STEP Past Papers by Topic

STEP Topic – Pulley

STEP I 1991 Question 11 (Mechanics)

11 A piledriver consists of a weight of mass M connected to a lighter counterweight of mass m by a light inextensible string passing over a smooth light fixed pulley. By considerations of energy or otherwise, show that if the weights are released from rest, and move vertically, then as long as the string remains taut and no collisions occur, the weights experience a constant acceleration of magnitude

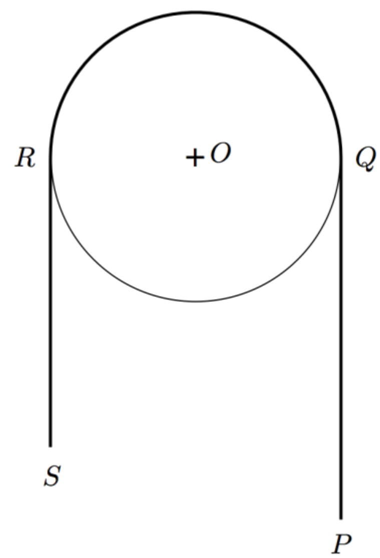
$$g\left(\frac{M-m}{M+m}\right).$$

Initially the weight is held vertically above the pile, and is released from rest. During the subsequent motion both weights move vertically and the only collisions are between the weight and the pile. Treating the pile as fixed and the collisions as completely inelastic, show that, if just before a collision the counterweight is moving with speed v, then just before the next collision it will be moving with speed $mv/(M+m)$. [You may assume that when the string becomes taut, the momentum lost by one weight equals that gained by the other.]

Further show that the times between successive collisions with the pile form a geometric progression. Show that the total time before the weight finally comes to rest is three times the time from the start to the first impact.

STEP III 1993 Question 13 (Mechanics)

13



A uniform circular disc with radius a, mass $4m$ and centre O is freely mounted on a fixed horizontal axis which is perpendicular to its plane and passes through O. A uniform heavy chain PS of length $(4+\pi)a$, mass $(4+\pi)m$ and negligible thickness is hung over the rim of the disc as shown in the diagram. Q and R are the points of the chain at the same level as O. The contact between the chain and the rim of the disc is sufficiently rough to prevent slipping. Initially, the system is at rest with $PQ = RS = 2a$. A particle of mass m is attached to the chain at P and the system is released. By considering the energy of the system, show that when P has descended a distance x, its speed v is given by

$$(\pi+7)av^2 = 2g(x^2+ax).$$

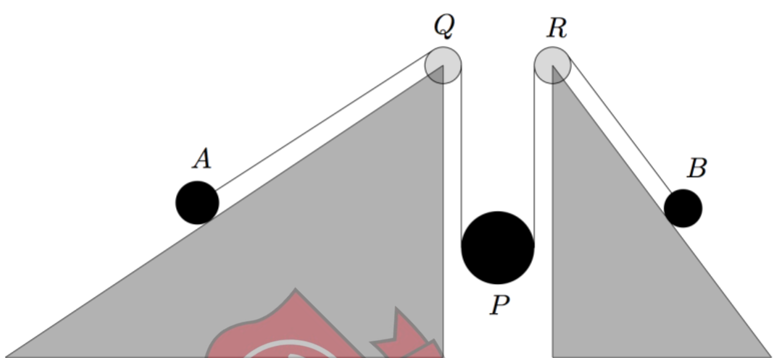
By considering the part PQ of the chain as a body of variable mass, show that when S reaches R the tension in the chain at Q is

$$\frac{5\pi-2}{\pi+7}mg.$$

STEP I 2012 Question 11 (Mechanics)

11 The diagram shows two particles, A of mass $5m$ and B of mass $3m$, connected by a light inextensible string which passes over two smooth, light, fixed pulleys, Q and R, and under a smooth pulley P which has mass M and is free to move vertically.

Particles A and B lie on fixed rough planes inclined to the horizontal at angles of $\arctan \frac{7}{24}$ and $\arctan \frac{4}{3}$ respectively. The segments AQ and RB of the string are parallel to their respective planes, and segments QP and PR are vertical. The coefficient of friction between each particle and its plane is μ.



(i) Given that the system is in equilibrium, with both A and B on the point of moving up their planes, determine the value of μ and show that $M = 6m$.

(ii) In the case when $M = 9m$, determine the initial accelerations of A, B and P in terms of g.

STEP II 2015 Question 10 (Pure)

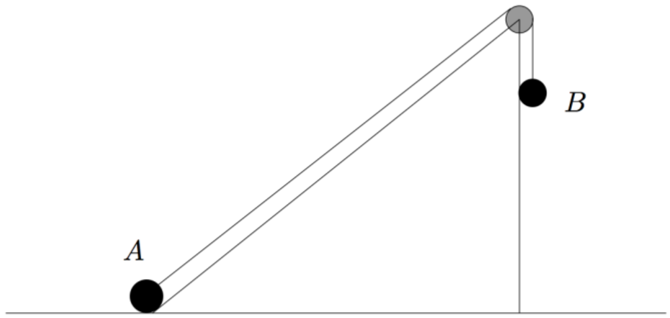
10 A particle of mass m is pulled along the floor of a room in a straight line by a light string which is pulled at constant speed V through a hole in the ceiling. The floor is smooth and horizontal, and the height of the room is h. Find, in terms of V and θ, the speed of the particle when the string makes an angle of θ with the vertical (and the particle is still in contact with the floor). Find also the acceleration, in terms of V, h and θ.

Find the tension in the string and hence show that the particle will leave the floor when

$$\tan^4\theta = \frac{V^2}{gh}\,.$$

STEP I 1994 Question 11 (Mechanics)

11



The diagram shows a small railway wagon A of mass m standing at the bottom of a smooth railway track of length d inclined at an angle θ to the horizontal. A light inextensible string, also of length d, is connected to the wagon and passes over a light frictionless pulley at the top of the incline. On the other end of the string is a ball B of mass M which hangs freely. The system is initially at rest and is then released.

(i) Find the condition which m, M and θ must satisfy to ensure that the ball will fall to the ground. Assuming that this condition is satisfied, show that the velocity v of the ball when it hits the ground satisfies

$$v^2 = \frac{2g(M - m\sin\theta)d\sin\theta}{M+m}.$$

(ii) Find the condition which m, M and θ must satisfy if the wagon is not to collide with the pulley at the top of the incline.

STEP I 1998 Question 11 (Mechanics)

11 Hank's Gold Mine has a very long vertical shaft of height l. A light chain of length l passes over a small smooth light fixed pulley at the top of the shaft. To one end of the chain is attached a bucket A of negligible mass and to the other a bucket B of mass m. The system is used to raise ore from the mine as follows. When bucket A is at the top it is filled with mass $2m$ of water and bucket B is filled with mass λm of ore, where $0 < \lambda < 1$. The buckets are then released, so that bucket A descends and bucket B ascends. When bucket B reaches the top both buckets are emptied and released, so that bucket B descends and bucket A ascends. The time to fill and empty the buckets is negligible. Find the time taken from the moment bucket A is released at the top until the first time it reaches the top again.

This process goes on for a very long time. Show that, if the greatest amount of ore is to be raised in that time, then λ must satisfy the condition $\mathrm{f}'(\lambda) = 0$ where

$$\mathrm{f}(\lambda) = \frac{\lambda(1-\lambda)^{1/2}}{(1-\lambda)^{1/2} + (3+\lambda)^{1/2}}.$$

STEP III 2012 Question 9 (Mechanics)

9 A pulley consists of a disc of radius r with centre O and a light thin axle through O perpendicular to the plane of the disc. The disc is non-uniform, its mass is M and its centre of mass is at O. The axle is fixed and horizontal.

Two particles, of masses m_1 and m_2 where $m_1 > m_2$, are connected by a light inextensible string which passes over the pulley. The contact between the string and the pulley is rough enough to prevent the string sliding. The pulley turns and the vertical force on the axle is found, by measurement, to be $P + Mg$.

(i) The moment of inertia of the pulley about its axle is calculated assuming that the pulley rotates without friction about its axle. Show that the calculated value is

$$\frac{((m_1 + m_2)P - 4m_1m_2g)r^2}{(m_1 + m_2)g - P}\,. \qquad (*)$$

(ii) Instead, the moment of inertia of the pulley about its axle is calculated assuming that a couple of magnitude C due to friction acts on the axle of the pulley. Determine whether this calculated value is greater or smaller than $(*)$.

Show that $C < (m_1 - m_2)rg$.

STEP II 2000 Question 10 (Pure)

10 A long light inextensible string passes over a fixed smooth light pulley. A particle of mass 4 kg is attached to one end A of this string and the other end is attached to a second smooth light pulley. A long light inextensible string BC passes over the second pulley and has a particle of mass 2 kg attached at B and a particle of mass of 1 kg attached at C. The system is held in equilibrium in a vertical plane. The string BC is then released from rest. Find the accelerations of the two moving particles.

After T seconds, the end A is released so that all three particles are now moving in a vertical plane. Find the accelerations of A, B and C in this second phase of the motion. Find also, in terms of g and T, the speed of A when B has moved through a total distance of $0.6gT^2$ metres.

STEP I 2006 Question 9 (Mechanics)

9 A block of mass 4 kg is at rest on a smooth, horizontal table. A smooth pulley P is fixed to one edge of the table and a smooth pulley Q is fixed to the opposite edge. The two pulleys and the block lie in a straight line.

Two horizontal strings are attached to the block. One string runs over pulley P; a particle of mass x kg hangs at the end of this string. The other string runs over pulley Q; a particle of mass y kg hangs at the end of this string, where $x > y$ and $x + y = 6$.

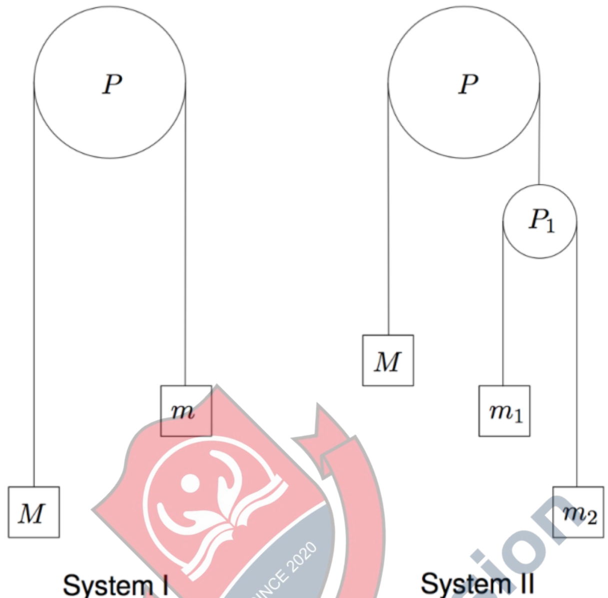
The system is released from rest with the strings taut. When the 4 kg block has moved a distance d, the string connecting it to the particle of mass x kg is cut. Show that the time taken by the block from the start of the motion until it first returns to rest (assuming that it does not reach the edge of the table) is $\sqrt{d/(5g)}\,\mathrm{f}(y)$, where

$$\mathrm{f}(y) = \frac{10}{\sqrt{6-2y}} + \left(1 + \frac{4}{y}\right)\sqrt{6-2y}.$$

Calculate the value of y for which $\mathrm{f}'(y) = 0$.

STEP I 2014 Question 11 (Mechanics)

11 The diagrams below show two separate systems of particles, strings and pulleys. In both systems, the pulleys are smooth and light, the strings are light and inextensible, the particles move vertically and the pulleys labelled with P are fixed. The masses of the particles are as indicated on the diagrams.



(i) For system I show that the acceleration, a_1, of the particle of mass M, measured in the downwards direction, is given by

$$a_1 = \frac{M-m}{M+m}\,g\,,$$

where g is the acceleration due to gravity. Give an expression for the force on the pulley due to the tension in the string.

(ii) For system II show that the acceleration, a_2, of the particle of mass M, measured in the downwards direction, is given by

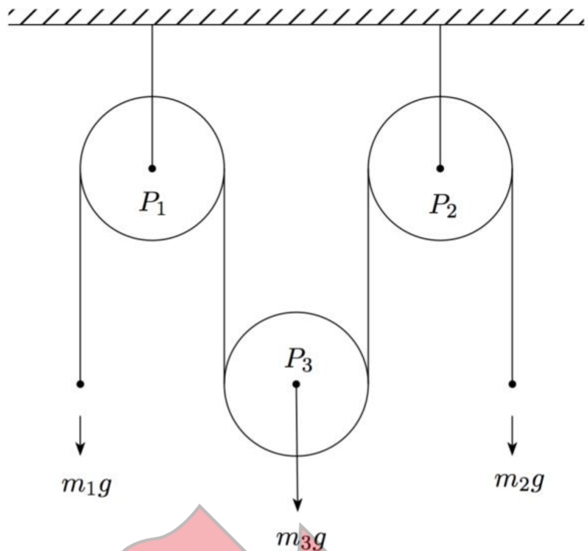
$$a_2 = \frac{M-4\mu}{M+4\mu}\,g\,,$$

where $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$.

In the case $m = m_1 + m_2$, show that $a_1 = a_2$ if and only if $m_1 = m_2$.

STEP II 1992 Question 14 (Mechanics)

14



In the diagram P_1 and P_2 are smooth light pulleys fixed at the same height, and P_3 is a third smooth light pulley, freely suspended. A smooth light inextensible string runs over P_1, under P_3 and over P_2, as shown: the parts of the string not in contact with any pulley are vertical. A particle of mass m_3 is attached to P_3. There is a particle of mass m_1 attached to the end of the string below P_1 and a particle of mass m_2 attached to the other end, below P_2. The system is released from rest. Find the tension in the string, and show that the pulley P_3 will remain at rest if

$$4m_1m_2 = m_3(m_1 + m_2).$$
