STEP Past Papers by Topic

STEP Topic - Friction

STEP I 1989 Question 13 (Mechanics)

13 A uniform ladder of mass M rests with its upper end against a smooth vertical wall, and with its lower end on a rough slope which rises upwards towards the wall and makes an angle of ϕ with the horizontal. The acute angle between the ladder and the wall is θ. If the ladder is in equilibrium, show that N and F, the normal reaction and frictional force at the foot of the ladder are given by

$$N = Mg\left(\cos\phi - \frac{\tan\theta\sin\phi}{2}\right),$$

$$F = Mg\left(\sin\phi + \frac{\tan\theta\cos\phi}{2}\right).$$

If the coefficient of friction between the ladder and the slope is 2, and $\phi = 45°$, what is the largest value of θ for which the ladder can rest in equilibrium?

STEP I Specimen Question 11 (Mechanics)

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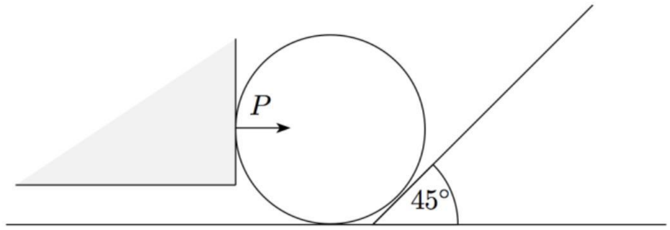


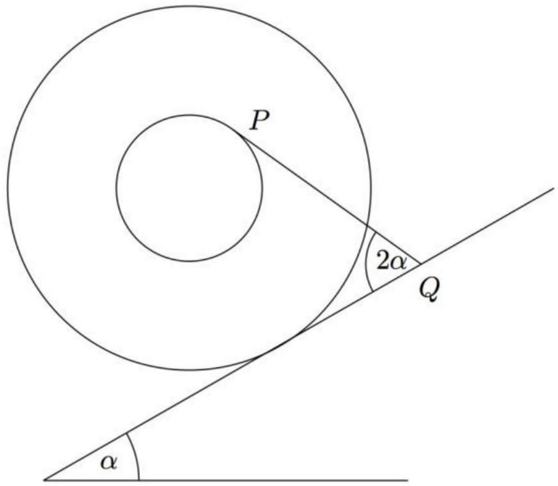
A uniform cylinder of mass M rests on a horizontal floor touching a loading ramp at $45°$ to the horizontal, as shown in the diagram. The cylinder is pushed from the side with a force of magnitude P by the vertical face of a piece of moving equipment. The coefficient of friction between the cylinder and the vertical face is μ and the coefficient of friction between the cylinder and the ramp is v. The value of P is such that the cylinder is just about to roll up the ramp. State on which surface the friction must be limiting, and hence show that

$$P = \frac{Mg}{1-\mu(1+\sqrt{2})}.$$

Show further that $\mu < \sqrt{2}-1$ and $v \geqslant \mu/(\sqrt{2}-\mu)$.

STEP III 1991 Question 11 (Mechanics)

11



A uniform circular cylinder of radius $2a$ with a groove of radius a cut in its central cross-section has mass M. It rests, as shown in the diagram, on a rough plane inclined at an acute angle α to the horizontal. It is supported by a light inextensible string would round the groove and attached to the cylinder at one end. The other end of the string is attached to the plane at Q, the free part of the string, PQ, making an angle 2α with the inclined plane. The coefficient of friction at the contact between the cylinder and the plane is μ. Show that $\mu \geqslant \frac{1}{3}\tan\alpha$.

The string PQ is now detached from the plane and the end Q is fastened to a particle of mass $3M$ which is placed on the plane, the position of the string remain unchanged. Given that $\tan\alpha = \frac{1}{2}$ and that the system remains in equilibrium, find the least value of the coefficient of friction between the particle and the plane.

STEP II 1987 Question 13 (Mechanics)

13 A uniform rod, of mass $3m$ and length $2a$, is freely hinged at one end and held by the other end in a horizontal position. A rough particle, of mass m, is placed on the rod at its mid-point. If the free end is then released, prove that, until the particle begins to slide on the rod, the inclination θ of the rod to the horizontal satisfies the equation

$$5a\dot{\theta}^2 = 8g\sin\theta.$$

The coefficient of friction between the particle and the rod is $\frac{1}{2}$. Show that, when the particle begins to slide, $\tan\theta = \frac{1}{26}$.

STEP I 1990 Question 13 (Mechanics)

13 A rough circular cylinder of mass M and radius a rests on a rough horizontal plane. The curved surface of the cylinder is in contact with a smooth rail, parallel to the axis of the cylinder, which touches the cylinder at a height $a/2$ above the plane. Initially the cylinder is held at rest. A particle of mass m rests in equilibrium on the cylinder, and the normal reaction of the cylinder on the particle makes an angle of θ with the upward vertical. The particle is on the same side of the centre of the cylinder as the rail. The coefficient of friction between the cylinder and the particle and between the cylinder and the plane are both μ. Obtain the condition on θ for the particle to rest in equilibrium. Show that, if the cylinder is released, equilibrium of both particle and cylinder is possible provided another inequality involving μ and θ (which should be found explicitly) is satisfied. Determine the largest possible value of θ for equilibrium, if $m = 7M$ and $\mu = 0.75$.

STEP II 1991 Question 13 (Mechanics)

13 A non-uniform rod AB of mass m is pivoted at one end A so that it can swing freely in a vertical plane. Its centre of mass is a distance d from A and its moment of inertia about any axis perpendicular to the rod through A is mk^2. A small ring of mass αm is free to slide along the rod and the coefficient of friction between the ring and rod is μ. The rod is initially held in a horizontal position with the ring a distance x from A. If $k^2 > xd$, show that when the rod is released, the ring will start to slide when the rod makes an angle θ with the downward vertical, where

$$\mu \tan\theta = \frac{3\alpha x^2 + k^2 + 2xd}{k^2 - xd}.$$

Explain what will happen if (i) $k^2 = xd$ and (ii) $k^2 < xd$.

STEP II 1992 Question 13 (Mechanics)

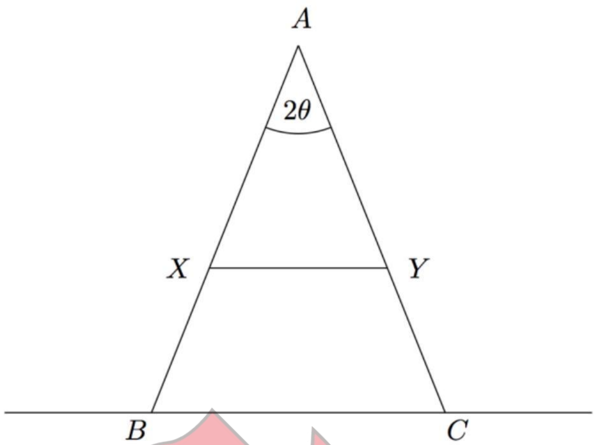
13 Two particles P_1 and P_2, each of mass m, are joined by a light smooth inextensible string of length ℓ. P_1 lies on a table top a distance d from the edge, and P_2 hangs over the edge of the table and is suspended a distance b above the ground. The coefficient of friction between P_1 and the table top is μ, and $\mu < 1$. The system is released from rest. Show that P_1 will fall off the edge of the table if and only if

$$\mu < \frac{b}{2d - b}.$$

Suppose that $\mu > b/(2d - b)$, so that P_1 comes to rest on the table, and that the coefficient of restitution between P_2 and the floor is e. Show that, if $e > 1/(2\mu)$, then P_1 comes to rest before P_2 bounces a second time.

STEP I 1992 Question 12 (Mechanics)

12 The diagram shows a crude step-ladder constructed by smoothly hinging-together two light ladders AB and AC, each of length l, at A. A uniform rod of wood, of mass m, is pin-jointed to X on AB and to Y on AC, where $AX = \frac{3}{4}l = AY$. The angle $\angle XAY$ is 2θ.



The rod XY will break if the tension in it exceeds T. The step-ladder stands on rough horizontal ground (coefficient of friction μ). Given that $\tan\theta > \mu$, find how large a mass M can safely be placed at A and show that if

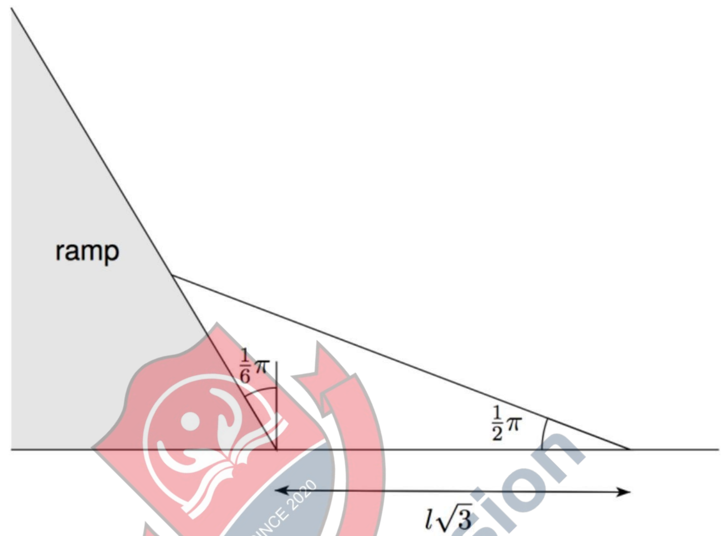
$$\tan\theta > \frac{6T}{mg} + 4\mu$$

the step-ladder will fail under its own weight.

[You may assume that friction is limiting at the moment of collapse.]

STEP III 1988 Question 11 (Mechanics)

11 A uniform ladder of length l and mass m rests with one end in contact with a smooth ramp inclined at an angle of $\pi/6$ to the vertical. The foot of the ladder rests, on horizontal ground, at a distance $l/\sqrt{3}$ from the foot of the ramp, and the coefficient of friction between the ladder and the ground is μ. The ladder is inclined at an angle $\pi/6$ to the horizontal, in the vertical plane containing a line of greatest slope of the ramp. A labourer of mass m intends to climb slowly to the top of the ladder.



(i) Find the value of μ if the ladder slips as soon as the labourer reaches the midpoint.

(ii) Find the minimum value of μ which will ensure that the labourer can reach the top of the ladder.

STEP II 1987 Question 11 (Mechanics)

11 A rough ring of radius a is fixed so that it lies in a plane inclined at an angle α to the horizontal. A uniform heavy rod of length $b(> a)$ has one end smoothly pivoted at the centre of the ring, so that the rod is free to move in any direction. It rests on the circumference of the ring, making an angle θ with the radius to the highest point on the circumference. Find the relation between α, θ and the coefficient of friction, μ, which must hold when the rod is in limiting equilibrium.

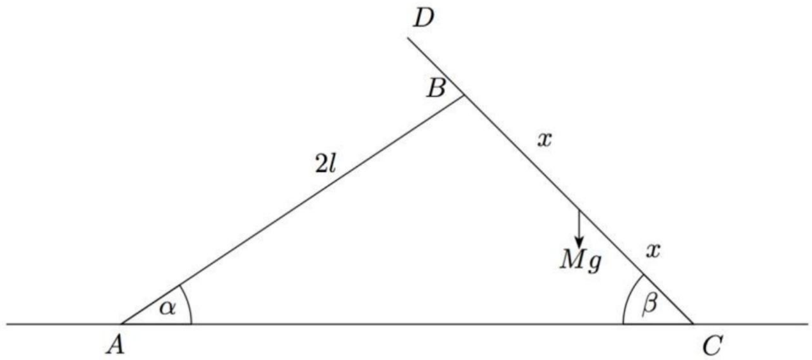
STEP II 1988 Question 13 (Mechanics)

13 A librarian wishes to pick up a row of identical books from a shelf, by pressing her hands on the outer covers of the two outermost books and lifting the whole row together. The covers of the books are all in parallel vertical planes, and the weight of each book is W. With each arm, the librarian can exert a maximum force of P in the vertical direction, and, independently, a maximum force of Q in the horizontal direction. The coefficient of friction between each pair of books and also between each hand and a book is μ. Derive an expression for the maximum number of books that can be picked up without slipping, using this method.

[You may assume that the books are thin enough for the rotational effect of the couple on each book to be ignored.]

STEP I 1991 Question 12 (Mechanics)

12



The above diagram illustrates a makeshift stepladder, made from two equal light planks AB and CD, each of length $2l$. The plank AB is smoothly hinged to the ground at A and makes an angle of α with the horizontal. The other plank CD has its bottom end C resting on the same horizontal ground and makes an angle β with the horizontal. It is pivoted smoothly to B at a point distance $2x$ from C. The coefficient of friction between CD and the ground is μ. A painter of mass M stands on CD, half between C and B. Show that, for equilibrium to be possible,

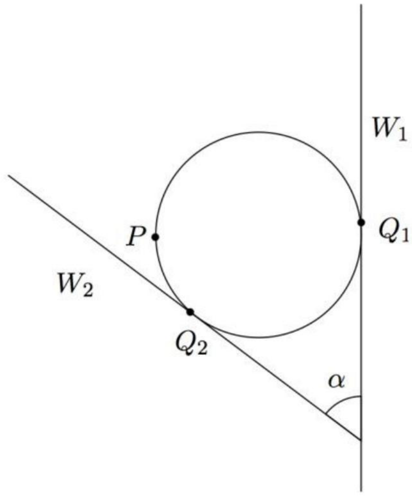
$$\mu \geqslant \frac{\cot\alpha\cot\beta}{2\cot\alpha + \cot\beta}.$$

Suppose now that B coincides with D. Show that, as α varies, the maximum distance from A at which the painter will be standing is

$$l\sqrt{\frac{1+81\mu^2}{1+9\mu^2}}.$$

STEP II 1993 Question 12 (Mechanics)

12



A uniform sphere of mass M and radius r rests between a vertical wall W_1 and an inclined plane W_2 that meets W_1 at an angle α. Q_1 and Q_2 are the points of contact of the sphere with W_1 and W_2 resectively, as shown in the diagram. A particle of mass m is attached to the sphere at P, where PQ_1 is a diameter, and the system is released. The sphere is on the point of slipping at Q_1 and at Q_2. Show that if the coefficients of friction between the sphere and W_1 and W_2 are μ_1 and μ_2 respectively, then

$$m = \frac{\mu_2 + \mu_1 \cos\alpha - \mu_1\mu_2 \sin\alpha}{(2\mu_1\mu_2 + 1)\sin\alpha + (\mu_2 - 2\mu_1)\cos\alpha - \mu_2} M.$$

If the sphere is on the point of rolling about Q_2 instead of slipping, show that

$$m = \frac{M}{\sec\alpha - 1}.$$

STEP III 1996 Question 10 (Mechanics)

10 Two rough solid circular cylinders, of equal radius and length and of uniform density, lie side by side on a rough plane inclined at an angle α to the horizontal, where $0 < \alpha < \pi/2$. Their axes are horizontal and they touch along their entire length. The weight of the upper cylinder is W_1 and the coefficient of friction between it and the plane is μ_1. The corresponding qunatities for the lower cylinder are W_2 and μ_2 respectively and the coefficient of friction between the two cylinders is μ. Show that for equilibrium to be possible:

(i) $W_1 \geqslant W_2$;

(ii) $\mu \geqslant \dfrac{W_1 + W_2}{W_1 - W_2}$;

(iii) $\mu_1 \geqslant \left(\dfrac{2W_1 \cot\alpha}{W_1 + W_2} - 1\right)^{-1}$.

Find the similar inequality to **(iii)** for μ_2.

STEP II 2003 Question 9 (Mechanics)

9 AB is a uniform rod of weight W. The point C on AB is such that $AC > CB$. The rod is in contact with a rough horizontal floor at A and with a cylinder at C. The cylinder is fixed to the floor with its axis horizontal. The rod makes an angle α with the horizontal and lies in a vertical plane perpendicular to the axis of the cylinder. The coefficient of friction between the rod and the floor is $\tan\lambda_1$ and the coefficient of friction between the rod and the cylinder is $\tan\lambda_2$.

Show that if friction is limiting both at A and at C, and $\alpha \neq \lambda_2 - \lambda_1$, then the frictional force acting on the rod at A has magnitude

$$\frac{W\sin\lambda_1\,\sin(\alpha-\lambda_2)}{\sin(\alpha+\lambda_1-\lambda_2)}\,.$$

STEP III 2001 Question 9 (Mechanics)

9 B_1 and B_2 are parallel, thin, horizontal fixed beams. B_1 is a vertical distance $d\sin\alpha$ above B_2, and a horizontal distance $d\cos\alpha$ from B_2, where $0 < \alpha < \pi/2$. A long heavy plank is held so that it rests on the two beams, perpendicular to each, with its centre of gravity at B_1. The coefficients of friction between the plank and B_1 and B_2 are μ_1 and μ_2, respectively, where $\mu_1 < \mu_2$ and $\mu_1 + \mu_2 = 2\tan\alpha$.

The plank is released and slips over the beams experiencing a force of resistance from each beam equal to the limiting frictional force (i.e. the product of the appropriate coefficient of friction and the normal reaction). Show that it will come to rest with its centre of gravity over B_2 in a time

$$\pi\left(\frac{d}{g(\mu_2-\mu_1)\cos\alpha}\right)^{\frac{1}{2}}.$$

STEP III 2000 Question 10 (Mechanics)

10 A sphere of radius a and weight W rests on horizontal ground. A thin uniform beam of weight $3\sqrt{3}\,W$ and length $2a$ is freely hinged to the ground at X, which is a distance $\sqrt{3}\,a$ from the point of contact of the sphere with the ground. The beam rests on the sphere, lying in the same vertical plane as the centre of the sphere. The coefficients of friction between the beam and the sphere and between the sphere and the ground are μ_1 and μ_2 respectively.

Given that the sphere is on the point of slipping at its contacts with both the ground and the beam, find the values of μ_1 and μ_2.

STEP II 1997 Question 9 (Mechanics)

9 A uniform solid sphere of diameter d and mass m is drawn slowly and without slipping from horizontal ground onto a step of height $d/4$ by a horizontal force which is always applied to the highest point of the sphere and is always perpendicular to the vertical plane which forms the face of the step. Find the maximum horizontal force throughout the movement, and prove that the coefficient of friction between the sphere and the edge of the step must exceed $1/\sqrt{3}$.

STEP I 2000 Question 11 (Mechanics)

11 A rod AB of length 0.81 m and mass 5 kg is in equilibrium with the end A on a rough floor and the end B against a very rough vertical wall. The rod is in a vertical plane perpendicular to the wall and is inclined at $45°$ to the horizontal. The centre of gravity of the rod is at G, where $AG = 0.21$ m. The coefficient of friction between the rod and the floor is 0.2, and the coefficient of friction between the rod and the wall is 1.0. Show that the friction cannot be limiting at both A and B.

A mass of 5 kg is attached to the rod at the point P such that now the friction is limiting at both A and B. Determine the length of AP.

STEP I 1995 Question 11 (Mechanics)

11 Two identical uniform cylinders, each of mass m, lie in contact with one another on a horizontal plane and a third identical cylinder rests symmetrically on them in such a way that the axes of the three cylinders are parallel. Assuming that all the surfaces in contact are equally rough, show that the minimum possible coefficient of friction is $2 - \sqrt{3}$.

STEP I 2004 Question 11 (Mechanics)

11 Two uniform ladders AB and BC of equal length are hinged smoothly at B. The weight of AB is W and the weight of BC is $4W$. The ladders stand on rough horizontal ground with $\angle ABC = 60^\circ$. The coefficient of friction between each ladder and the ground is μ.

A decorator of weight $7W$ begins to climb the ladder AB slowly. When she has climbed up $\frac{1}{3}$ of the ladder, one of the ladders slips. Which ladder slips, and what is the value of μ?

STEP III 2002 Question 10 (Mechanics)

10 A light hollow cylinder of radius a can rotate freely about its axis of symmetry, which is fixed and horizontal. A particle of mass m is fixed to the cylinder, and a second particle, also of mass m, moves on the rough inside surface of the cylinder. Initially, the cylinder is at rest, with the fixed particle on the same horizontal level as its axis and the second particle at rest vertically below this axis. The system is then released. Show that, if θ is the angle through which the cylinder has rotated, then

$$\ddot{\theta} = \frac{g}{2a}\left(\cos\theta - \sin\theta\right) ,$$

provided that the second particle does not slip.

Given that the coefficient of friction is $(3+\sqrt{3})/6$, show that the second particle starts to slip when the cylinder has rotated through $60°$.

STEP III 2000 Question 11 (Mechanics)

11 A thin beam is fixed at a height $2a$ above a horizontal plane. A uniform straight rod ACB of length $9a$ and mass m is supported by the beam at C. Initially, the rod is held so that it is horizontal and perpendicular to the beam. The distance AC is $3a$, and the coefficient of friction between the beam and the rod is μ.

The rod is now released. Find the minimum value of μ for which B strikes the horizontal plane before slipping takes place at C.

STEP I 2010 Question 9 (Mechanics)

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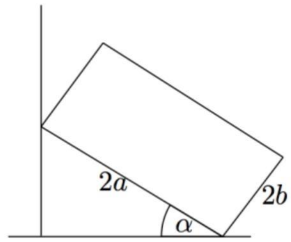


The diagram shows a uniform rectangular lamina with sides of lengths $2a$ and $2b$ leaning against a rough vertical wall, with one corner resting on a rough horizontal plane. The plane of the lamina is vertical and perpendicular to the wall, and one edge makes an angle of α with the horizontal plane. Show that the centre of mass of the lamina is a distance $a\cos\alpha + b\sin\alpha$ from the wall.

The coefficients of friction at the two points of contact are each μ and the friction is limiting at both contacts. Show that

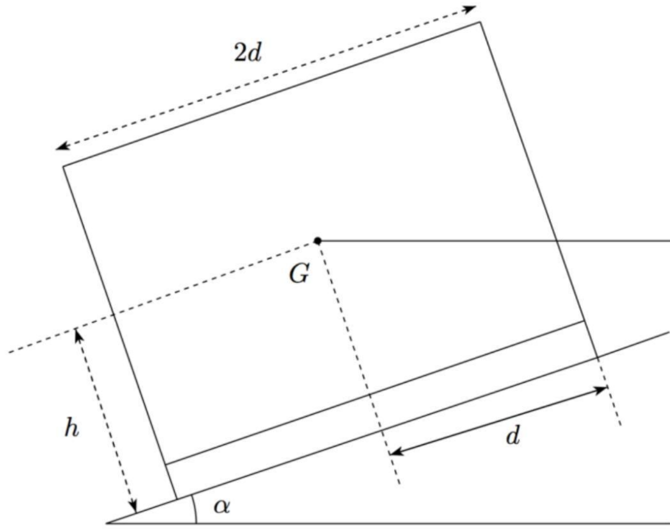
$$a\cos(2\lambda + \alpha) = b\sin\alpha\,,$$

where $\tan\lambda = \mu$.

Show also that if the lamina is square, then $\lambda = \frac{1}{4}\pi - \alpha$.

STEP I 2002 Question 9 (Mechanics)

9



A lorry of weight W stands on a plane inclined at an angle α to the horizontal. Its wheels are a distance $2d$ apart, and its centre of gravity G is at a distance h from the plane, and halfway between the sides of the lorry. A horizontal force P acts on the lorry through G, as shown.

(i) If the normal reactions on the lower and higher wheels of the lorry are equal, show that the sum of the frictional forces between the wheels and the ground is zero.

(ii) If P is such that the lorry does not tip over (but the normal reactions on the lower and higher wheels of the lorry need not be equal), show that

$$W\tan(\alpha-\beta) \leqslant P \leqslant W\tan(\alpha+\beta),$$

where $\tan\beta = d/h$.

STEP II 2005 Question 9 (Mechanics)

9 Two particles, A and B, of masses m and $2m$, respectively, are placed on a line of greatest slope, ℓ, of a rough inclined plane which makes an angle of $30°$ with the horizontal. The coefficient of friction between A and the plane is $\frac{1}{6}\sqrt{3}$ and the coefficient of friction between B and the plane is $\frac{1}{3}\sqrt{3}$. The particles are at rest with B higher up ℓ than A and are connected by a light inextensible string which is taut. A force P is applied to B.

(i) Show that the least magnitude of P for which the two particles move upwards along ℓ is $\frac{11}{8}\sqrt{3}\,mg$ and give, in this case, the direction in which P acts.

(ii) Find the least magnitude of P for which the particles do not slip downwards along ℓ.

STEP II 2007 Question 10 (Pure)

10 A solid figure is composed of a uniform solid cylinder of density ρ and a uniform solid hemisphere of density 3ρ. The cylinder has circular cross-section, with radius r, and height $3r$, and the hemisphere has radius r. The flat face of the hemisphere is joined to one end of the cylinder, so that their centres coincide.

The figure is held in equilibrium by a force P so that one point of its flat base is in contact with a rough horizontal plane and its base is inclined at an angle α to the horizontal. The force P is horizontal and acts through the highest point of the base. The coefficient of friction between the solid and the plane is μ. Show that

$$\mu \geqslant \left|\tfrac{9}{8} - \tfrac{1}{2}\cot\alpha\right| .$$

STEP II 2002 Question 11 (Mechanics)

11 A rigid straight beam AB has length l and weight W. Its weight per unit length at a distance x from B is $\alpha W l^{-1}(x/l)^{\alpha-1}$, where α is a positive constant. Show that the centre of mass of the beam is at a distance $\alpha l/(\alpha+1)$ from B.

The beam is placed with the end A on a rough horizontal floor and the end B resting against a rough vertical wall. The beam is in a vertical plane at right angles to the plane of the wall and makes an angle of θ with the floor. The coefficient of friction between the floor and the beam is μ and the coefficient of friction between the wall and the beam is also μ. Show that, if the equilibrium is limiting at both A and B, then

$$\tan\theta = \frac{1-\alpha\mu^2}{(1+\alpha)\mu}\,.$$

Given that $\alpha = 3/2$ and given also that the beam slides for any $\theta < \pi/4$ find the greatest possible value of μ.

STEP I 2008 Question 11 (Mechanics)

11 A straight uniform rod has mass m. Its ends P_1 and P_2 are attached to small light rings that are constrained to move on a rough rigid circular wire with centre O fixed in a vertical plane, and the angle P_1OP_2 is a right angle. The rod rests with P_1 lower than P_2, and with both ends lower than O. The coefficient of friction between each of the rings and the wire is μ. Given that the rod is in limiting equilibrium (i.e. on the point of slipping at both ends), show that

$$\tan\alpha = \frac{1-2\mu-\mu^2}{1+2\mu-\mu^2},$$

where α is the angle between P_1O and the vertical ($0 < \alpha < 45°$).

Let θ be the acute angle between the rod and the horizontal. Show that $\theta = 2\lambda$, where λ is defined by $\tan\lambda = \mu$ and $0 < \lambda < 22.5°$.

STEP III 2007 Question 11 (Mechanics)

11 **(i)** A wheel consists of a thin light circular rim attached by light spokes of length a to a small hub of mass m. The wheel rolls without slipping on a rough horizontal table directly towards a straight edge of the table. The plane of the wheel is vertical throughout the motion. The speed of the wheel is u, where $u^2 < ag$.

Show that, after the wheel reaches the edge of the table and while it is still in contact with the table, the frictional force on the wheel is zero. Show also that the hub will fall a vertical distance $(ag - u^2)/(3g)$ before the rim loses contact with the table.

(ii) Two particles, each of mass $m/2$, are attached to a light circular hoop of radius a, at the ends of a diameter. The hoop rolls without slipping on a rough horizontal table directly towards a straight edge of the table. The plane of the hoop is vertical throughout the motion. When the centre of the hoop is vertically above the edge of the table it has speed u, where $u^2 < ag$, and one particle is vertically above the other.

Show that, after the hoop reaches the edge of the table and while it is still in contact with the table, the frictional force on the hoop is non-zero and deduce that the hoop will slip before it loses contact with the table.

STEP I 2015 Question 11 (Mechanics)

11 Two long circular cylinders of equal radius lie in equilibrium on an inclined plane, in contact with one another and with their axes horizontal. The weights of the upper and lower cylinders are W_1 and W_2, respectively, where $W_1 > W_2$. The coefficients of friction between the inclined plane and the upper and lower cylinders are μ_1 and μ_2, respectively, and the coefficient of friction between the two cylinders is μ. The angle of inclination of the plane is α (which is positive).

(i) Let F be the magnitude of the frictional force between the two cylinders, and let F_1 and F_2 be the magnitudes of the frictional forces between the upper cylinder and the plane, and the lower cylinder and the plane, respectively. Show that $F = F_1 = F_2$.

(ii) Show that

$$\mu \geqslant \frac{W_1 + W_2}{W_1 - W_2},$$

and that

$$\tan\alpha \leqslant \frac{2\mu_1 W_1}{(1+\mu_1)(W_1 + W_2)}.$$

STEP I 2007 Question 9 (Mechanics)

9 A particle of weight W is placed on a rough plane inclined at an angle of θ to the horizontal. The coefficient of friction between the particle and the plane is μ. A horizontal force X acting on the particle is just sufficient to prevent the particle from sliding down the plane; when a horizontal force kX acts on the particle, the particle is about to slide up the plane. Both horizontal forces act in the vertical plane containing the line of greatest slope.

Prove that

$$(k-1)\left(1+\mu^2\right)\sin\theta\cos\theta = \mu\,(k+1)$$

and hence that $k \geqslant \dfrac{(1+\mu)^2}{(1-\mu)^2}$.

STEP II 1995 Question 9 (Mechanics)

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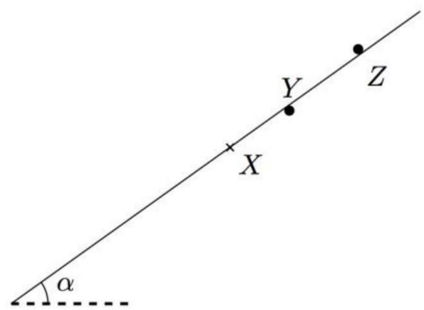

Two thin horizontal bars are parallel and fixed at a distance d apart, and the plane containing them is at an angle α to the horizontal. A thin uniform rod rests in equilibrium in contact with the bars under one and above the other and perpendicular to both. The diagram shows the bards (in cross section and exaggerated in size) with the rod over one bar at Y and under the other at Z. (Thus YZ has length d.) The centre of the rod is at X and XZ has length l. The coefficient of friction between the rod and each bar is μ. Explain why we must have $l \leqslant d$.

Find, in terms of d, l and α, the least possible value of μ. Verify that, when $l = 2d$, your result shows that

$$\mu \geqslant \tfrac{1}{3}\tan\alpha.$$

STEP I 2013 Question 11 (Mechanics)

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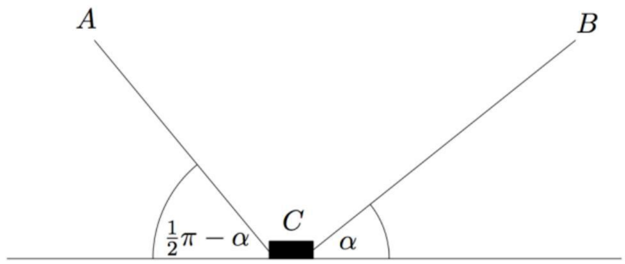


The diagram shows a small block C of weight W initially at rest on a rough horizontal surface. The coefficient of friction between the block and the surface is μ. Two light strings, AC and BC, are attached to the block, making angles $\frac{1}{2}\pi - \alpha$ and α to the horizontal, respectively. The tensions in AC and BC are $T\sin\beta$ and $T\cos\beta$ respectively, where $0 < \alpha + \beta < \frac{1}{2}\pi$.

(i) In the case $W > T\sin(\alpha+\beta)$, show that the block will remain at rest provided

$$W \sin\lambda \geqslant T\cos(\alpha+\beta-\lambda)\,,$$

where λ is the acute angle such that $\tan\lambda = \mu$.

(ii) In the case $W = T\tan\phi$, where $2\phi = \alpha + \beta$, show that the block will start to move in a direction that makes an angle ϕ with the horizontal.

STEP I 2005 Question 9 (Mechanics)

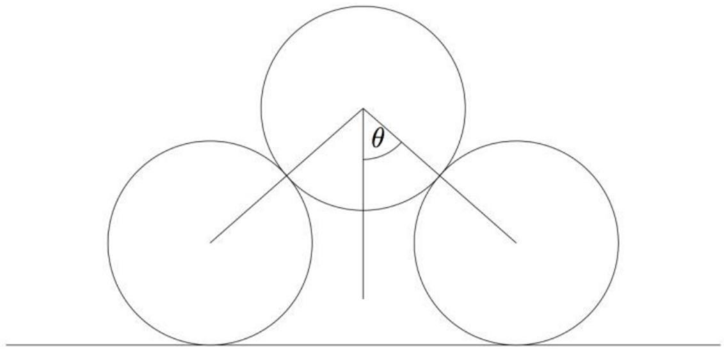
9 A non-uniform rod AB has weight W and length $3l$. When the rod is suspended horizontally in equilibrium by vertical strings attached to the ends A and B, the tension in the string attached to A is T.

When instead the rod is held in equilibrium in a horizontal position by means of a smooth pivot at a distance l from A and a vertical string attached to B, the tension in the string is T. Show that $5T = 2W$.

When instead the end B of the rod rests on rough horizontal ground and the rod is held in equilibrium at an angle θ to the horizontal by means of a string that is perpendicular to the rod and attached to A, the tension in the string is $\frac{1}{2}T$. Calculate θ and find the smallest value of the coefficient of friction between the rod and the ground that will prevent slipping.

STEP II 2013 Question 9 (Mechanics)

9 The diagram shows three identical discs in equilibrium in a vertical plane. Two discs rest, not in contact with each other, on a horizontal surface and the third disc rests on the other two. The angle at the upper vertex of the triangle joining the centres of the discs is 2θ.



The weight of each disc is W. The coefficient of friction between a disc and the horizontal surface is μ and the coefficient of friction between the discs is also μ.

(i) Show that the normal reaction between the horizontal surface and a disc in contact with the surface is $\frac{3}{2}W$.

(ii) Find the normal reaction between two discs in contact and show that the magnitude of the frictional force between two discs in contact is $\dfrac{W\sin\theta}{2(1+\cos\theta)}$.

(iii) Show that if $\mu < 2 - \sqrt{3}$ there is no value of θ for which equilibrium is possible.

STEP II 2012 Question 10 (Pure)

10 A hollow circular cylinder of internal radius r is held fixed with its axis horizontal. A uniform rod of length $2a$ (where $a < r$) rests in equilibrium inside the cylinder inclined at an angle of θ to the horizontal, where $\theta \neq 0$. The vertical plane containing the rod is perpendicular to the axis of the cylinder. The coefficient of friction between the cylinder and each end of the rod is μ, where $\mu > 0$.

Show that, if the rod is on the point of slipping, then the normal reactions R_1 and R_2 of the lower and higher ends of the rod, respectively, on the cylinder are related by

$$\mu(R_1 + R_2) = (R_1 - R_2)\tan\phi$$

where ϕ is the angle between the rod and the radius to an end of the rod.

Show further that

$$\tan\theta = \frac{\mu r^2}{r^2 - a^2(1 + \mu^2)}\,.$$

Deduce that $\lambda < \phi$, where $\tan\lambda = \mu$.

STEP II 2014 Question 9 (Mechanics)

9 A uniform rectangular lamina $ABCD$ rests in equilibrium in a vertical plane with the corner A in contact with a rough vertical wall. The plane of the lamina is perpendicular to the wall. It is supported by a light inextensible string attached to the side AB at a distance d from A. The other end of the string is attached to a point on the wall above A where it makes an acute angle θ with the downwards vertical. The side AB makes an acute angle ϕ with the upwards vertical at A. The sides BC and AB have lengths $2a$ and $2b$ respectively. The coefficient of friction between the lamina and the wall is μ.

(i) Show that, when the lamina is in limiting equilibrium with the frictional force acting upwards,

$$d\sin(\theta+\phi) = (\cos\theta + \mu\sin\theta)(a\cos\phi + b\sin\phi)\,. \qquad (*)$$

(ii) How should $(*)$ be modified if the lamina is in limiting equilibrium with the frictional force acting downwards?

(iii) Find a condition on d, in terms of a, b, $\tan\theta$ and $\tan\phi$, which is necessary and sufficient for the frictional force to act upwards. Show that this condition cannot be satisfied if $b(2\tan\theta + \tan\phi) < a$.

STEP II 2008 Question 11 (Mechanics)

11 A wedge of mass km has the shape (in cross-section) of a right-angled triangle. It stands on a smooth horizontal surface with one face vertical. The inclined face makes an angle θ with the horizontal surface. A particle P, of mass m, is placed on the inclined face and released from rest. The horizontal face of the wedge is smooth, but the inclined face is rough and the coefficient of friction between P and this face is μ.

(i) When P is released, it slides down the inclined plane at an acceleration a relative to the wedge. Show that the acceleration of the wedge is

$$\frac{a\cos\theta}{k+1}\,.$$

To a stationary observer, P appears to descend along a straight line inclined at an angle $45°$ to the horizontal. Show that

$$\tan\theta = \frac{k}{k+1}\,.$$

In the case $k = 3$, find an expression for a in terms of g and μ.

(ii) What happens when P is released if $\tan\theta \leqslant \mu$?
