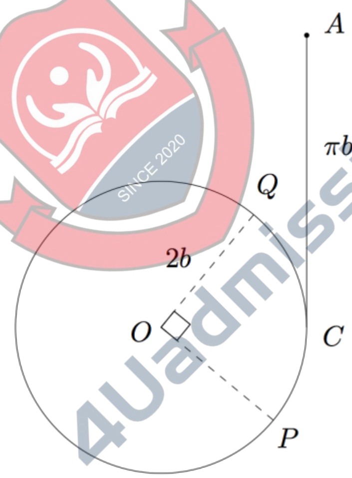
STEP Past Papers by Topic

STEP Topic – Hooke's Law

STEP III 1992 Question 13 (Mechanics)

13



A uniform circular disc of radius $2b$, mass m and centre O is free to turn about a fixed horizontal axis through O perpendicular to the plane of the disc. A light elastic string of modulus kmg, where $k > 4/\pi$, has one end attached to a fixed point A and the other end to the rim of the disc at P. The string is in contact with the rim of the disc along the arc PC, and OC is horizontal. The natural length of the string and the length of the line AC are each πb and AC is vertical. A particle Q of mass m is attached to the rim of the disc and $\angle POQ = 90°$ as shown in the diagram. The system is released from rest with OP vertical and P below O. Show that P reaches C and that then the upward vertical component of the reaction on the axis is $mg(10 - \pi k)/3$.

STEP II 1991 Question 12 (Mechanics)

12 A particle is attached to one end B of a light elastic string of unstretched length a. Initially the other end A is at rest and the particle hangs at rest at a distance $a + c$ vertically below A. At time $t = 0$, the end A is forced to oscillate vertically, its downwards displacement at time t being $b \sin pt$. Let $x(t)$ be the downwards displacement of the particle at time t from its initial equilibrium position. Show that, while the string remains taut, $x(t)$ satisfies

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -n^2(x - b\sin pt),$$

where $n^2 = g/c$, and that if $0 < p < n$, $x(t)$ is given by

$$x(t) = \frac{bn}{n^2 - p^2}(n \sin pt - p \sin nt).$$

Write down a necessary and sufficient condition that the string remains taut throughout the subsequent motion, and show that it is satisfied if $pb < (n - p)c$.

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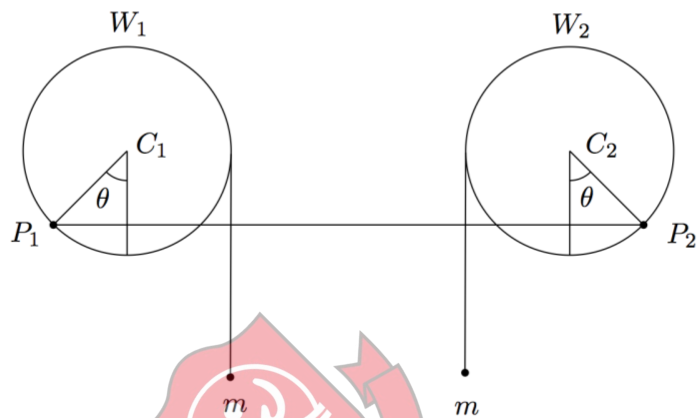
STEP II 1992 Question 12 (Mechanics)

12 In the figure, W_1 and W_2 are wheels, both of radius r. Their centres C_1 and C_2 are fixed at the same height, a distance d apart, and each wheel is free to rotate, without friction, about its centre. Both wheels are in the same vertical plane. Particles of mass m are suspended from W_1 and W_2 as shown, by light inextensible strings would round the wheels. A light elastic string of natural length d and modulus elasticity λ is fixed to the rims of the wheels at the points P_1 and P_2. The lines joining C_1 to P_1 and C_2 to P_2 both make an angle θ with the vertical. The system is in equilibrium.



Show that

$$\sin 2\theta = \frac{mgd}{\lambda r}.$$

For what value or values of λ (in terms of m, d, r and g) are there

(i) no equilibrium positions,

(ii) just one equilibrium position,

(iii) exactly two equilibrium positions,

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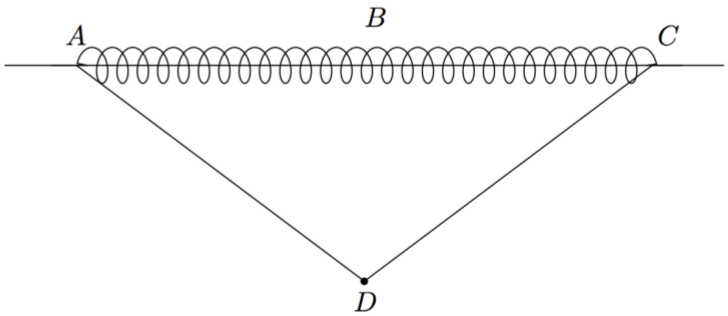
STEP I 1990 Question 12 (Mechanics)

12



In the above diagram, ABC represents a light spring of natural length $2l$ and modulus of elasticity λ, which is coiled round a smooth fixed horizontal rod. B is the midpoint of AC. The two ends of a light inelastic string of length $2l$ are attached to the spring at A and C. A particle of mass m is fixed to the string at D, the midpoint of the string. The system can be in equilibrium with the angle CAD equal to $\pi/6$. Show that

$$mg = \lambda\left(\frac{2}{\sqrt{3}} - 1\right).$$

Write the length AC as $2xl$, obtain an expression for the potential energy of the system as a function of x.

The particle is held at B, and the spring is restored to its natural length $2l$. The particle is then released and falls vertically. Obtain an equation satisfied by x when the particle next comes to rest. Verify numerically that a possible solution for x is approximately 0.66.

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STEP I 1987 Question 10 (Mechanics)

10 A rubber band band of length 2π and modulus of elasticity λ encircles a smooth cylinder of unit radius, whose axis is horizontal. A particle of mass m is attached to the lowest point of the band, and hangs in equilibrium at a distance x below the axis of the cylinder. Obtain an expression in terms of x for the stretched length of the band in equilibrium.

What is the value of λ if $x = 2$?

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STEP III 1993 Question 12 (Mechanics)

12 $ABCD$ is a horizontal line with $AB = CD = a$ and $BC = 6a$. There are fixed smooth pegs at B and C. A uniform string of natural length $2a$ and modulus of elasticity kmg is stretched from A to D, passing over the pegs at B and C. A particle of mass m is attached to the midpoint P of the string. When the system is in equilibrium, P is a distance $a/4$ below BC. Evaluate k.

The particle is pulled down to a point Q, which is at a distance pa below the mid-point of BC, and is released from rest. P rises to a point R, which is at a distance $3a$ above BC. Show that $2p^2 - p - 17 = 0$.

Show also that the tension in the strings is less when the particle is at R than when the particle is at Q.

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STEP I 1988 Question 13 (Mechanics)

13 A piece of circus apparatus consists of a rigid uniform plank of mass 1000 kg, suspended in a horizontal position by two equal light vertical ropes attached to the ends. The ropes each have natural length 10 m and modulus of elasticity 490 000 N. Initially the plank is hanging in equilibrium. Nellie, an elephant of mass 4000 kg, lands in the middle of the plank while travelling vertically downwards at speed 5 ms^{-1}. While carrying Nellie, the plank comes instantaneously to rest at a negligible height above the floor, and at this instant Nellie steps nimbly and gently off the plank onto the floor. Assuming that the plank remains horizontal, and the rope remain vertical, throughout the motion, find to three significant figures its initial height above the floor.

During the motion after Nellie alights, do the ropes ever become slack?

[Take g to be 9.8 ms^{-1}.]

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STEP II 2000 Question 11 (Mechanics)

11 The string AP has a natural length of $1{\cdot}5$metres and modulus of elasticity equal to $5g$ newtons. The end A is attached to the ceiling of a room of height $2\cdot 5$ metres and a particle of mass $0{\cdot}5$ kg is attached to the end P. The end P is released from rest at a point $0{\cdot}5$ metres above the floor and vertically below A. Show that the string becomes slack, but that P does not reach the ceiling.

Show also that while the string is in tension, P executes simple harmonic motion, and that the time in seconds that elapses from the instant when P is released to the instant when P first returns to its original position is

$$\left(\frac{8}{3g}\right)^{\frac{1}{2}} + \left(\frac{3}{5g}\right)^{\frac{1}{2}}\Big(\pi - \arccos(3/7)\Big).$$

[Note that $\arccos x$ is another notation for $\cos^{-1} x$.]

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STEP III 2004 Question 10 (Mechanics)

10 A particle P of mass m is attached to points A and B, where A is a distance $9a$ vertically above B, by elastic strings, each of which has modulus of elasticity $6mg$. The string AP has natural length $6a$ and the string BP has natural length $2a$. Let x be the distance AP.

The system is released from rest with P on the vertical line AB and $x = 6a$. Show that the acceleration $\ddot{x}$ of P is $\dfrac{4g}{a}(7a - x)$ for $6a < x < 7a$ and $\dfrac{g}{a}(7a - x)$ for $7a < x < 9a$.

Find the time taken for the particle to reach B.

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STEP I 2001 Question 11 (Mechanics)

11 A smooth cylinder with circular cross-section of radius a is held with its axis horizontal. A light elastic band of unstretched length $2\pi a$ and modulus of elasticity λ is wrapped round the circumference of the cylinder, so that it forms a circle in a plane perpendicular to the axis of the cylinder. A particle of mass m is then attached to the rubber band at its lowest point and released from rest.

(i) Given that the particle falls to a distance $2a$ below the below the axis of the cylinder, but no further, show that

$$\lambda = \frac{9\pi mg}{(3\sqrt{3} - \pi)^2} \,.$$

(ii) Given instead that the particle reaches its maximum speed at a distance $2a$ below the axis of the cylinder, find a similar expression for λ.

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STEP I 1996 Question 9 (Mechanics)

9 A bungee-jumper of mass m is attached by means of a light rope of natural length l and modulus of elasticity mg/k, where k is a constant, to a bridge over a ravine. She jumps from the bridge and falls vertically towards the ground. If she only just avoids hitting the ground, show that the height h of the bridge above the floor of the ravine satisfies

$$h^2 - 2hl(k+1) + l^2 = 0,$$

and hence find h. Show that the maximum speed v which she attains during her fall satisfies

$$v^2 = (k+2)gl.$$

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STEP I 1995 Question 10 (Mechanics)

10 A small ball of mass m is suspended in equilibrium by a light elastic string of natural length l and modulus of elasticity λ. Show that the total length of the string in equilibrium is $l(1+mg/\lambda)$.

If the ball is now projected downwards from the equilibrium position with speed u_0, show that the speed v of the ball at distance x below the equilibrium position is given by

$$v^2 + \frac{\lambda}{lm}x^2 = u_0^2.$$

At distance h, where $\lambda h^2 < lmu_0^2$, below the equilibrium position is a horizontal surface on which the ball bounces with a coefficient of restitution e. Show that after one bounce the velocity u_1 at $x = 0$ is given by

$$u_1^2 = e^2 u_0^2 + \frac{\lambda}{lm}h^2(1 - e^2),$$

and that after the second bounce the velocity u_2 at $x = 0$ is given by

$$u_2^2 = e^4 u_0^2 + \frac{\lambda}{lm}h^2(1 - e^4).$$

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STEP III 2014 Question 10 (Mechanics)

10 Two particles X and Y, of equal mass m, lie on a smooth horizontal table and are connected by a light elastic spring of natural length a and modulus of elasticity λ. Two more springs, identical to the first, connect X to a point P on the table and Y to a point Q on the table. The distance between P and Q is $3a$.

Initially, the particles are held so that $XP = a$, $YQ = \frac{1}{2}a$, and $PXYQ$ is a straight line. The particles are then released.

At time t, the particle X is a distance $a + x$ from P and the particle Y is a distance $a + y$ from Q. Show that

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{\lambda}{a}(2x + y)$$

and find a similar expression involving $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$. Deduce that

$$x - y = A\cos\omega t + B\sin\omega t$$

where A and B are constants to be determined and $ma\omega^2 = \lambda$. Find a similar expression for $x + y$.

Show that Y will never return to its initial position.

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STEP III 1997 Question 9 (Mechanics)

9 A uniform rigid rod BC is suspended from a fixed point A by light stretched springs AB, AC. The springs are of different natural lengths but the ratio of tension to extension is the same constant κ for each. The rod is *not* hanging vertically. Show that the ratio of the lengths of the stretched springs is equal to the ratio of the natural lengths of the unstretched springs.

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STEP III 20131 Question 11 (Mechanics)

11 An equilateral triangle, comprising three light rods each of length $\sqrt{3}a$, has a particle of mass m attached to each of its vertices. The triangle is suspended horizontally from a point vertically above its centre by three identical springs, so that the springs and rods form a tetrahedron. Each spring has natural length a and modulus of elasticity kmg, and is light. Show that when the springs make an angle θ with the horizontal the tension in each spring is

$$\frac{kmg(1-\cos\theta)}{\cos\theta}\,.$$

Given that the triangle is in equilibrium when $\theta = \frac{1}{6}\pi$, show that $k = 4\sqrt{3} + 6$.

The triangle is released from rest from the position at which $\theta = \frac{1}{3}\pi$. Show that when it passes through the equilibrium position its speed V satisfies

$$V^2 = \frac{4ag}{3}(6+\sqrt{3})\,.$$

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STEP III 2008 Question 10 (Mechanics)

10 A long string consists of n short light strings joined together, each of natural length ℓ and modulus of elasticity λ. It hangs vertically at rest, suspended from one end. Each of the short strings has a particle of mass m attached to its lower end. The short strings are numbered 1 to n, the nth short string being at the top. By considering the tension in the rth short string, determine the length of the long string. Find also the elastic energy stored in the long string.

A uniform heavy rope of mass M and natural length L_0 has modulus of elasticity λ. The rope hangs vertically at rest, suspended from one end. Show that the length, L, of the rope is given by

$$L = L_0\left(1 + \frac{Mg}{2\lambda}\right),$$

and find an expression in terms of L, L_0 and λ for the elastic energy stored in the rope.

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STEP II 1987 Question 14 (Mechanics)

14 A thin uniform elastic band of mass m, length l and modulus of elasticity λ is pushed on to a smooth circular cone of vertex angle 2α, in such a way that all elements of the band are the same distance from the vertex. It is then released from rest. Let $x(t)$ be the length of the band at time t after release, and let t_0 be the time at which the band becomes slack.

Assuming that a small element of the band which subtends an angle $\delta\theta$ at the axis of the cone experiences a force, due to the tension T in the band, of magnitude $T\delta\theta$ directed towards the axis, and ignoring the effects of gravity, show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{4\pi^2\lambda}{ml}(x-l)\sin^2\alpha = 0, \qquad (0 < t < t_0).$$

Find the value of t_0.

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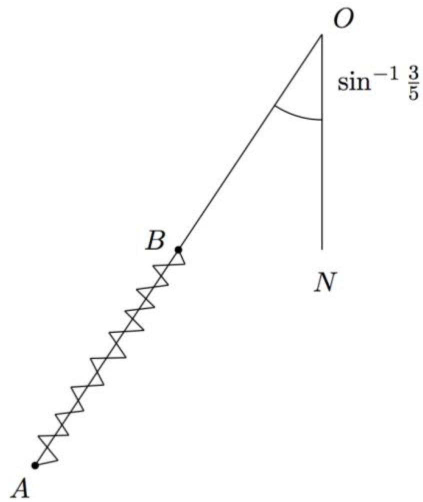
STEP III 1991 Question 14 (Mechanics)

14



The end O of a smooth light rod OA of length $2a$ is a fixed point. The rod OA makes a fixed angle $\sin^{-1}\frac{3}{5}$ with the downward vertical ON, but is free to rotate about ON. A particle of mass m is attached to the rod at A and a small ring B of mass m is free to slide on the rod but is joined to a spring of natural length a and modulus of elasticity kmg. The vertical plane containing the rod OA rotates about ON with constant angular velocity $\sqrt{5g/2a}$ and B is at rest relative to the rod. Show that the length of OB is

$$\frac{(10k+8)a}{10k-9}.$$

Given that the reaction of the rod on the particle at A makes an angle $\tan^{-1}\frac{13}{21}$ with the horizontal, find the value of k. Find also the magnitude of the reaction between the rod and the ring B.

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