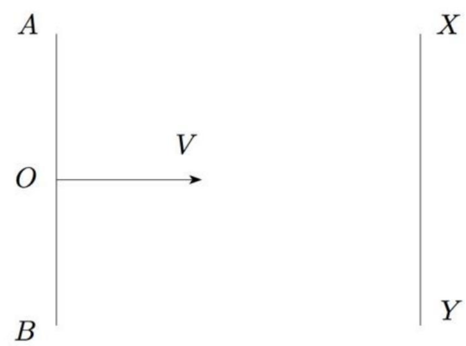

AOB represents a smooth vertical wall and XY represents a parallel smooth vertical barrier, both standing on a smooth horizontal table. A particle P is projected along the table from O with speed V in a direction perpendicular to the wall. At the time of projection, the distance between the wall and the barrier is $(75/32)VT$, where T is a constant. The barrier moves directly towards the wall, remaining parallel to the wall, with initial speed $4V$ and with constant acceleration $4V/T$ directly away from the wall. The particle strikes the barrier XY and rebounds. Show that this impact takes place at time $5T/8$.

The barrier is sufficiently massive for its motion to be unaffected by the impact. Given that the coefficient of restitution is $1/2$, find the speed of P immediately after impact.

P strikes AB and rebounds. Given that the coefficient of restitution for this collision is also $1/2$, show that the next collision of P with the barrier is at time $9T/8$ from the start of the motion.

STEP II 1997 Question 10 (Pure)

10 *In this question the effect of gravity is to be neglected.*

A small body of mass M is moving with velocity v along the axis of a long, smooth, fixed, circular cylinder of radius L. An internal explosion splits the body into two spherical fragments, with masses qM and $(1-q)M$, where $q \leqslant \frac{1}{2}$. After bouncing perfectly elastically off the cylinder (one bounce each) the fragments collide and coalesce at a point $\frac{1}{2}L$ from the axis. Show that $q = \frac{3}{8}$.

The collision occurs at a time $5L/v$ after the explosion. Find the energy imparted to the fragments by the explosion, and find the velocity after coalescence.

STEP III 1994 Question 9 (Mechanics)

9 A smooth, axially symmetric bowl has its vertical cross-sections determined by $s = 2\sqrt{ky}$, where s is the arc-length measured from its lowest point V, and y is the height above V. A particle is released from rest at a point on the surface at a height h above V. Explain why

$$\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2 + 2gy$$

is constant.

Show that the time for the particle to reach V is

$$\pi\sqrt{\frac{k}{2g}}.$$

Two elastic particles of mass m and αm, where $\alpha < 1$, are released simultaneously from opposite sides of the bowl at heights $\alpha^2 h$ and h respectively. If the coefficient of restitution between the particles is α, describe the subsequent motion.

STEP I 2003 Question 11 (Mechanics)

11 A smooth plane is inclined at an angle α to the horizontal. A and B are two points a distance d apart on a line of greatest slope of the plane, with B higher than A. A particle is projected up the plane from A towards B with initial speed u, and simultaneously another particle is released from rest at B. Show that they collide after a time d/u.

The coefficient of restitution between the two particles is e and both particles have mass m. Show that the loss of kinetic energy in the collision is $\frac{1}{4}mu^2(1-e^2)$.

STEP II 1999 Question 10 (Pure)

10 N particles $P_1, P_2, P_3, \ldots, P_N$ with masses $m, qm, q^2m, \ldots, q^{N-1}m$, respectively, are at rest at distinct points along a straight line in gravity-free space. The particle P_1 is set in motion towards P_2 with velocity V and in every subsequent impact the coefficient of restitution is e, where $0 < e < 1$. Show that after the first impact the velocities of P_1 and P_2 are

$$\left(\frac{1-eq}{1+q}\right)V \quad \text{and} \quad \left(\frac{1+e}{1+q}\right)V,$$

respectively.

Show that if $q \leqslant e$, then there are exactly $N-1$ impacts and that if $q = e$, then the total loss of kinetic energy after all impacts have occurred is equal to

$$\frac{1}{2}me(1-e^{N-1})V^2.$$

STEP III 2006 Question 11 (Mechanics)

11 A lift of mass M and its counterweight of mass M are connected by a light inextensible cable which passes over a fixed frictionless pulley. The lift is constrained to move vertically between smooth guides. The distance between the floor and the ceiling of the lift is h. Initially, the lift is at rest, and the distance between the top of the lift and the pulley is greater than h. A small tile of mass m becomes detached from the ceiling of the lift and falls to the floor of the lift. Show that the speed of the tile just before the impact is

$$\sqrt{\frac{(2M-m)gh}{M}}\,.$$

The coefficient of restitution between the tile and the floor of the lift is e. Given that the magnitude of the impulsive force on the lift due to tension in the cable is equal to the magnitude of the impulsive force on the counterweight due to tension in the cable, show that the loss of energy of the system due to the impact is $mgh(1-e^2)$. Comment on this result.

STEP III 2000 Question 9 (Mechanics)

9 Two small discs of masses m and μm lie on a smooth horizontal surface. The disc of mass μm is at rest, and the disc of mass m is projected towards it with velocity $\mathbf{u}$. After the collision, the disc of mass μm moves in the direction given by unit vector $\mathbf{n}$. The collision is perfectly elastic.

(i) Show that the speed of the disc of mass μm after the collision is $\dfrac{2\mathbf{u}\cdot\mathbf{n}}{1+\mu}$.

(ii) Given that the two discs have equal kinetic energy after the collision, find an expression for the cosine of the angle between $\mathbf{n}$ and $\mathbf{u}$ and show that $3-\sqrt{8} \leqslant \mu \leqslant 3+\sqrt{8}$.

STEP II 2007 Question 9 (Mechanics)

9 A solid right circular cone, of mass M, has semi-vertical angle α and smooth surfaces. It stands with its base on a smooth horizontal table. A particle of mass m is projected so that it strikes the curved surface of the cone at speed u. The coefficient of restitution between the particle and the cone is e. The impact has no rotational effect on the cone and the cone has no vertical velocity after the impact.

(i) The particle strikes the cone in the direction of the normal at the point of impact. Explain why the trajectory of the particle immediately after the impact is parallel to the normal to the surface of the cone. Find an expression, in terms of M, m, α, e and u, for the speed at which the cone slides along the table immediately after impact.

(ii) If instead the particle falls vertically onto the cone, show that the speed w at which the cone slides along the table immediately after impact is given by

$$w = \frac{mu(1+e)\sin\alpha\cos\alpha}{M + m\cos^2\alpha}.$$

Show also that the value of α for which w is greatest is given by

$$\cos\alpha = \sqrt{\frac{M}{2M+m}}.$$

STEP II 2001 Question 10 (Pure)

10 Two particles A and B of masses m and km, respectively, are at rest on a smooth horizontal surface. The direction of the line passing through A and B is perpendicular to a vertical wall which is on the other side of B from A. The particle A is now set in motion towards B with speed u. The coefficient of restitution between A and B is e_1 and between B and the wall is e_2. Show that there will be a second collision between A and B provided

$$k < \frac{1 + e_2(1 + e_1)}{e_1} \,.$$

Show that, if $e_1 = \frac{1}{3}$, $e_2 = \frac{1}{2}$ and $k < 5$, then the kinetic energy of A and B immediately after B rebounds from the wall is greater than $mu^2/27$.

STEP III 2005 Question 10 (Mechanics)

10 Two thin discs, each of radius r and mass m, are held on a rough horizontal surface with their centres a distance $6r$ apart. A thin light elastic band, of natural length $2\pi r$ and modulus $\dfrac{\pi mg}{12}$, is wrapped once round the discs, its straight sections being parallel. The contact between the elastic band and the discs is smooth. The coefficient of static friction between each disc and the horizontal surface is μ, and each disc experiences a force due to friction equal to μmg when it is sliding.

The discs are released simultaneously. If the discs collide, they rebound and a half of their total kinetic energy is lost in the collision.

(i) Show that the discs start sliding, but come to rest before colliding, if and only if $\frac{2}{3} < \mu < 1$.

(ii) Show that, if the discs collide at least once, their total kinetic energy just before the first collision is $\frac{4}{3}mgr(2 - 3\mu)$.

(iii) Show that if $\frac{4}{9} > \mu^2 > \frac{5}{27}$ the discs come to rest exactly once after the first collision.

STEP I 2000 Question 10 (Mechanics)

10 Three particles P_1, P_2 and P_3 of masses m_1, m_2 and m_3 respectively lie at rest in a straight line on a smooth horizontal table. P_1 is projected with speed v towards P_2 and brought to rest by the collision. After P_2 collides with P_3, the latter moves forward with speed v. The coefficients of restitution in the first and second collisions are e and e', respectively. Show that

$$e' = \frac{m_2 + m_3 - m_1}{m_1}.$$

Show that $2m_1 \geqslant m_2 + m_3 \geqslant m_1$ for such collisions to be possible.

If m_1, m_3 and v are fixed, find, in terms of m_1, m_3 and v, the largest and smallest possible values for the final energy of the system.

STEP II 2006 Question 10 (Pure)

10 Three particles, A, B and C, of masses m, km and $3m$ respectively, are initially at rest lying in a straight line on a smooth horizontal surface. Then A is projected towards B at speed u. After the collision, B collides with C. The coefficient of restitution between A and B is $\frac{1}{2}$ and the coefficient of restitution between B and C is $\frac{1}{4}$.

(i) Find the range of values of k for which A and B collide for a second time.

(ii) Given that $k = 1$ and that B and C are initially a distance d apart, show that the time that elapses between the two collisions of A and B is $\dfrac{60d}{13u}$.

STEP III 2002 Question 11 (Mechanics)

11 A particle moves on a smooth triangular horizontal surface AOB with angle $AOB = 30°$. The surface is bounded by two vertical walls OA and OB and the coefficient of restitution between the particle and the walls is e, where $e < 1$. The particle, which is initially at point P on the surface and moving with velocity u_1, strikes the wall OA at M_1, with angle $PM_1A = \theta$, and rebounds, with velocity v_1, to strike the wall OB at N_1, with angle $M_1N_1B = \theta$. Find e and $\dfrac{v_1}{u_1}$ in terms of θ.

The motion continues, with the particle striking side OA at M_2, M_3, ... and striking side OB at N_2, N_3, ... Show that, if $\theta < 60°$, the particle reaches O in a finite time.

STEP I 2002 Question 11 (Mechanics)

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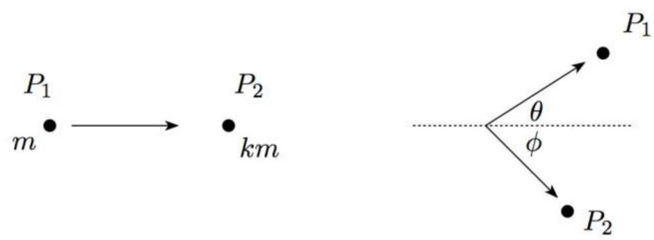


A particle P_1 of mass m collides with a particle P_2 of mass km which is at rest. No energy is lost in the collision. The direction of motion of P_1 and P_2 after the collision make non-zero angles of θ and ϕ, respectively, with the direction of motion of P_1 before the collision, as shown. Show that

$$\sin^2\theta + k\sin^2\phi = k\sin^2(\theta+\phi)\,.$$

Show that, if the angle between the particles after the collision is a right angle, then $k = 1$.

STEP II 1994 Question 10 (Pure)

10 A truck is towing a trailer of mass m across level ground by means of an elastic rope of natural length l whose modulus of elasticity is λ. At first the rope is slack and the trailer stationary. The truck then accelerates until the rope becomes taut and thereafter the truck travels in a straight line at a constant speed u. Assuming that the effect of friction on the trailer is negligible, show that the trailer will collide with the truck at a time

$$\pi\left(\frac{lm}{\lambda}\right)^{\frac{1}{2}}+\frac{l}{u}$$

after the rope first becomes taut.

STEP I 1996 Question 10 (Mechanics)

10 A spaceship of mass M is at rest. It separates into two parts in an explosion in which the total kinetic energy released is E. Immediately after the explosion the two parts have masses m_1 and m_2 and speeds v_1 and v_2 respectively. Show that the minimum possible relative speed $v_1 + v_2$ of the two parts of the spaceship after the explosion is $(8E/M)^{1/2}$.

STEP II 2012 Question 11 (Mechanics)

11 A small block of mass km is initially at rest on a smooth horizontal surface. Particles P_1, P_2, P_3, ... are fired, in order, along the surface from a fixed point towards the block. The mass of the ith particle is im (i = 1, 2, ...) and the speed at which it is fired is u/i. Each particle that collides with the block is embedded in it. Show that, if the nth particle collides with the block, the speed of the block after the collision is

$$\frac{2nu}{2k + n(n+1)}.$$

In the case $2k = N(N+1)$, where N is a positive integer, determine the number of collisions that occur. Show that the total kinetic energy lost in all the collisions is

$$\tfrac{1}{2}mu^2\left(\sum_{n=2}^{N+1}\frac{1}{n}\right).$$

STEP III 1998 Question 10 (Mechanics)

10 Two identical spherical balls, moving on a horizontal, smooth table, collide in such a way that both momentum and kinetic energy are conserved. Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the velocities of the balls before the collision and let $\mathbf{v}'_1$ and $\mathbf{v}'_2$ be the velocities of the balls after the collision, where $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}'_1$ and $\mathbf{v}'_2$ are two-dimensional vectors. Write down the equations for conservation of momentum and kinetic energy in terms of these vectors. Hence show that their relative speed is also conserved.

Show that, if one ball is initially at rest but after the collision both balls are moving, their final velocities are perpendicular.

Now suppose that one ball is initially at rest, and the second is moving with speed V. After a collision in which they lose a proportion k of their original kinetic energy ($0 \leqslant k \leqslant 1$), the direction of motion of the second ball has changed by an angle θ. Find a quadratic equation satisfied by the final speed of the second ball, with coefficients depending on k, V and θ. Hence show that $k \leqslant \frac{1}{2}$.

STEP II 2010 Question 10 (Pure)

10 **(i)** In an experiment, a particle A of mass m is at rest on a smooth horizontal table. A particle B of mass bm, where $b > 1$, is projected along the table directly towards A with speed u. The collision is perfectly elastic.

Find an expression for the speed of A after the collision in terms of b and u, and show that, irrespective of the relative masses of the particles, A cannot be made to move at twice the initial speed of B.

(ii) In a second experiment, a particle B_1 is projected along the table directly towards A with speed u. This time, particles $B_2, B_3, \ldots, B_n$ are at rest in order on the line between B_1 and A. The mass of B_i ($i = 1, 2, \ldots, n$) is $\lambda^{n+1-i}m$, where $\lambda > 1$. All collisions are perfectly elastic. Show that, by choosing n sufficiently large, there is no upper limit on the speed at which A can be made to move.

In the case $\lambda = 4$, determine the least value of n for which A moves at more than $20u$. You may use the approximation $\log_{10} 2 \approx 0.30103$.

STEP II 2011 Question 9 (Mechanics)

9 Two particles, A of mass $2m$ and B of mass m, are moving towards each other in a straight line on a smooth horizontal plane, with speeds $2u$ and u respectively. They collide directly. Given that the coefficient of restitution between the particles is e, where $0 < e \leqslant 1$, determine the speeds of the particles after the collision.

After the collision, B collides directly with a smooth vertical wall, rebounding and then colliding directly with A for a second time. The coefficient of restitution between B and the wall is f, where $0 < f \leqslant 1$. Show that the velocity of B after its second collision with A is

$$\tfrac{2}{3}(1-e^2)u - \tfrac{1}{3}(1-4e^2)fu$$

towards the wall and that B moves towards (not away from) the wall for all values of e and f.

STEP II 1995 Question 10 (Pure)

10 Three small spheres of masses m_1, m_2 and m_3, move in a straight line on a smooth horizontal table. (Their order on the straight line is the order given.) The coefficient of restitution between any two spheres is e. The first moves with velocity u towards the second whilst the second and third are at rest. After the first collision the second sphere hits the third after which the velocity of the second sphere is u. Find m_1 in terms of m_2, m_3 and e. deduce that

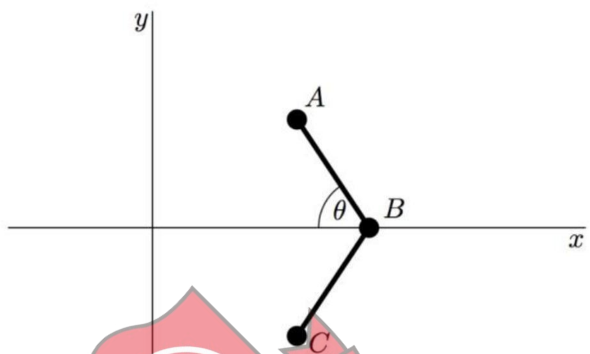
$$m_2 e > m_3(1 + e + e^2).$$

Suppose that the relation between m_1, m_2 and m_3 is that in the formula you found above, but that now the first sphere initially moves with velocity u and the other two spheres with velocity v, all in the same direction along the line. If $u > v > 0$ use the first part to find the velocity of the second sphere after two collisions have taken place. (You should not need to make any substantial computations but you should state your argument clearly.)

STEP II 2015 Question 11 (Mechanics)

11 Three particles, A, B and C, each of mass m, lie on a smooth horizontal table. Particles A and C are attached to the two ends of a light inextensible string of length $2a$ and particle B is attached to the midpoint of the string. Initially, A, B and C are at rest at points $(0, a)$, $(0, 0)$ and $(0, -a)$, respectively.

An impulse is delivered to B, imparting to it a speed u in the positive x direction. The string remains taut throughout the subsequent motion.



(i) At time t, the angle between the x-axis and the string joining A and B is θ, as shown in the diagram, and B is at $(x, 0)$. Write down the coordinates of A in terms of x, a and θ. Given that the velocity of B is $(v, 0)$, show that the velocity of A is $(\dot{x} + a\sin\theta\,\dot{\theta}\,,\ a\cos\theta\,\dot{\theta})$, where the dot denotes differentiation with respect to time.

(ii) Show that, before particles A and C first collide,

$$3\dot{x} + 2a\dot{\theta}\sin\theta = v \quad \text{and} \quad \dot{\theta}^2 = \frac{v^2}{a^2(3 - 2\sin^2\theta)}\,.$$

(iii) When A and C collide, the collision is elastic (no energy is lost). At what value of θ does the second collision between particles A and C occur? (You should justify your answer.)

(iv) When $v = 0$, what are the possible values of θ? Is $v = 0$ whenever θ takes these values?

STEP III 2005 Question 9 (Mechanics)

9 Two particles, A and B, move without friction along a horizontal line which is perpendicular to a vertical wall. The coefficient of restitution between the two particles is e and the coefficient of restitution between particle B and the wall is also e, where $0 < e < 1$. The mass of particle A is $4em$ (with $m > 0$), and the mass of particle B is $(1-e)^2m$.

Initially, A is moving towards the wall with speed $(1-e)v$ (where $v > 0$) and B is moving away from the wall and towards A with speed $2ev$. The two particles collide at a distance d from the wall. Find the speeds of A and B after the collision.

When B strikes the wall, it rebounds along the same line. Show that a second collision will take place, at a distance de from the wall.

Deduce that further collisions will take place. Find the distance from the wall at which the nth collision takes place, and show that the times between successive collisions are equal.

STEP I 2010 Question 11 (Mechanics)

11 Two particles of masses m and M, with $M > m$, lie in a smooth circular groove on a horizontal plane. The coefficient of restitution between the particles is e. The particles are initially projected round the groove with the same speed u but in opposite directions. Find the speeds of the particles after they collide for the first time and show that they will both change direction if $2em > M - m$.

After a further $2n$ collisions, the speed of the particle of mass m is v and the speed of the particle of mass M is V. Given that at each collision both particles change their directions of motion, explain why

$$mv - MV = u(M - m),$$

and find v and V in terms of m, M, e, u and n.

STEP I 2011 Question 10 (Mechanics)

10 A particle, A, is dropped from a point P which is at a height h above a horizontal plane. A second particle, B, is dropped from P and first collides with A after A has bounced on the plane and before A reaches P again. The bounce and the collision are both perfectly elastic. Explain why the speeds of A and B immediately before the first collision are the same.

The masses of A and B are M and m, respectively, where $M > 3m$, and the speed of the particles immediately before the first collision is u. Show that both particles move upwards after their first collision and that the maximum height of B above the plane after the first collision and before the second collision is

$$h + \frac{4M(M-m)u^2}{(M+m)^2 g}\,.$$

STEP I 2006 Question 11 (Mechanics)

11 Particles A_1, A_2, A_3, ..., A_n (where $n \geqslant 2$) lie at rest in that order in a smooth straight horizontal trough. The mass of A_{n-1} is m and the mass of A_n is λm, where $\lambda > 1$. Another particle, A_0, of mass m, slides along the trough with speed u towards the particles and collides with A_1. Momentum and energy are conserved in all collisions.

(i) Show that it is not possible for there to be exactly one particle moving after all collisions have taken place.

(ii) Show that it is not possible for A_{n-1} and A_n to be the only particles moving after all collisions have taken place.

(iii) Show that it is not possible for A_{n-2}, A_{n-1} and A_n to be the only particles moving after all collisions have taken place.

(iv) Given that there are exactly two particles moving after all collisions have taken place, find the speeds of these particles in terms of u and λ.

STEP II 2008 Question 10 (Pure)

10 The lengths of the sides of a rectangular billiards table $ABCD$ are given by $AB = DC = a$ and $AD = BC = 2b$. There are small pockets at the midpoints M and N of the sides AD and BC, respectively. The sides of the table may be taken as smooth vertical walls.

A small ball is projected along the table from the corner A. It strikes the side BC at X, then the side DC at Y and then goes directly into the pocket at M. The angles BAX, CXY and DYM are α, β and γ respectively. On each stage of its path, the ball moves with constant speed in a straight line, the speeds being u, v and w respectively. The coefficient of restitution between the ball and the sides is e, where $e > 0$.

(i) Show that $\tan\alpha\tan\beta = e$ and find γ in terms of α.

(ii) Show that $\tan\alpha = \dfrac{(1+2e)b}{(1+e)a}$ and deduce that the shot is possible whatever the value of e.

(iii) Find an expression in terms of e for the fraction of the kinetic energy of the ball that is lost during the motion.

STEP I 1998 Question 9 (Mechanics)

9 Two small spheres A and B of equal mass m are suspended in contact by two light inextensible strings of equal length so that the strings are vertical and the line of centres is horizontal. The coefficient of restitution between the spheres is e. The sphere A is drawn aside through a very small distance in the plane of the strings and allowed to fall back and collide with the other sphere B, its speed on impact being u. Explain briefly why the succeeding collisions will all occur at the lowest point. (Hint: Consider the periods of the two pendulums involved.)

Show that the speed of sphere A immediately after the second impact is $\frac{1}{2}u(1+e^2)$ and find the speed, then, of sphere B.

STEP I 2014 Question 10 (Mechanics)

10 **(i)** A uniform spherical ball of mass M and radius R is released from rest with its centre a distance $H + R$ above horizontal ground. The coefficient of restitution between the ball and the ground is e. Show that, after bouncing, the centre of the ball reaches a height $R + He^2$ above the ground.

(ii) A second uniform spherical ball, of mass m and radius r, is now released from rest together with the first ball (whose centre is again a distance $H + R$ above the ground when it is released). The two balls are initially one on top of the other, with the second ball (of mass m) above the first. The two balls separate slightly during their fall, with their centres remaining in the same vertical line, so that they collide immediately after the first ball has bounced on the ground. The coefficient of restitution between the balls is also e. The centre of the second ball attains a height h above the ground.

Given that $R = 0.2$, $r = 0.05$, $H = 1.8$, $h = 4.5$ and $e = \frac{2}{3}$, determine the value of M/m.

STEP II 2013 Question 11 (Mechanics)

11 Three identical particles lie, not touching one another, in a straight line on a smooth horizontal surface. One particle is projected with speed u directly towards the other two which are at rest. The coefficient of restitution in all collisions is e, where $0 < e < 1$.

(i) Show that, after the second collision, the speeds of the particles are $\frac{1}{2}u(1-e)$, $\frac{1}{4}u(1-e^2)$ and $\frac{1}{4}u(1+e)^2$. Deduce that there will be a third collision whatever the value of e.

(ii) Show that there will be a fourth collision if and only if e is less than a particular value which you should determine.

STEP I 2009 Question 11 (Mechanics)

11 Two particles move on a smooth horizontal table and collide. The masses of the particles are m and M. Their velocities before the collision are $u\mathbf{i}$ and $v\mathbf{i}$, respectively, where $\mathbf{i}$ is a unit vector and $u > v$. Their velocities after the collision are $p\mathbf{i}$ and $q\mathbf{i}$, respectively. The coefficient of restitution between the two particles is e, where $e < 1$.

(i) Show that the loss of kinetic energy due to the collision is

$$\tfrac{1}{2}m(u-p)(u-v)(1-e)\,,$$

and deduce that $u \geqslant p$.

(ii) Given that each particle loses the same (non-zero) amount of kinetic energy in the collision, show that

$$u+v+p+q=0\,,$$

and that, if $m \neq M$,

$$e = \frac{(M+3m)u+(3M+m)v}{(M-m)(u-v)}\,.$$

STEP I 2005 Question 10 (Mechanics)

10 Three collinear, non-touching particles A, B and C have masses a, b and c, respectively, and are at rest on a smooth horizontal surface. The particle A is given an initial velocity u towards B. These particles collide, giving B a velocity v towards C. These two particles then collide, giving C a velocity w.

The coefficient of restitution is e in both collisions. Determine an expression for v, and show that

$$w = \frac{abu\,(1+e)^2}{(a+b)\,(b+c)}\,.$$

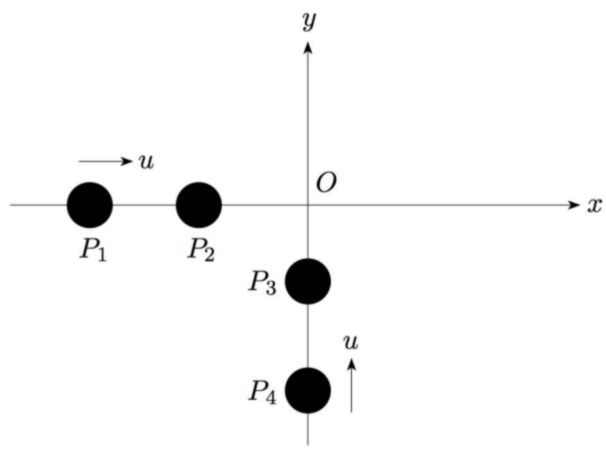
Determine the final velocities of each of the three particles in the cases:

(i) $\dfrac{a}{b} = \dfrac{b}{c} = e\,;$

(ii) $\dfrac{b}{a} = \dfrac{c}{b} = e\,.$

STEP II 2009 Question 10 (Mechanics)

10



Four particles P_1, P_2, P_3 and P_4, of masses m_1, m_2, m_3 and m_4, respectively, are arranged on smooth horizontal axes as shown in the diagram.

Initially, P_2 and P_3 are stationary, and both P_1 and P_4 are moving towards O with speed u. Then P_1 and P_2 collide, at the same moment as P_4 and P_3 collide. Subsequently, P_2 and P_3 collide at O, as do P_1 and P_4 some time later. The coefficient of restitution between each pair of particles is e, and $e > 0$.

Show that initially P_2 and P_3 are equidistant from O.

STEP III 2011 Question 10 (Mechanics)

10 Particles P and Q, each of mass m, lie initially at rest a distance a apart on a smooth horizontal plane. They are connected by a light elastic string of natural length a and modulus of elasticity $\frac{1}{2}ma\omega^2$, where ω is a constant.

Then P receives an impulse which gives it a velocity u directly away from Q. Show that when the string next returns to length a, the particles have travelled a distance $\frac{1}{2}\pi u/\omega$, and find the speed of each particle.

Find also the total time between the impulse and the subsequent collision of the particles.

STEP I 2002 Question 10 (Mechanics)

10 A bicycle pump consists of a cylinder and a piston. The piston is pushed in with steady speed u. A particle of air moves to and fro between the piston and the end of the cylinder, colliding perfectly elastically with the piston and the end of the cylinder, and always moving parallel with the axis of the cylinder. Initially, the particle is moving towards the piston at speed v. Show that the speed, v_n, of the particle just after the nth collision with the piston is given by $v_n = v + 2nu$.

Let d_n be the distance between the piston and the end of the cylinder at the nth collision, and let t_n be the time between the nth and $(n+1)$th collisions. Express $d_n - d_{n+1}$ in terms of u and t_n, and show that

$$d_{n+1} = \frac{v + (2n-1)u}{v + (2n+1)u}\, d_n \, .$$

Express d_n in terms of d_1, u, v and n.

In the case $v = u$, show that $ut_n = \dfrac{d_1}{n(n+1)}$.

STEP III Specimen Question 14 (Mechanics)

14 A uniform straight rod of mass m and length $4a$ can rotate freely about its midpoint on a smooth horizontal table. Initially the rod is at rest. A particle of mass m travelling on the table with speed u at right angles to the rod collides perfectly elastically with the rod at a distance a from the centre of the rod. Show that the angular speed, ω, of the rod after the collision is given by

$$a\omega = 6u/7.$$

Show also that the particle and rod undergo a subsequent collision.

STEP II 1990 Question 14 (Mechanics)

14 The identical uniform smooth spherical marbles $A_1, A_2, \ldots, A_n$, where $n \geqslant 3$, each of mass m, lie in that order in a smooth straight trough, with each marble touching the next. The marble A_{n+1}, which is similar to A_n but has mass λm, is placed in the trough so that it touches A_n. Another marble A_0, identical to A_n, slides along the trough with speed u and hits A_1. It is given that kinetic energy is conserved throughout.

(i) Show that if $\lambda < 1$, there is a possible subsequent motion in which only A_n and A_{n+1} move (and A_0 is reduced to rest), but that if $\lambda > 1$, such a motion is not possible.

(ii) If $\lambda > 1$, show that a subsequent motion in which only A_{n-1}, A_n and A_{n+1} move is not possible.

(iii) If $\lambda > 1$, find a possible subsequent motion in which only two marbles move.
