STEP Past Papers by Topic

STEP Topic – SHM

STEP III 1989 Question 13 (Mechanics)

13 The points A, B, C, D and E lie on a thin smooth horizontal table and are equally spaced on a circle with centre O and radius a. At each of these points there is a small smooth hole in the table. Five elastic strings are threaded through the holes, one end of each beging attached at O under the table and the other end of each being attached to a particle P of mass m on top of the table. Each of the string has natural length a and modulus of elasticity λ. If P is displaced from O to any point F on the table and released from rest, show that P moves with simple harmonic motion of period T, where

$$T = 2\pi\sqrt{\frac{am}{5\lambda}}.$$

The string PAO is replaced by one of natural length a and modulus $k\lambda$. P is displaced along OA from its equilibrium position and released. Show that P still moves in a straight line with simple harmonic motion, and, given that the period is $T/2$, find k.

STEP II 1993 Question 13 (Mechanics)

13 The force F of repulsion between two particles with positive charges Q and Q' is given by $F = kQQ'/r^2$, where k is a positive constant and r is the distance between the particles. Two small beads P_1 and P_2 are fixed to a straight horizontal smooth wire, a distance d apart. A third bead P_3 of mass m is free to move along the wire between P_1 and P_3. The beads carry positive electrical charges Q_1, Q_2 and Q_3. If P_3 is in equilibrium at a distance a from P_1, show that

$$a = \frac{d\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}}.$$

Suppose that P_3 is displaced slightly from its equilibrium position and released from rest. Show that it performs approximate simple harmonic motion with period

$$\frac{\pi d}{(\sqrt{Q_1} + \sqrt{Q_2})^2}\sqrt{\frac{2md\sqrt{Q_1 Q_2}}{kQ_3}}.$$

[You may use the fact that $\dfrac{1}{(a+y)^2} \approx \dfrac{1}{a^2} - \dfrac{2y}{a^3}$ for small y.]

STEP III 1996 Question 11 (Mechanics)

11 A smooth circular wire of radius a is held fixed in a vertical plane with light elastic strings of natural length a and modulus λ attached to the upper and lower extremities, A and C respectively, of the vertical diameter. The other ends of the two strings are attached to a small ring B which is free to slide on the wire. Show that, while both strings remain taut, the equation for the motion of the ring is

$$2ma\ddot{\theta} = \lambda(\cos\theta - \sin\theta) - mg\sin\theta,$$

where θ is the angle $\angle CAB$.

Initially the system is at rest in equilibrium with $\sin\theta = \frac{3}{5}$. Deduce that $5\lambda = 24mg$.

The ring is now displaced slightly. Show that, in the ensuing motion, it will oscillate with period approximately

$$10\pi\sqrt{\frac{a}{91g}}\,.$$

STEP III 2003 Question 9 (Mechanics)

9 A particle P of mass m is constrained to move on a vertical circle of smooth wire with centre O and of radius a. L is the lowest point of the circle and H the highest and $\angle LOP = \theta$. The particle is attached to H by an elastic string of natural length a and modulus of elasticity αmg, where $\alpha > 1$. Show that, if $\alpha > 2$, there is an equilibrium position with $0 < \theta < \pi$.

Given that $\alpha = 2 + \sqrt{2}$, and that $\theta = \frac{1}{2}\pi + \phi$, show that

$$\ddot{\phi} \approx -\frac{g(\sqrt{2}+1)}{2a}\,\phi$$

when ϕ is small.

For this value of α, explain briefly what happens to the particle if it is given a small displacement when $\theta = \frac{1}{2}\pi$.

STEP II 1996 Question 11 (Mechanics)

11 A particle hangs in equilibrium from the ceiling of a stationary lift, to which it is attached by an elastic string of natural length l extended to a length $l + a$. The lift now descends with constant acceleration f such that $0 < f < g/2$. Show that the extension y of the string from its equilibrium length satisfies the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \frac{g}{a} y = g - f.$$

Hence show that the string never becomes slack and the amplitude of the oscillation of the particle is af/g.

After a time T the lift stops accelerating and moves with constant velocity. Show that the string never becomes slack and the amplitude of the oscillation is now

$$\frac{2af}{g} |\sin \tfrac{1}{2}\omega T|,$$

where $\omega^2 = g/a$.

STEP III 1998 Question 9 (Mechanics)

9 A uniform right circular cone of mass m has base of radius a and perpendicular height h from base to apex. Show that its moment of inertia about its axis is $\frac{3}{10}ma^2$, and calculate its moment of inertia about an axis through its apex parallel to its base.
[*Any theorems used should be stated clearly.*]

The cone is now suspended from its apex and allowed to perform small oscillations. Show that their period is

$$2\pi\sqrt{\frac{4h^2+a^2}{5gh}}\,.$$

[*You may assume that the centre of mass of the cone is a distance $\frac{3}{4}h$ from its apex.*]

STEP III 2007 Question 9 (Mechanics)

9 Two small beads, A and B, each of mass m, are threaded on a smooth horizontal circular hoop of radius a and centre O. The angle θ is the acute angle determined by $2\theta = \angle AOB$.

The beads are connected by a light straight spring. The energy stored in the spring is

$$mk^2a^2(\theta - \alpha)^2,$$

where k and α are constants satisfying $k > 0$ and $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$.

The spring is held in compression with $\theta = \beta$ and then released. Find the period of oscillations in the two cases that arise according to the value of β and state the value of β for which oscillations do not occur.

STEP III 2010 Question 10 (Mechanics)

10 A small bead B, of mass m, slides without friction on a fixed horizontal ring of radius a. The centre of the ring is at O. The bead is attached by a light elastic string to a fixed point P in the plane of the ring such that $OP = b$, where $b > a$. The natural length of the elastic string is c, where $c < b - a$, and its modulus of elasticity is λ. Show that the equation of motion of the bead is

$$ma\ddot{\phi} = -\lambda\left(\frac{a\sin\phi}{c\sin\theta} - 1\right)\sin(\theta+\phi)\,,$$

where $\theta = \angle BPO$ and $\phi = \angle BOP$.

Given that θ and ϕ are small, show that $a(\theta + \phi) \approx b\theta$. Hence find the period of small oscillations about the equilibrium position $\theta = \phi = 0$.

STEP I 1999 Question 10 (Mechanics)

10 A particle is attached to a point P of an unstretched light uniform spring AB of modulus of elasticity λ in such a way that AP has length a and PB has length b. The ends A and B of the spring are now fixed to points in a vertical line a distance l apart, The particle oscillates along this line. Show that the motion is simple harmonic. Show also that the period is the same whatever the value of l and whichever end of the string is uppermost.

STEP III 2009 Question 10 (Mechanics)

10 A light spring is fixed at its lower end and its axis is vertical. When a certain particle P rests on the top of the spring, the compression is d. When, instead, P is dropped onto the top of the spring from a height h above it, the compression at time t after P hits the top of the spring is x. Obtain a second-order differential equation relating x and t for $0 \leqslant t \leqslant T$, where T is the time at which P first loses contact with the spring.

Find the solution of this equation in the form

$$x = A + B\cos(\omega t) + C\sin(\omega t)\,,$$

where the constants A, B, C and ω are to be given in terms of d, g and h as appropriate.

Show that

$$T = \sqrt{d/g}\,\left(2\pi - 2\arctan\sqrt{2h/d}\,\right)\,.$$

STEP III 2013 Question 9 (Mechanics)

9 A sphere of radius R and uniform density ρ_{s} is floating in a large tank of liquid of uniform density ρ. Given that the centre of the sphere is a distance x above the level of the liquid, where $x < R$, show that the volume of liquid displaced is

$$\frac{\pi}{3}(2R^3 - 3R^2x + x^3)\,.$$

The sphere is acted upon by two forces only: its weight and an upward force equal in magnitude to the weight of the liquid it has displaced. Show that

$$4R^3\rho_{\mathsf{s}}(g + \ddot{x}) = (2R^3 - 3R^2x + x^3)\rho g\,.$$

Given that the sphere is in equilibrium when $x = \frac{1}{2}R$, find ρ_{s} in terms of ρ. Find, in terms of R and g, the period of small oscillations about this equilibrium position.

STEP II 1988 Question 14 (Mechanics)

14 Two particles of mass M and m $(M > m)$ are attached to the ends of a light rod of length $2l$. The rod is fixed at its midpoint to a point O on a horizontal axle so that the rod can swing freely about O in a vertical plane normal to the axle. The axle rotates about a *vertical* axis through O at a constant angular speed ω such that the rod makes a constant angle α $(0 < \alpha < \frac{1}{2}\pi)$ with the vertical. Show that

$$\omega^2 = \left(\frac{M-m}{M+m}\right)\frac{g}{l\cos\alpha}.$$

Show also that the force of reaction of the rod on the axle is inclined at an angle

$$\tan^{-1}\left[\left(\frac{M-m}{M+m}\right)^2 \tan\alpha\right]$$

with the downward vertical.
