STEP Past Papers by Topic

STEP Topic – Equilibrium

STEP II 1987 Question 12 (Mechanics)

12 A long, inextensible string passes through a small fixed ring. One end of the string is attached to a particle of mass m, which hangs freely. The other end is attached to a bead also of mass m which is threaded on a smooth rigid wire fixed in the same vertical plane as the ring. The curve of the wire is such that the system can be in static equilibrium for all positions of the bead. The shortest distance between the wire and the ring is $d(>0)$. Using plane polar coordinates centred on the ring, find the equation of the curve.

The bead is set in motion. Assuming that the string remains taut, show that the speed of the bead when it is a distance r from the ring is

$$\left(\frac{r}{2r-d}\right)^{\frac{1}{2}}v,$$

where v is the speed of the bead when $r=d$.

STEP III Specimen Question 12 (Mechanics)

12 One end A of a uniform straight rod AB of mass M and length L rests against a smooth vertical wall. The other end B is attached to a light inextensible string BC of length αL which is fixed to the wall at a point C vertically above A. The rod is in equilibrium with the points A, B and C not collinear. Determine the inclination of the rod to the vertical and the set of possible values of α.

Show that the tension in the string is

$$\frac{Mg\alpha}{2}\left(\frac{3}{\alpha^2-1}\right)^{\frac{1}{2}}.$$

STEP II 1990 Question 13 (Mechanics)

13 A thin non-uniform rod PQ of length $2a$ has its centre of gravity a distance $a + d$ from P. It hangs (not vertically) in equilibrium suspended from a small smooth peg O by means of a light inextensible string of length $2b$ which passes over the peg and is attached at its ends to P and Q. Express OP and OQ in terms of a, b and d. By considering the angle POQ, or otherwise, show that $d < a^2/b$.

STEP III 1990 Question 11 (Mechanics)

11 The points O, A, B and C are the vertices of a uniform square lamina of mass M. The lamina can turn freely under gravity about a horizontal axis perpendicular to the plane of the lamina through O. The sides of the lamina are of length $2a$. When the lamina is haning at rest with the diagonal OB vertically downwards it is struck at the midpoint of OC by a particle of mass $6M$ moving horizontally in the plane of the lamina with speed V. The particle adheres to the lamina. Find, in terms of a, M and g, the value which V^2 must exceed for the lamina and particle to make complete revolutions about the axis.

STEP II 1989 Question 12 (Mechanics)

12 A uniform rectangular lamina of sides $2a$ and $2b$ rests in a vertical plane. It is supported in equilibrium by two smooth pegs fixed in the same horizontal plane, a distance d apart, so that one corner of the lamina is below the level of the pegs. Show that if the distance between this (lowest) corner and the peg upon which the side of length $2a$ rests is less than a, then the distance between this corner and the other peg is less than b.

Show also that

$$b\cos\theta - a\sin\theta = d\cos 2\theta,$$

where θ is the acute angle which the sides of length $2b$ make with the horizontal.

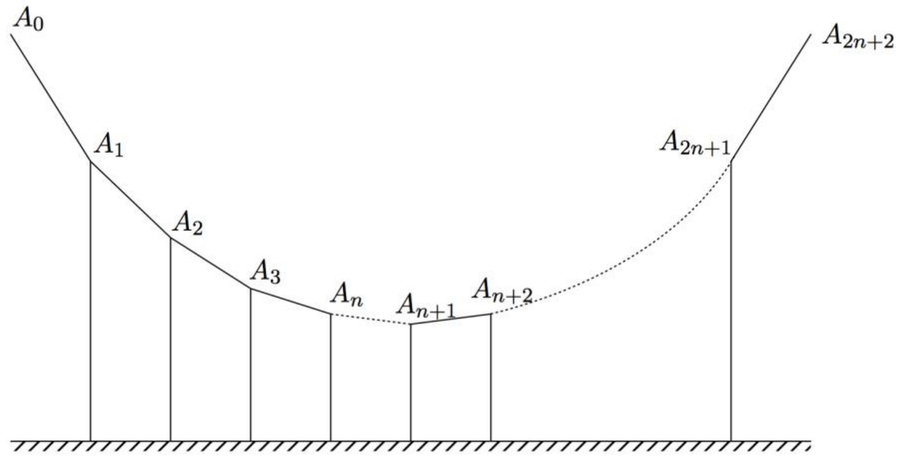
STEP III 1990 Question 12 (Mechanics)

12 A uniform smooth wedge of mass m has congruent triangular end faces $A_1B_1C_1$ and $A_2B_2C_2$, and A_1A_2, B_1B_2 and C_1C_2 are perpendicular to these faces. The points A, B and C are the midpoints of A_1A_2, B_1B_2 and C_1C_2 respectively. The sides of the triangle ABC have lengths $AB = AC = 5a$ and $BC = 6a$. The wedge is placed with BC on a smooth horizontal table, a particle of mass $2m$ is placed at A on AC, and the system is released from rest. The particle slides down AC, strikes the table, bounces perfectly elastically and lands again on the table at D. At this time the point C of the wedge has reached the point E.

Show that $DE = \frac{192}{19}a$.

STEP I 1991 Question 10 (Mechanics)

10



The above diagram represents a suspension bridge. A heavy uniform horizontal roadway is attached by vertical struts to a light flexible chain at points $A_1 = (x_1, y_1)$, $A_2 = (x_2, y_2), \ldots,$ $A_{2n+1} = (x_{2n+1}, y_{2n+1})$, where the coordinates are referred to horizontal and vertically upward axes Ox, Oy. The chain is fixed to external supports at points

$$A_0 = (x_0, y_0) \quad \text{and} \quad A_{2n+2} = (x_{2n+2}, y_{2n+2})$$

at the same height. The weight of the chain and struts may be neglected. Each strut carries the same weight w. The horizontal spacing h between A_i and A_{i+1} (for $0 \leqslant i \leqslant 2n+1$) is constant. Write down equations satisfied by the tensions T_i in the portion $A_{i-1}A_i$ of the chain for $1 \leqslant i \leqslant n+1$. Hence or otherwise show that

$$\frac{h}{y_n - y_{n+1}} = \frac{3h}{y_{n-1} - y_n} = \cdots = \frac{(2n+1)y}{y_0 - y_1}.$$

Verify that the points $A_0, A_1, \ldots, A_{2n+1}, A_{2n+2}$ lie on a parabola.

STEP I 1988 Question 12 (Mechanics)

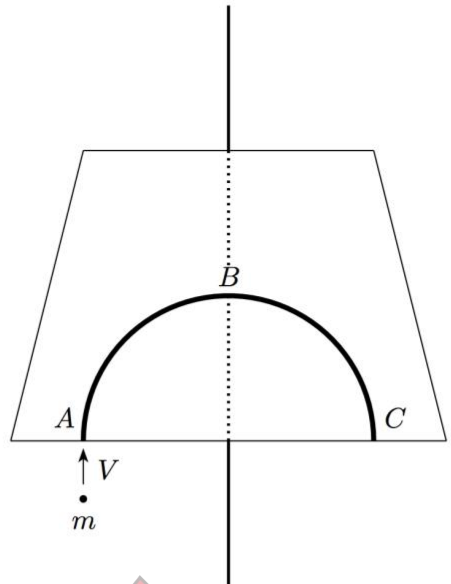
12 A skater of mass M is skating inattentively on a smooth frozen canal. She suddenly realises that she is heading perpendicularly towards the straight canal bank at speed V. She is at a distance d from the bank and can choose one of two methods of trying to avoid it; either she can apply a force of constant magnitude F, acting at right-angles to her velocity, so that she travels in a circle; or she can apply a force of magnitude $\frac{1}{2}F(V^2+v^2)/V^2$ directly backwards, where v is her instantaneous speed. Treating the skater as a particle, find the set of values of d for which she can avoid hitting the bank. Comment **briefly** on the assumption that the skater is a particle.

STEP I 1993 Question 11 (Mechanics)

11 A piece of uniform wire is bent into three sides of a square $ABCD$ so that the side AD is missing. Show that if it is first hung up by the point A and then by the point B then the angle between the two directions of BC is $\tan^{-1} 18$.

STEP I 1991 Question 13 (Mechanics)

13



A heavy smooth lamina of mass M is free to slide without rotation along a straight line on a fixed smooth horizontal table. A smooth groove ABC is inscribed in the lamina, as indicated in the above diagram. The tangents to the groove at A and at B are parallel to the line. When the lamina is stationary, a particle of mass m (where $m < M$) enters the groove at A. The particle is travelling, with speed V, parallel to the line and in the plane of the lamina and table. Calculate the speeds of the particle and of the lamina, when the particle leaves the groove at C.

Suppose now that the lamina is held fixed by a peg attached to the line. Supposing that the groove ABC is a semicircle of radius r, obtain the value of the average force per unit time exerted on the peg by the lamina between the instant that the particle enters the groove and the instant that it leaves it.

STEP II 1994 Question 9 (Mechanics)

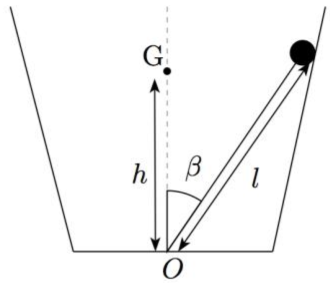
9 A light rod of length $2a$ is hung from a point O by two light inextensible strings OA and OB each of length b and each fixed at O. A particle of mass m is attached to the end A and a particle of mass $2m$ is attached to the end B. Show that, in equilibrium, the angle θ that the rod makes the horizontal satisfies the equation

$$\tan\theta = \frac{a}{3\sqrt{b^2 - a^2}}.$$

Express the tension in the string AO in terms of m, g, a and b.

STEP III 1997 Question 11 (Mechanics)

11

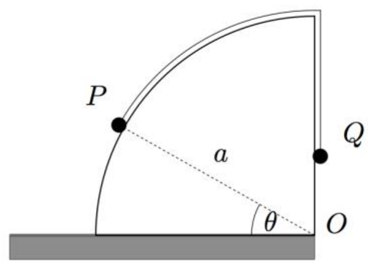


A heavy symmetrical bell and clapper can both swung freely in a vertical plane about a point O on a horizontal beam at the apex of the bell. The mass of the bell is M and its moment of inertia about the beam is Mk^2. Its centre of mass, G, is a distance h from O. The clapper may be regarded as a small heavy ball on a light rod of length l. Initially the bell is held with its axis vertical and its mouth above the beam. The clapper ball rests against the side of the bell, with the rod making an angle β with the axis. The bell is then released. Show that, at the moment when the clapper and bell separate, the clapper rod makes an angle α with the upwards vertical, where

$$\cot\alpha = \cot\beta - \frac{k^2}{hl}\operatorname{cosec}\beta.$$

STEP III 2010 Question 9 (Mechanics)

9



The diagram shows two particles, P and Q, connected by a light inextensible string which passes over a smooth block fixed to a horizontal table. The cross-section of the block is a quarter circle with centre O, which is at the edge of the table, and radius a. The angle between OP and the table is θ. The masses of P and Q are m and M, respectively, where $m < M$.

Initially, P is held at rest on the table and in contact with the block, Q is vertically above O, and the string is taut. Then P is released. Given that, in the subsequent motion, P remains in contact with the block as θ increases from 0 to $\frac{1}{2}\pi$, find an expression, in terms of m, M, θ and g, for the normal reaction of the block on P and show that

$$\frac{m}{M} \geqslant \frac{\pi - 1}{3}.$$

STEP III 1994 Question 11 (Mechanics)

11 A step-ladder has two sections AB and AC, each of length $4a$, smoothly hinged at A and connected by a light elastic rope DE, of natural length $a/4$ and modulus W, where D is on AB, E is on AC and $AD = AE = a$. The section AB, which contains the steps, is uniform and of weight W and the weight of AC is negligible.

The step-ladder rests on a smooth horizontal floor and a man of weight $4W$ carefully ascends it to stand on a rung distant βa from the end of the ladder resting on the floor. Find the height above the floor of the rung on which the man is standing when β is the maximum value at which equilibrium is possible.

STEP II 2010 Question 11 (Mechanics)

11 A uniform rod AB of length $4L$ and weight W is inclined at an angle θ to the horizontal. Its lower end A rests on a fixed support and the rod is held in equilibrium by a string attached to the rod at a point C which is $3L$ from A. The reaction of the support on the rod acts in a direction α to AC and the string is inclined at an angle β to CA. Show that

$$\cot\alpha = 3\tan\theta + 2\cot\beta\,.$$

Given that $\theta = 30°$ and $\beta = 45°$, show that $\alpha = 15°$.

STEP III 2011 Question 9 (Mechanics)

9 Particles P and Q have masses $3m$ and $4m$, respectively. They lie on the outer curved surface of a smooth circular cylinder of radius a which is fixed with its axis horizontal. They are connected by a light inextensible string of length $\frac{1}{2}\pi a$, which passes over the surface of the cylinder. The particles and the string all lie in a vertical plane perpendicular to the axis of the cylinder, and the axis intersects this plane at O. Initially, the particles are in equilibrium.

Equilibrium is slightly disturbed and Q begins to move downwards. Show that while the two particles are still in contact with the cylinder the angle θ between OQ and the vertical satisfies

$$7a\dot{\theta}^2 + 8g\cos\theta + 6g\sin\theta = 10g\,.$$

(i) Given that Q loses contact with the cylinder first, show that it does so when $\theta = \beta$, where β satisfies

$$15\cos\beta + 6\sin\beta = 10.$$

(ii) Show also that while P and Q are still in contact with the cylinder the tension in the string is $\frac{12}{7}mg(\sin\theta + \cos\theta)$.

STEP I 2011 Question 11 (Mechanics)

11 A thin non-uniform bar AB of length $7d$ has centre of mass at a point G, where $AG = 3d$. A light inextensible string has one end attached to A and the other end attached to B. The string is hung over a smooth peg P and the bar hangs freely in equilibrium with B lower than A. Show that

$$3\sin\alpha = 4\sin\beta\,,$$

where α and β are the angles PAB and PBA, respectively.

Given that $\cos\beta = \frac{4}{5}$ and that α is acute, find in terms of d the length of the string and show that the angle of inclination of the bar to the horizontal is $\arctan\frac{1}{7}$.

STEP II 1989 Question 14 (Mechanics)

14 One end of a light inextrnsible string of length l is fixed to a point on the upper surface of a thin, smooth, horizontal table-top, at a distance $(l - a)$ from one edge of the table-top. A particle of mass m is fixed to the other end of the string, and held a distance a away from this edge of the table-top, so that the string is horizontal and taut. The particle is then released. Find the tension in the string after the string has rotated through an angle θ, and show that the largest magnitude of the force on the edge of the table top is $8mg/\sqrt{3}$.
