Worked Solutions for ENGAA Past Papers

ENGAA S1 2016 - Question 1

Find the complete set of solutions to $-8 < 6 - \frac{x}{2}$

A $x < 4$

B $x > 4$

C $x < 20$

D $x > 20$

E $x < 22$

F $x > 22$

G $x < 28$

H $x > 28$

ENGAA S1 2016 - Question 1 - Worked Solution

$$-8 < 6 - \frac{x}{2}$$

$$-14 < -\frac{x}{2}$$

$$-28 < -x$$

$$x < 28$$

Answer is G

ENGAA S1 2016 - Question 2

A nuclide ${}^{214}_{82}\text{Pb}$ changes by radioactive decay into the nuclide ${}^{210}_{82}\text{Pb}$.

Which combination of emissions produces this change?

A 3 alpha

B 2 alpha and 1 beta

C 2 alpha and 2 beta

D 1 alpha and 2 beta

E 3 beta

ENGAA S1 2016 - Question 2 - Worked Solution

$$82214Pb \rightarrow ... \rightarrow 82210Pb \ + \ ...$$

The lead-214 nuclide loses 4 nucleons so it emits 1 alpha particle. This would result in a nuclide

$$80210X$$

The proton number then increases by 2. This means 2 beta decays occur

$$1\ \alpha\ and\ 2\ \beta$$

Answer is D

ENGAA S1 2016 - Question 3

Which one of the following is a simplification of $(\sqrt{3}-\sqrt{2})^2$?

A $1-2\sqrt{3}\sqrt{2}$

B $5-2\sqrt{2}\sqrt{3}$

C $2\sqrt{3}-2\sqrt{2}$

D 1

E $5-\sqrt{2}\sqrt{3}$

F $13-2\sqrt{2}\sqrt{3}$

G $5+2\sqrt{2}\sqrt{3}$

H 5

ENGAA S1 2016 - Question 3 - Worked Solution

$$(\sqrt{3}-\sqrt{2})^2 = \sqrt{3}^2 + \sqrt{2}^2 - 2\sqrt{3}\sqrt{2}$$

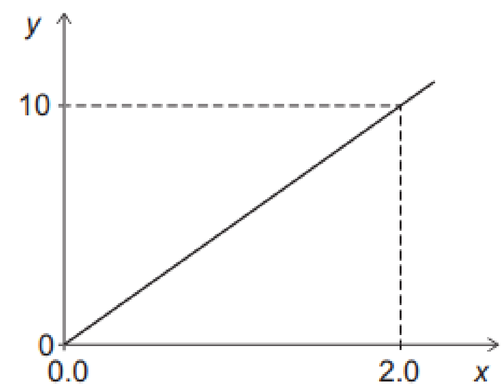
$$= 3 + 2 - 2\sqrt{6}$$

$$= 5 - 2\sqrt{2}\sqrt{3}$$

Answer is B

ENGAA S1 2016 - Question 4

The graph shown of quantity y against quantity x represents the motion of a body.



(The scales on both axes are in the appropriate S.I. units, and the gravitational field strength g is $10\,N\,kg^{-1}$.)

Which two of the following could the graph represent?

1 kinetic energy against velocity for an object of mass 10 kg undergoing free-fall

2 potential energy against height for an object of mass 20 kg being lifted by a constant external force

3 velocity against time for an object of mass 20 kg being accelerated by a resultant force of 100 N

4 work done by an external force of 5 N against distance moved for an object of mass 12 kg being moved at constant speed by (and in the direction of) the external force

A 1 and 2

B 1 and 3

C 1 and 4

D 2 and 3

E 2 and 4

F 3 and 4

ENGAA S1 2016 - Question 4 - Worked Solution

1)

$$Kinetic\ energy \propto velocity^2$$

So 1 can't be true

2)

Potential energy = mgh

If the x-axis were height the gradient would be $mg = 20 \times 10 = 200$.

The gradient is 5 so 2 can't be true.

3)

F=ma a=change in velocity/change in time

If the force is constant 100N and the mass 20kg the gradient will be 5 , so 3 can be true.

4)

Work done = force x distance (does not depend on moon)

If force is 5N , the gradient will be 5

So 4 could be true.

3 & 4

Answer is F

ENGAA S1 2016 - Question 5

5 The ratio of Q:R is 5:2 and the ratio of R:S is 3:10

Which one of the following gives the ratio Q:S in its simplest form?

A 1:2

B 2:1

C 3:4

D 3:25

E 4:3

F 25:3

ENGAA S1 2016 - Question 5 - Worked Solution

$$Q : R = 5 : 2$$

$$R : S = 3 : 10$$

$$R = \frac{3}{10} S$$

$$Q = \frac{5}{2} R$$

$$Q = \frac{5}{2} \times \frac{3}{10} S$$

$$Q = \frac{15}{20} S$$

$$Q = \frac{3}{4} S$$

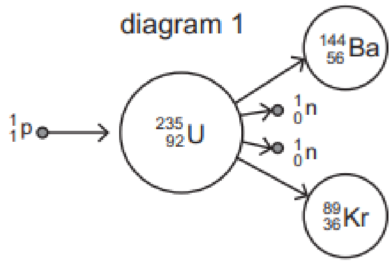
$$Q : S = 3 : 4$$

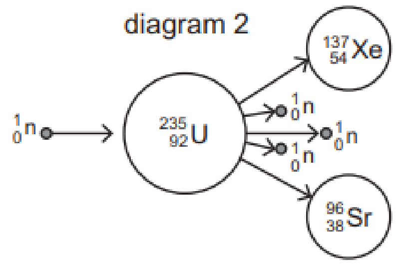
Answer is C

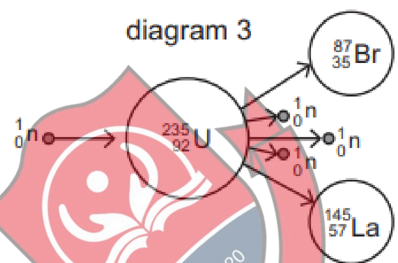
ENGAA S1 2016 - Question 6

6 A uranium-235 nucleus can undergo fission to produce two smaller nuclei.

Which of the diagrams, if any, could represent this process?







A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

ENGAA S1 2016 - Question 6 - Worked Solution

Diagram 1

$$92235U + 11P \rightarrow 2\times 01n + 55144Ba + 3689Kr$$

The number of protons is not constant,
The nucleon number is also not constant
This is not possible
Diagram 2

$$92235U + 01n \rightarrow 3\times 01n + 3896Sr + 54137Xe$$

The number of proton and nucleons is constant
This is possible
Diagram 3

$$01n + 92235U \rightarrow 3\times 01n + 3587Br + 57145La$$

This is not possible
 2 only
Answer is C

ENGAA S1 2016 - Question 7

7 The mean age of the twenty members of a running club is exactly 28.

The mean age increases by exactly 2 years when two new members join.

What is the mean age of the two new members?

A 20 years

B 22 years

C 30 years

D 40 years

E 50 years

F 52 years

ENGAA S1 2016 - Question 7 - Worked Solution

$$Total\ combined\ age\ =\ mean\ age \times number\ of\ people$$

$$Before\ =\ 28\times 20\ =\ 560$$

$$After\ =\ 30\times 22\ =\ 660$$

$$The\ combined\ age\ increased\ by\ 100$$

$$The\ two\ new\ members\ have\ a\ mean\ age\ of\ 50$$

Answer is E

ENGAA S1 2016 - Question 8

8 A circuit consists of a 5.0 Ω resistor and a variable resistor connected in series with a 24 V battery. The variable resistor has a minimum resistance of 3.0 Ω and a maximum resistance of 15 Ω. The battery and the connecting wires have negligible resistance.

What is the maximum power dissipated in the 5.0 Ω resistor?

A 7.2 W

B 18 W

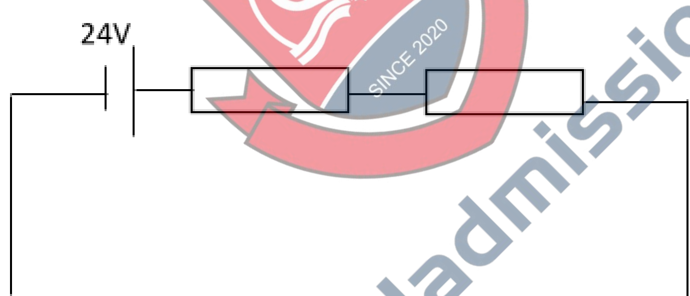
C 27 W

D 45 W

E 72 W

F 75 W

ENGAA S1 2016 - Question 8 - Worked Solution



$$3.0 \leq R \leq 15.0$$

For the maximum power in the 5.0Ω resistor, find the maximum voltage across it
By Kirchoff's voltage law the sum of voltages across the resistors will be 24V

$$V_i = V_{in} \frac{R_i}{R_T}$$

For maximum voltage across 5Ω resistor, $\frac{5}{R_T}$ must be maximum so the variable resistor must have the minimum resistance

$$V_1 = 24 \times \frac{5}{8} = 15V$$

$$P = \frac{V^2}{R} = 45W$$

Answer is D

ENGAA S1 2016 - Question 9

9 A medical scanner is bought for £15 000.

The value of the scanner depreciates by 20% every year.

By how much has the scanner reduced in value after 2 years?

A £600

B £3000

C £5400

D £6000

E £9000

F £9600

G £12 000

ENGAA S1 2016 - Question 9 - Worked Solution

$$V_{t+1} = 0.8V_t$$

The value one year after is 0.8 x the value of the previous year

$$After\ 2\ years:\ V_2 = 0.8^2 \times V_o = ζ9600$$

$$\Delta V = V_o - V_1 = ζ5400$$

Answer is C

ENGAA S1 2016 - Question 10

10 The total power P radiated by a star is given by:

$$P = kR^2T^4$$

where R is the radius of the star, T is its surface temperature and k is a constant.

The power currently radiated by the Sun is 4.0×10^{26} W. Towards the end of the Sun's life its radius will increase by a factor of a hundred and its surface temperature will decrease by a factor of two.

What will be the power radiated by the Sun when these changes have occurred?

A 2.5×10^{27} W

B 1.0×10^{28} W

C 2.0×10^{28} W

D 2.5×10^{29} W

E 1.0×10^{30} W

F 2.0×10^{30} W

G 2.5×10^{33} W

H 1.0×10^{34} W

ENGAA S1 2016 - Question 10 - Worked Solution

$$P = KR^2T^4$$

$$R^1 = 100R$$

$$T^1 = \frac{T}{2}$$

$$P^1 = K(100R)^2\left(\frac{T}{2}\right)^4$$

$$P^1 = KR^2T^4 \times \frac{100^2}{2^4}$$

$$P^1 = 635P$$

$$P = 4.0\times10^{26}w$$

$$P^{1} = 2.5\times10^{29} w$$

Answer is D

ENGAA S1 2016 - Question 11

11 The point A is 4 km due East of the point B.

The bearing of the point C from A is 330° and the bearing of C from B is 060°

Find the distance BC.

A 2 km

B $2\sqrt{3}$ km

C 4 km

D $2\sqrt{5}$ km

E $4\sqrt{2}$ km

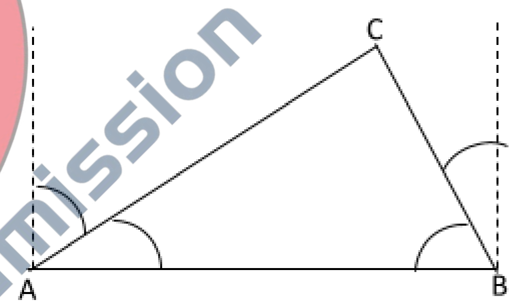
ENGAA S1 2016 - Question 11 - Worked Solution

$BC = AB \sin sin\, 60$

$BC = 4km \times \frac{\sqrt{3}}{2}$

$BC = 2\sqrt{3}km$

Answer is B



ENGAA S1 2016 - Question 12

12 A transverse wave travelling through a medium has a frequency of 5.0 Hz, a wavelength of 4.0 cm and an amplitude of 3.0 cm.

What is the total distance travelled by a particle of the medium in one minute?

A 900 cm

B 1200 cm

C 1800 cm

D 2400 cm

E 3600 cm

F 4800 cm

ENGAA S1 2016 - Question 12 - Worked Solution

$v = f\lambda$ in the direction of the wave's motion
BUT particles in the motion have no net movement
During one complete cycle a particle in the medium will be displaced from 0 to A back to 0_1 to -A and finally back to 0.
This is a total distance of 4A.
This transverse v = 4Af

$$d = vt$$

$$d = 4\times 3.0\times 5.0\times 60$$

$$d = 3600cm$$

Answer is E

ENGAA S1 2016 - Question 13

13 The quantities x and y are positive.

x is inversely proportional to the square root of y.

When $x = 8$, $y = 9$.

What is the value of y when $x = 6$?

A $\frac{3}{2}$

B 2

C $\frac{81}{16}$

D $\frac{27}{14}$

E 12

F 16

ENGAA S1 2016 - Question 13 - Worked Solution

$$x \propto \frac{1}{\sqrt{y}}$$

$$x = \frac{K}{\sqrt{y}}$$

$$8 = \frac{K}{\sqrt{9}}$$

$$8 = \frac{K}{3}$$

$$K = 24$$

$$6 = \frac{K}{\sqrt{y}}$$

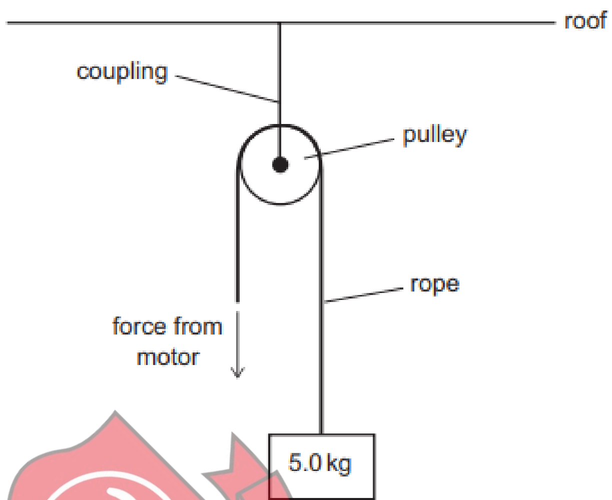
$$6 = \frac{24}{\sqrt{y}}$$

$$y = \left(\frac{24}{6}\right)^2 = 16$$

Answer is F

ENGAA S1 2016 - Question 14

14 A motor is used to lift a mass of 5.0 kg using a pulley system as shown in the diagram. The pulley is secured to the roof using a coupling.



The motor needs to cause the mass to accelerate upwards at $0.80\,m\,s^{-2}$.

What is the minimum tension force that the coupling must be able to withstand without breaking?

(The gravitational field strength g is $10\,N\,kg^{-1}$. The pulley system is frictionless and has negligible mass. The rope has negligible mass and is inextensible.)

A 4.0 N

B 8.0 N

C 46 N

D 50 N

E 54 N

F 92 N

G 104 N

H 108 N

ENGAA S1 2016 - Question 14 - Worked Solution

$$g = 10$$

$$m = 5.0kg$$

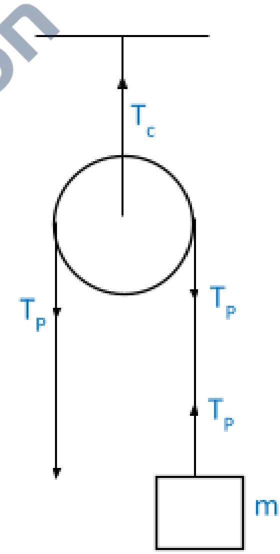
$$a = 0.8ms^{-1}$$

$$T_P - mg = ma$$

$$T_P - 50 = 4.0$$

$$T_P = 54N$$

$$T_C = 2T_P$$



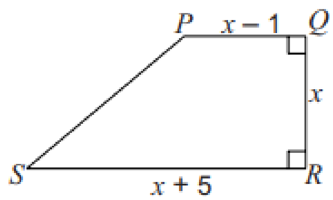
The coupling must be able to with stand at least 108N

Answer is H

ENGAA S1 2016 - Question 15

15 In a trapezium $PQRS$, the parallel sides are PQ and RS.

$PQ = (x - 1)$ cm, $RS = (x + 5)$ cm and the vertical height $QR = x$ cm.



[diagram not to scale]

The area of the trapezium is 120 cm^2.

What is the length of RS?

A 9 cm

B 10 cm

C 11 cm

D 12 cm

E 15 cm

F 17 cm

ENGAA S1 2016 - Question 15 - Worked Solution

$A = \frac{1}{2} \times (x + 5 + x - 1)$

$A = \frac{1}{2} \times (2x + 4)$

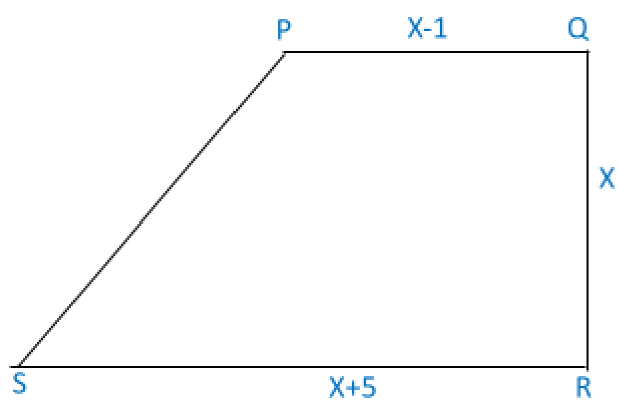
$A = x(x + 2)$

$x^2 + 2 = 120$

$(x + 1)^2 - 1^2 = 120$

$x + 1 = \sqrt{121}$

$x = - 1 \pm 11$



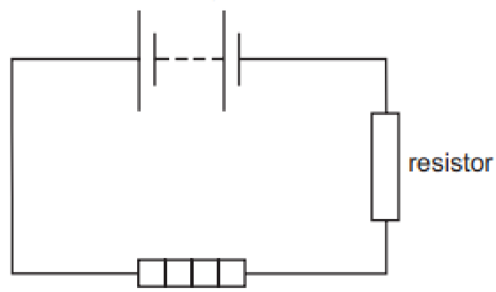
$x = 10 \quad (must\ be\ +\ ve)$

$RS = x + 5 = 15cm$

Answer is E

ENGAA S1 2016 - Question 16

16 A heater is connected in series with a resistor and a 6.0 V battery in the circuit shown.



The total resistance of the circuit is 15 Ω. In 3.0 minutes, 180 J of electrical energy is transferred into other forms in the heater.

How much charge flows through the heater in the 3.0 minutes and what is the voltage across the heater?

	charge / C	*voltage* / V
A	1.2	150
B	1.2	216
C	7.5	0.041
D	7.5	24
E	72	0.40
F	72	2.5
G	450	0.40
H	450	2.5

ENGAA S1 2016 - Question 16 - Worked Solution

The heater is R_1

$P = \frac{V^2}{R} = \frac{\Delta E}{\Delta t}$

$\frac{180J}{180s} = \frac{V_1^{\ 2}}{R_1} = IV_1$

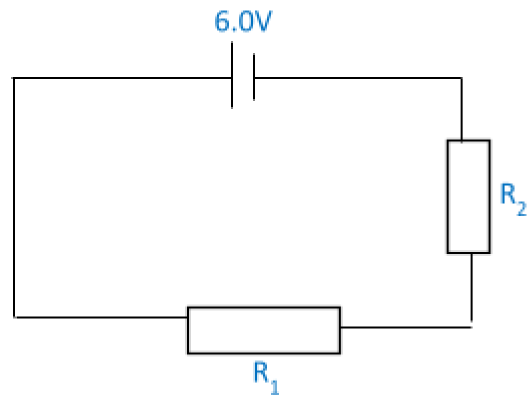
$I = \frac{6.0V}{15\Omega}$

$V_1 = \frac{15}{6}$ *voltage across the heater*

$\Delta Q = I\Delta t = \frac{6}{15} \times 180 = 72C$

$V_1 = 2.5V$

Answer is F



ENGAA S1 2016 - Question 17

17 Make b the subject of the formula:

$$a = \frac{b^2 + 2}{3b^2 - 1}$$

A $b = \pm\sqrt{\left(\frac{a+2}{3a+1}\right)}$

B $b = \pm\sqrt{\left(\frac{a+2}{3a-1}\right)}$

C $b = \pm\sqrt{\left(\frac{2-a}{3a+1}\right)}$

D $b = \pm\sqrt{\left(\frac{2-a}{3a-1}\right)}$

E $b = \pm\sqrt{\left(\frac{3}{3a+1}\right)}$

F $b = \pm\sqrt{\left(\frac{3}{3a-1}\right)}$

ENGAA S1 2016 - Question 17 - Worked Solution

$$a = \frac{b^2+2}{3b^2-1}$$

$$a\left(3b^2 - 1\right) = b^2 + 2$$

$$3ab^2 - a = b^2 + 2$$

$$(3a - 1)b^2 - a - 2 = 0$$

$$(3a - 1)b^2 = a + 2$$

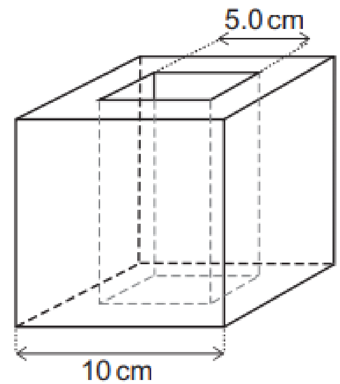
$$b^2 = \frac{a+2}{3a-1}$$

$$b = \pm\sqrt{\frac{a+2}{3a-1}}$$

Answer is B

ENGAA S1 2016 - Question 18

18 A cubic block has a hole through it with a square cross-section. The dimensions are shown on the diagram. The weight of the block is 30 N.



What is the density of the material from which the block is made?

(The gravitational field strength g is 10 N kg^{-1}.)

A 0.30 g cm^{-3}

B 0.40 g cm^{-3}

C 0.60 g cm^{-3}

D 1.2 g cm^{-3}

E 3.0 g cm^{-3}

F 4.0 g cm^{-3}

G 6.0 g cm^{-3}

H 12 g cm^{-3}

ENGAA S1 2016 - Question 18 - Worked Solution

$$V_{cube} = 10^3 - 5^2 \times 10 = 750cm^3$$

$$750cm^3 \times P \times 10\frac{N}{kg} = 30N$$

$$P = 0.004kgcm^{-3}$$

$$P = 4gcm^{-3}$$

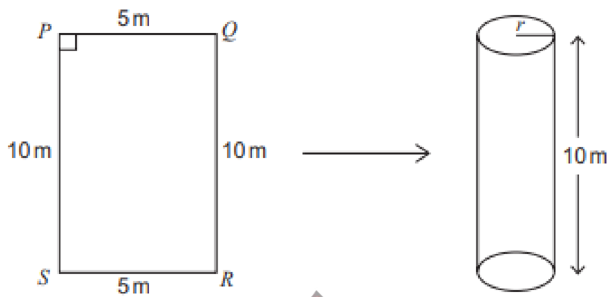
Answer is F

ENGAA S1 2016 - Question 19

19 A thin rectangular sheet of metal 10 m by 5 m is made into an open ended cylinder by joining the edges PS and QR.

The height of the cylinder is 10 m.

What is the volume, in cubic metres, enclosed by this cylinder?



[diagram not to scale]

A $\frac{5}{2\pi}$

B $\frac{25}{4\pi}$

C $\frac{125}{2\pi}$

D 62.5π

E $\frac{125}{\pi}$

F 250π

ENGAA S1 2016 - Question 19 - Worked Solution

$$Circumference\ of\ cylinder = 5m$$

$$2\pi r = 5$$

$$r = \frac{5}{2\pi}$$

$$v = \pi r^2 h$$

$$h = 10m$$

$$v = \left(\frac{5}{2\pi}\right)^2 \times \pi \times 10$$

$$v = \frac{250}{4\pi}$$

$$v = \frac{125}{2\pi}$$

Answer is C

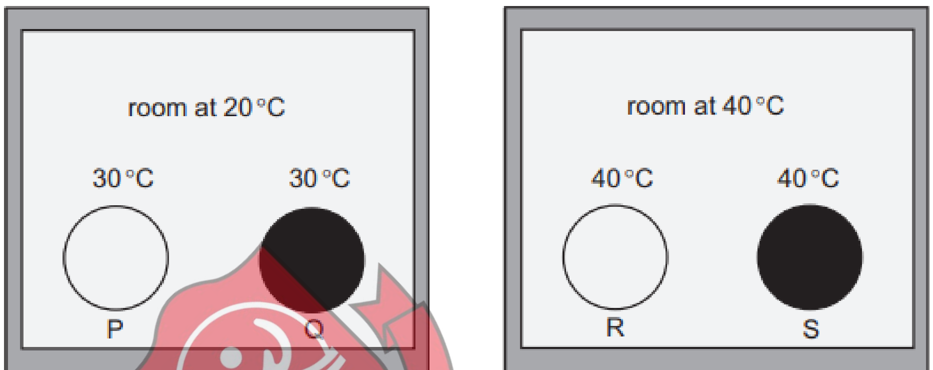
ENGAA S1 2016 - Question 20

20 The diagram shows four solid steel balls P, Q, R and S which are of identical size.

Balls P and R have shiny surfaces. Balls Q and S have dull surfaces.

Balls P and Q are in a room at 20 °C. Balls R and S are in a room at 40 °C.

The temperature of each ball at a given moment in time is shown on the diagram.



Which two balls lose thermal energy by convection, and which ball emits thermal radiation at the greatest rate?

	lose thermal energy by convection	*greatest rate of emission of thermal radiation*
A	P and Q	P
B	P and Q	Q
C	P and Q	R
D	P and Q	S
E	R and S	P
F	R and S	Q
G	R and S	R
H	R and S	S

ENGAA S1 2016 - Question 20 - Worked Solution

As P & Q are hotter than their surroundings, they lose heat by convection.
As S & Q are dull, they act more like black bodies than P & R. Therefore they radiate more heat than if they were shiny.
S radiates heat at the highest rare as it has the highest temperature.
NB: S will gain the heat it radiates back by convection, so although it radiates heat it will maintained a constant temperature
Answer is D

ENGAA S1 2016 - Question 21

21 Which one of the following is a simplification of $4+\frac{4-x^2}{x^2-2x}$?

A $3-\frac{2}{x}$

B $3+\frac{2}{x}$

C $4-\frac{2}{x}$

D $4+\frac{2}{x}$

E $5-\frac{2}{x}$

F $5+\frac{2}{x}$

ENGAA S1 2016 - Question 21 - Worked Solution

$$4+\frac{4-x^2}{x^2-2x}$$

$$=4+\frac{(2+x)(2-x)}{x(x-2)}$$

$$=4-\frac{2+x}{x}$$

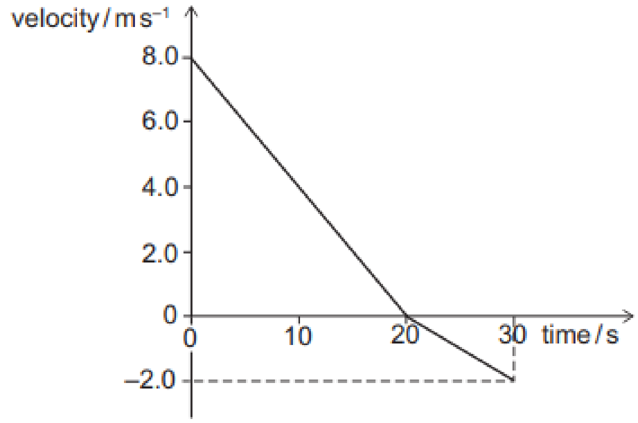
$$=4-\frac{2}{x}-1$$

$$=3-\frac{2}{x}$$

Answer is A

ENGAA S1 2016 - Question 22

22 The diagram shows the velocity-time graph for an object travelling in a straight line over a period of 30 s.



What total distance did the object travel in the 30 s, how far from its starting position was it at the end of the 30 s, and what was its average speed over the 30 s?

	total distance travelled / m	distance from starting position / m	average speed / $m\,s^{-1}$
A	90	70	3.0
B	90	70	5.0
C	90	90	3.0
D	90	90	5.0
E	180	140	5.0
F	180	140	6.0
G	180	180	5.0
H	180	180	6.0

ENGAA S1 2016 - Question 22 - Worked Solution

Distance = area between v – t curve and t – axis regardless of above or below t – axis

$$distance = \frac{1}{2}\times 8\times 20 + \frac{1}{2}\times 2\times 10$$

$$distance = 80 + 10 = 90m$$

$$Average\ speed = distance \div time = \frac{90}{30} = 3ms^{-1}$$

Distance from starting position = area between v – t curve and t – axis taking area under the t – axis as – ve

$$= \frac{1}{2}\times 8\times 20 - \frac{1}{2}\times 2\times 10$$

$$= 70m$$

Answer is A

ENGAA S1 2016 - Question 23

23 During summer activities week 120 students each chose one activity from swimming, archery, and tennis.

46 of the students were girls.

36 of the students chose tennis, and $\frac{2}{3}$ of these were boys; 25 girls chose swimming, and 27 students chose archery.

A boy is picked at random. What is the probability that he chose swimming?

A $\frac{3}{20}$

B $\frac{9}{37}$

C $\frac{4}{15}$

D $\frac{16}{37}$

E $\frac{32}{57}$

ENGAA S1 2016 - Question 23 - Worked Solution

120 students

$$46\ girls \quad \therefore 74\ boys$$

36 choose tennis and 27 choose archery

$$\therefore 120 - 36 - 27 = 57 \quad choose\ swiming$$

$25\ girls\ choose\ swiming \therefore 57 - 25 = 32\ boys$ choose swimming

$$P\left(\frac{swimming}{boy} = \frac{32}{74} = \frac{16}{37}\right)$$

Answer is D

ENGAA S1 2016 - Question 24

24 Bronze is a mixture of tin and copper.

A particular sample of bronze contains 10% tin by volume. (In other words, 10% of the total volume of the sample is tin and 90% of it is copper.)

What percentage of the mass of the sample is tin?

(Density of tin $= Y$ and density of copper $= X$.)

A $\frac{X}{9X-Y}\times 100$

B $\frac{X}{9Y-X}\times 100$

C $\frac{Y}{9X-Y}\times 100$

D $\frac{Y}{9Y-X}\times 100$

E $\frac{X}{9X+Y}\times 100$

F $\frac{X}{9Y+X}\times 100$

G $\frac{Y}{9X+Y}\times 100$

H $\frac{Y}{9Y+X}\times 100$

ENGAA S1 2016 - Question 24 - Worked Solution

Let the volume = V

The mass of tin = 0.1VY = M_T

The mass of copper = 0.9 VX = M_c

The total mass = (0.1Y+0.9X)V = M

$$\frac{M_T}{M} = \frac{0.1YV}{(0.1Y+0.9X)V}$$

$$\frac{M_T}{M} = \frac{Y}{Y+9X}\times 100\%$$

Answer is G

ENGAA S1 2016 - Question 25

25 Which one of the following expressions is equivalent to $\frac{9^{2n+1} \times 3^{4-3n}}{27^{2-n}}$?

A 3^9

B 3^{-2n}

C 3^{2-2n}

D 3^{4n}

E 3^{6n-2}

F 3^6

ENGAA S1 2016 - Question 25 - Worked Solution

$$\frac{9^{2n+1} \times 3^{4-3n}}{27^{2-n}}$$

$$= \frac{\left(3^2\right)^{2n+1} \times 3^{4-3n}}{\left(3^3\right)^{2-n}}$$

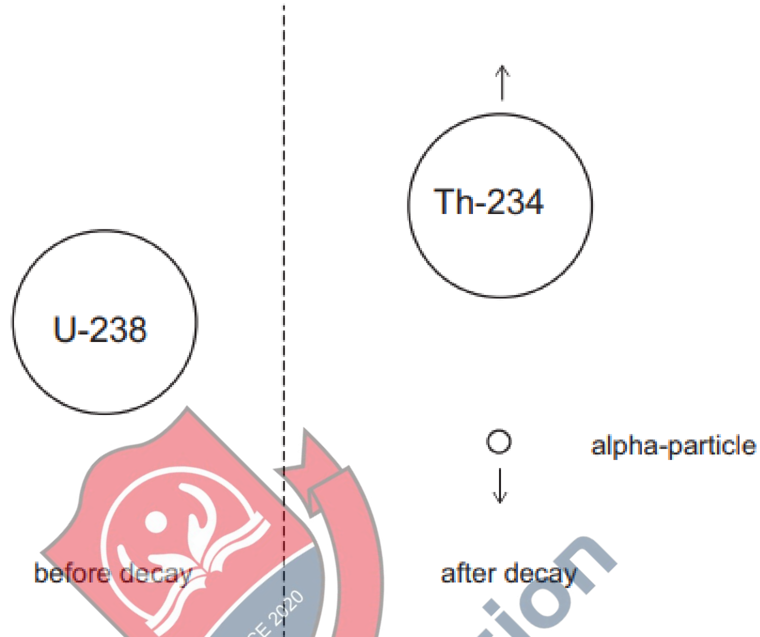
$$= 3^{4n+1+4-3n-6+3n}$$

$$= 3^{4n}$$

Answer is D

ENGAA S1 2016 - Question 26

26 When a stationary uranium-238 nucleus decays by alpha emission it forms a nucleus of thorium-234. The total kinetic energy produced by the decay is E.



What is the kinetic energy of the alpha particle?

A $\frac{4E}{238}$

B $\frac{4E}{234}$

C $\frac{E}{2}$

D $\frac{234E}{238}$

E E

ENGAA S1 2016 - Question 26 - Worked Solution

The α particles has a relative mass of 4 and the Th – 234 has a relative mass of 234
By conservation of momentum

$$4v_{\alpha} = 234v_{T}$$

$$v_{T} = \frac{4}{234}v_{\alpha}$$

$$\frac{1}{2}\times4\times{v_{\alpha}}^{2} + \frac{1}{2}\times234\times{v_{T}}^{2} = E$$

$$2{v_{\alpha}}^{2} + \frac{1}{2}\times234\times\left(\frac{4}{234}\right)^{2}{v_{\alpha}}^{2} = E$$

$$E_{\alpha} = 2{v_{\alpha}}^{2}$$

$$E = {v_{\alpha}}^{2}\left(2 + \frac{8}{234}\right)$$

$$E_{\alpha} = \frac{2E}{2+\frac{8}{234}} = \frac{234}{238}E$$

Answer is D

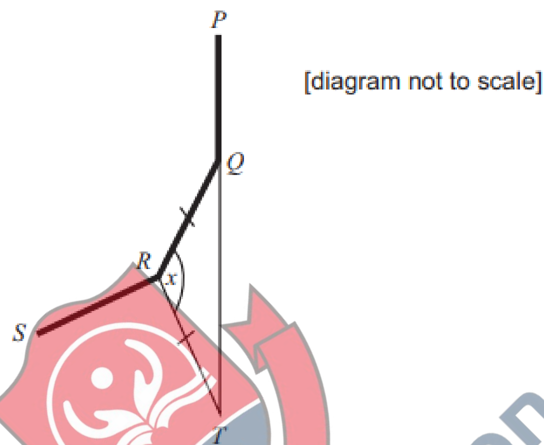
ENGAA S1 2016 - Question 27

27 In the diagram below, $PQRS$ is part of a regular polygon.

The polygon has n sides.

The side PQ is extended to T such that PQT is a straight line.

The length of RQ is the same as the length of RT.



Find an equation for n in terms of x, where x is the size of angle $\angle$QRT in degrees.

A $n = \dfrac{180}{x - 90}$

B $n = \dfrac{180 - x}{720}$

C $n = \dfrac{360 - x}{90}$

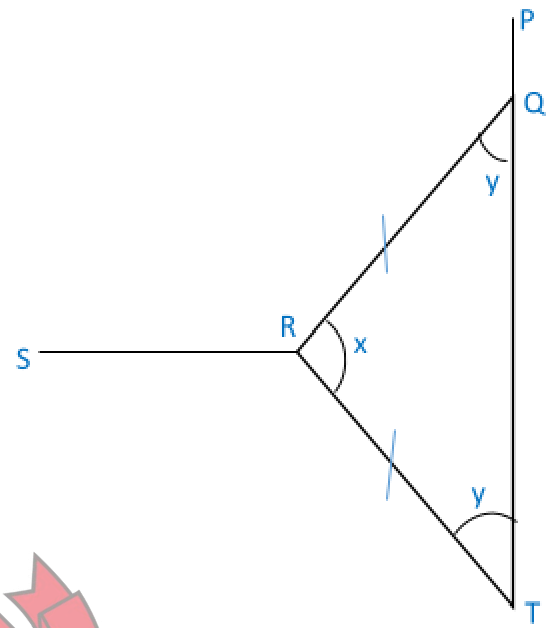
D $n = \dfrac{360}{180 - x}$

E $n = \dfrac{720}{180 - x}$

F $n = \dfrac{720}{360 - x}$

G $n = \dfrac{360}{360 - x}$

ENGAA S1 2016 - Question 27 - Worked Solution



$\angle RQT = \angle QTR = y$

$2y + x = 180$

$\angle RQP + \angle RQT = 180$

$\angle RQP = \frac{(n-2)(180)}{n} = 180\left(1 - \frac{2}{n}\right)$

$\angle RQT = 180 - \angle RQP$

$= 180 - 180\left(1 - \frac{2}{n}\right)$

$= \frac{2}{n} \times 180°$

$y = \frac{360}{n}$

$x = 180 - 2y$

$x = 180 - \frac{720}{n}$

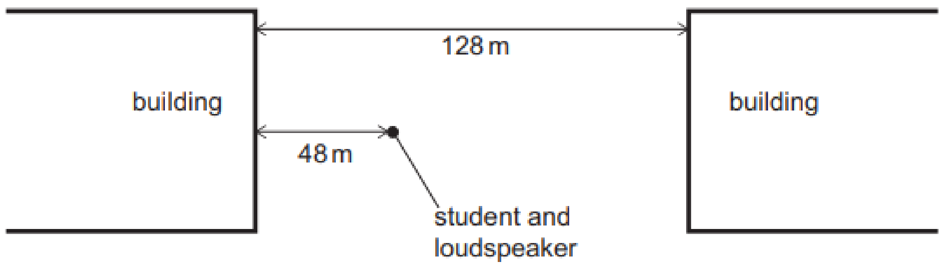
$\frac{720}{n} = 180 - x$

$n = \frac{720}{180-x}$

Answer is E

ENGAA S1 2016 - Question 28

28 A student carries out an experiment to measure the speed of sound. A loudspeaker that emits sound in all directions is placed between two buildings that are 128 m apart as shown. The student and loudspeaker are 48 m from one of the buildings.



The loudspeaker is connected to a signal generator that causes it to emit regular clicks. The student notices that each click results in two echoes, one from each building. The rate at which the clicks are produced is gradually increased from zero until each echo coincides with a new click being emitted by the loudspeaker.

What is the frequency of emission of clicks when this happens?

(The speed of sound in air = $320\,\text{m}\,\text{s}^{-1}$.)

A 2.0 Hz

B 2.5 Hz

C 3.3 Hz

D 4.0 Hz

E 5.3 Hz

F 6.7 Hz

G 10 Hz

ENGAA S1 2016 - Question 28 - Worked Solution

The two buildings are 48m and 80m away from the student
Sound will have to travel 96 or 160m to echo which will take 0.3s or 0.5s
The time between clicks therefore has 0.3s or 0.5s.
The minimum frequency when a click coincides with an echo is 2.0Hz

However for both echoes to coincide with a click, the clicks must occur every 0.1s or at a frequency of 10.0Hz.

Answer is G

ENGAA S1 2016 - Question 29

29 When $x = 2$ is substituted in the expression $x^3 + px^2 + qx + p^2$ the result is 0.

When $x = 1$ is substituted into the same expression, the result is -3.5.

Find all possible value(s) of p.

A $p = -1 \pm \frac{\sqrt{6}}{3}$

B $p = 1$ or $p = -3$

C $p = 1$

D $p = 1 \pm \sqrt{7}$

E there are no values for p

ENGAA S1 2016 - Question 29 - Worked Solution

$$2^3 + p \times 2^2 + 2q + p^2 = 0 \quad --- ①$$

$$1^3 + p \times 1^2 + 1q + p^2 = -3.5 \quad ---- ②$$

$$① - 2 \times ②$$

$$6 + 2p - p^2 = 7$$

$$p^2 - 2p + 1 = 0$$

$$(p - 1)^2 - 1^2 + 1 = 0$$

$$(p - 1)^2 = 0$$

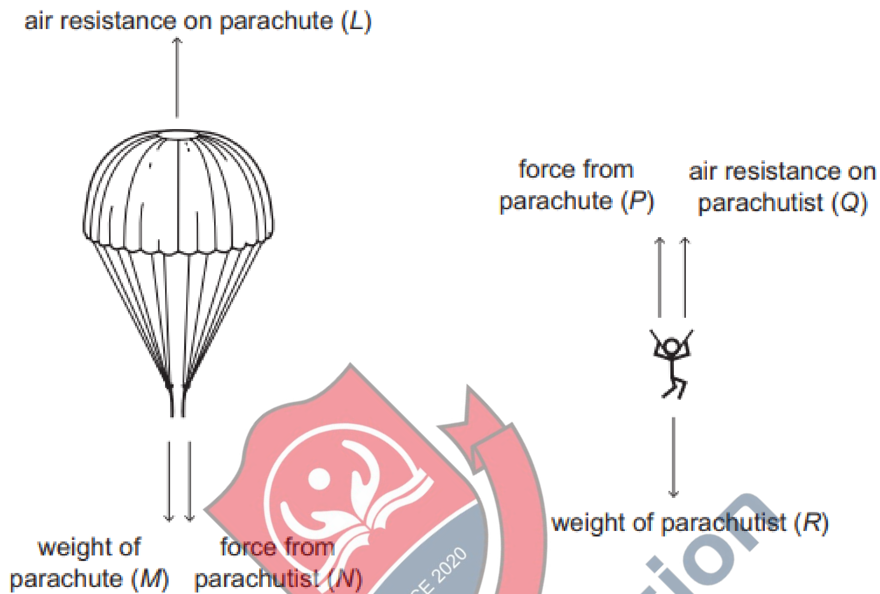
$$p = 1$$

Answer is C

ENGAA S1 2016 - Question 30

30 A parachutist is falling at terminal speed with his parachute open. The diagrams show, separately, the vertical forces acting on the parachute and the vertical forces acting on the parachutist.

The letters *L*, *M*, *N*, *P*, *Q* and *R* represent the magnitude of each force as indicated.



Consider the following equations:

Equation 1: $L = M + N$
Equation 2: $R = P + Q$
Equation 3: $L = Q$
Equation 4: $N = P$
Equation 5: $M + R = L + Q$

Which of these equations, if any, is/are the direct result of the application of Newton's Third Law to this situation?

A none of them

B 3 only

C 4 only

D 5 only

E 1 and 2 only

F 3 and 4 only

G 1, 2 and 5 only

H 1, 2, 3, 4 and 5

ENGAA S1 2016 - Question 30 - Worked Solution

1 is a result of $F = ma$, Newton's second law
2 is a result of $\Sigma F = ma$
3 states that the air resistance force is equal for the parachute and parachutist.
4 states that the force from the parachute on the parachutist is equal to the force from the parachutist on the parachute, which is Newton's third law.
S is a result of $\Sigma F = ma$
Only 4 is a direct result of the application of Newton's third law.
Answer is C

ENGAA S1 2016 - Question 31

31 A square PQRS is drawn above the x-axis with the side PQ on the x-axis.

P is the point $(-5, 0)$ and Q is the point $(1, 0)$.

A circle is drawn inside the square with diameter equal in length to the side of the square.

Which one of the following is an equation of the circle?

A $x^2 + y^2 - 4x + 6y + 4 = 0$

B $x^2 + y^2 - 4x + 6y + 9 = 0$

C $x^2 + y^2 + 4x - 6y + 4 = 0$

D $x^2 + y^2 + 4x - 6y + 9 = 0$

E $x^2 + y^2 - 6x - 4y + 9 = 0$

F $x^2 + y^2 - 6x + 4y + 4 = 0$

G $x^2 + y^2 + 6x - 4y + 4 = 0$

H $x^2 + y^2 + 6x + 4y + 9 = 0$

ENGAA S1 2016 - Question 31 - Worked Solution

The square has a side length 6.
If the circle is fully inside the square and its diameter is equal to the side length of the square then its center must be at the center of the square.
The center of the square is either at (- 2 , 3) or (- 2 , - 3)

$$\textit{the circle has equation}: (x + 2)^2 + (y - 3)^2 = 3^2, \; or\; (x + 2)^2 + (y + 3)^2 = 3^2$$

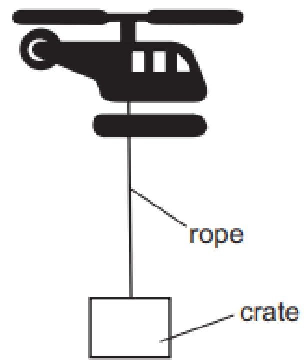
$$x^2 + 4x + 4 + y^2 - 6y + 9 = 9$$

$$x^2 + 4x + 4 + y^2 + 6y + 9 = 9$$

Only C is either of the possible equations
Answer is C

ENGAA S1 2016 - Question 32

32 A crate has a total mass of 800 kg, including its contents. A helicopter of mass 4200 kg is carrying the crate using a light inextensible rope as shown:



The helicopter and crate are accelerating upwards at 2.0 m s^{-2}.

What is the tension in the rope?

(The gravitational field strength g is 10 N kg^{-1}; air resistance can be ignored.)

A 6400 N

B 8000 N

C 9600 N

D 18 000 N

E 40 000 N

F 42 000 N

G 50 000 N

H 60 000 N

ENGAA S1 2016 - Question 32 - Worked Solution

$$T - mg = ma$$

$$T = 800 \times 10 + 800 \times 2$$

$$T = 9600N$$

Answer is C

ENGAA S1 2016 - Question 33

33 The first term of a convergent geometric series is 8.

The fifth term is 2.

The sixth term is real and positive.

What is the sum to infinity of this series?

(The sum to infinity of a convergent geometric series is given by $\frac{a}{1-r}$, where a is the first term and r is the common ratio.)

A $8(1+\sqrt{2})$

B $8(1-\sqrt{2})$

C $8(2+\sqrt{2})$

D $8(2-\sqrt{2})$

E 16

F $\frac{8\sqrt[5]{4}}{\sqrt[5]{4}-1}$

G $\frac{8\sqrt[5]{4}}{\sqrt[5]{4}+1}$

ENGAA S1 2016 - Question 33 - Worked Solution

$$a = 8$$

$$ar^4 = 2$$

$$r > 0 , r \in R$$

$$8r^4 = 2$$

$$r^4 = \frac{1}{4}$$

$$r = \left(\frac{1}{4}\right)^{\frac{1}{4}}$$

$$r = \frac{\sqrt{2}}{2}$$

$$S_\infty = \frac{a}{1-r}$$

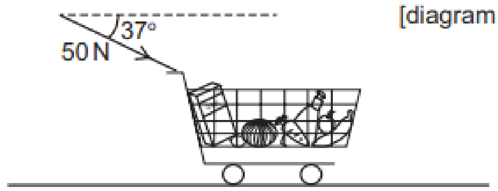
$$S_{\infty} = 16 + 8\sqrt{2}$$

$$S_{\infty} = 8\left(2 + \sqrt{2}\right)$$

Answer is C

ENGAA S1 2016 - Question 34

34 A shopper pushes a supermarket trolley a distance of 15 m in a straight line across a level, horizontal surface. The shopper applies a constant force of 50 N at an angle of 37° below the horizontal. The total weight of the trolley and its contents is 350 N.



[diagram not to scale]

What is the magnitude of the total vertical force that the surface exerts on the trolley and how much work is done by the pushing force?

(You may use the approximations $\sin 37° = 0.60$; $\cos 37° = 0.80$.)

	vertical force / N	work done / J
A	380	600
B	380	750
C	390	450
D	390	750
E	400	450
F	400	600

ENGAA S1 2016 - Question 34 - Worked Solution

$$W = Fd \cos cos\, \theta$$

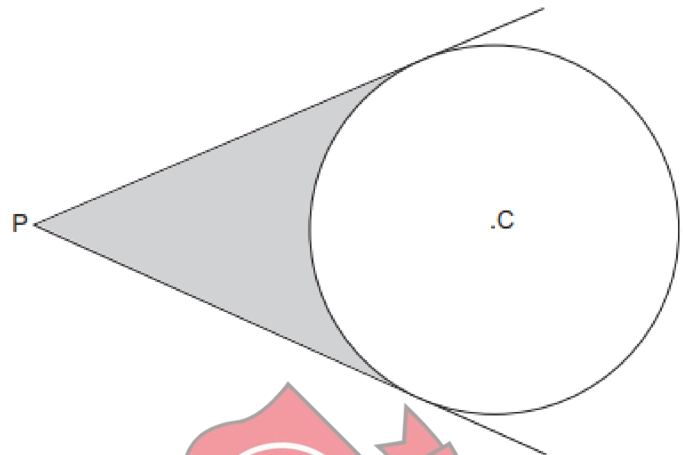
$$W = 15 \times 50 \times 0.80 = 600J$$

The vertical component of the force = 50 sin 37 = 30N
The total downward force on the trolley = 380N
Thus the reaction force of the surface on the trolley = 380N
Answer is A

ENGAA S1 2016 - Question 35

35 Tangents are drawn from a point P to a circle of radius 10 cm.

The centre of the circle is C and the distance PC is 20 cm.



[diagram not to scale]

Which one of the following is an expression for the shaded area in square centimetres?

A $\frac{100}{3}\left(3\sqrt{3}-\pi\right)$

B $\frac{100}{3}\left(3\sqrt{5}-\pi\right)$

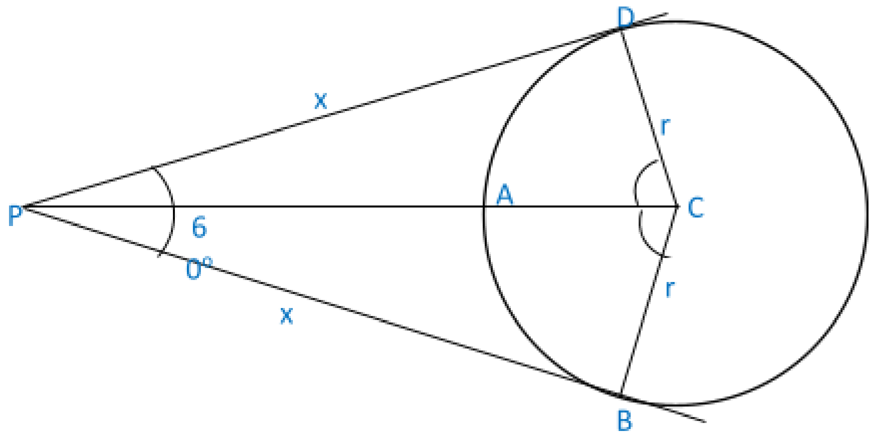
C $\frac{50}{3}\left(6\sqrt{3}-\pi\right)$

D $\frac{50}{3}\left(6\sqrt{5}-\pi\right)$

E $\frac{50}{3}\left(\sqrt{3}-2\pi\right)$

F $\frac{50}{3}\left(2\pi-\sqrt{3}\right)$

ENGAA S1 2016 - Question 35 - Worked Solution



$$AP = 10cm \quad AC = 10cm$$

$$PB = PD = x$$

$$x^2 + r^2 = 20^2$$

$$x^2 = 300$$

$$x = 10\sqrt{3}cm$$

$$\cos cos\ \frac{\theta}{2} = \frac{1}{2}$$

$$\frac{\theta}{2} = \frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3}$$

The area of the shaded area is equal to the area of the triangle with vertices PBD subtract the area of the segment of the circle formed by ABD, denoted by V.
That is equal to the sector of the circle formed by BCD subtract the area of the triangle with vertices BCD, shown as W.

$$A_{PBD} = \frac{1}{2}x^2 \sin sin\ 60°$$

$$= \frac{1}{2} \times 300 \times \frac{\sqrt{3}}{2}$$

$$= 75\sqrt{3}$$

$$A_{sector} = \pi r^2 \times \frac{2\pi/3}{2\pi} = \frac{\pi r^2}{3} = \frac{100\pi}{3}$$

$$A_W = \frac{1}{2} r^2 \sin sin \frac{2\pi}{3} = \frac{1}{2} \times 100 \times \frac{\sqrt{3}}{2} = 25\sqrt{3}$$

$$A_V = 100\frac{\pi}{3} - 25\sqrt{3}$$

$$A_{shaded} = 75\sqrt{3} - \left(100\frac{\pi}{3} - 25\sqrt{3}\right)$$

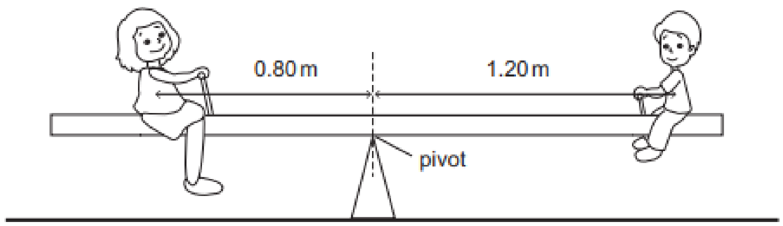
$$= 100\sqrt{3} - 100\frac{\pi}{3}$$

$$= \frac{100}{3}\left(3\sqrt{3} - \pi\right)$$

Answer is A

ENGAA S1 2016 - Question 36

36 A plank of non-uniform density which has a mass of 15 kg is used to make a see-saw. A pivot is placed under the centre of the plank as shown on the diagram.



[diagram not to scale]

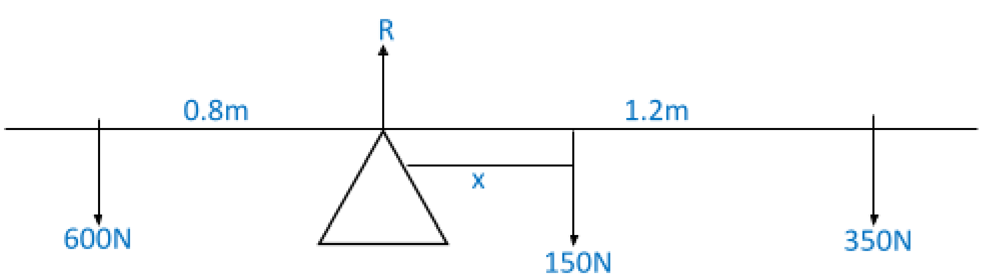
A boy of mass 35 kg sits at one end of the plank with his centre of gravity 1.20 m from the pivot. The see-saw balances when a woman of mass 60 kg sits on the plank on the other side of the pivot. Her centre of gravity is 0.80 m from the pivot.

Where is the centre of gravity of the plank and what is the magnitude of the force between the pivot and the plank?

(The gravitational field strength g is $10\,\text{N kg}^{-1}$.)

	distance from pivot	*force* / N
A	0.40 m on left of pivot	100
B	0.40 m on left of pivot	1100
C	at the pivot	100
D	at the pivot	1100
E	0.20 m on right of pivot	100
F	0.20 m on right of pivot	1100
G	0.40 m on right of pivot	100
H	0.40 m on right of pivot	1100

ENGAA S1 2016 - Question 36 - Worked Solution



$$R = 600 + 150 + 350 = 1100N$$

Take moments about the pivot

$$0.8\times600 = 1.2\times350 + x\times150$$

$$480 = 420 + 150x$$

$$x = 0.4m$$

The center of gravity of the beam is 0.4m to the right of the pivot
NB : If CoG were to the left , x would have been negative.
Answer is H

ENGAA S1 2016 - Question 37

37 Given that $7\cos\theta - 3\tan\theta\sin\theta = 1$, which one of the following is true?

A $\cos\theta = -\frac{3}{5}$ or $-\frac{1}{2}$

B $\cos\theta = -\frac{3}{5}$ or $\frac{1}{2}$

C $\cos\theta = \frac{3}{5}$ or $\frac{1}{2}$

D $\cos\theta = \frac{3}{5}$ or $-\frac{1}{2}$

ENGAA S1 2016 - Question 37 - Worked Solution

$$7\cos cos\,\theta - 3\tan tan\,\theta \sin sin\,\theta = 1$$

$$7\cos cos\,\theta - 3\frac{\sin sin\,\theta}{\cos cos\,\theta}\sin sin\,\theta = 1$$

$$7cos^2\theta - 3sin^2\theta = \cos cos\,\theta$$

$$7cos^2\theta - 3\left(1 - cos^2\theta\right) = \cos cos\,\theta$$

$$Let\ u = \cos cos\,\theta$$

$$7u^2 - 3 + 3u^2 = u$$

$$10u^2 - u - 3 = 0$$

$$u = \frac{1 \pm \sqrt{1^2 - 4(10)(-3)}}{20}$$

$$u = \frac{1 \pm \sqrt{121}}{20}$$

$$u = \frac{1 \pm 11}{20}$$

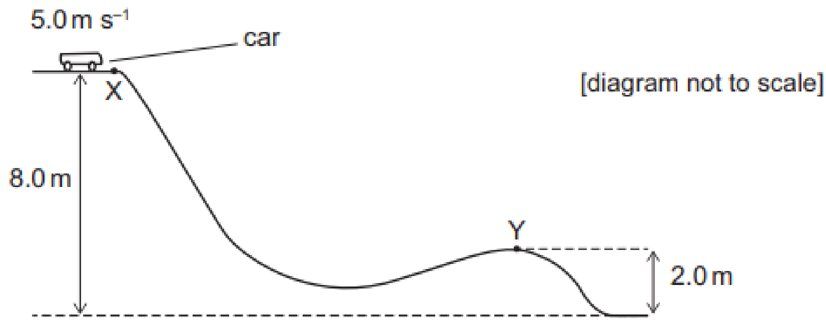
$$u = \frac{12}{20}\ ,\quad u = \frac{10}{20}$$

$$\cos cos\,\theta = \frac{3}{5}\ ,\quad \cos cos\,\theta = -\frac{1}{2}$$

Answer is D

ENGAA S1 2016 - Question 38

38 A car of mass 200 kg on a fairground ride travels at a speed of $5.0\,m\,s^{-1}$ at point X. The car is allowed to move down a sloping section of track without any energy input. The heights above the ground of points X and Y are shown. When the car reaches point Y its speed is $9.0\,m\,s^{-1}$.



How much energy is transferred in overcoming resistive forces as the car travels from X to Y?

(The gravitational field strength g is $10\,N\,kg^{-1}$.)

A 3900 J

B 6400 J

C 7900 J

D 10 400 J

E 11 200 J

ENGAA S1 2016 - Question 38 - Worked Solution

$$\Delta GPE_{XY} = mg\Delta h$$

$$= 200 \times 10 \times 6$$

$$= 12000J$$

$$\Delta EK_{XY} = \frac{1}{2} \times 200 \times \left(9^2 - 5^2\right)$$

$$= 5600J$$

$$12000 - 5600 = 6400J \;\; are\ lost\ to\ resistive\ forces$$

Answer is B

ENGAA S1 2016 - Question 39

39 The complete set of values of a for which the equation $3x^2 = (a+2)x - 3$ has two real distinct roots is

A no values of a

B $-4\sqrt{2} < a < 4\sqrt{2}$

C $a < -4\sqrt{2},\ a > 4\sqrt{2}$

D $-4 < a < 8$

E $a < -4,\ a > 8$

F $-8 < a < 4$

G $a < -8,\ a > 4$

H all values of a

ENGAA S1 2016 - Question 39 - Worked Solution

$$3x^2 = (a+2)x - 3$$

$$3x^2 - (a+2)x + 3 = 0$$

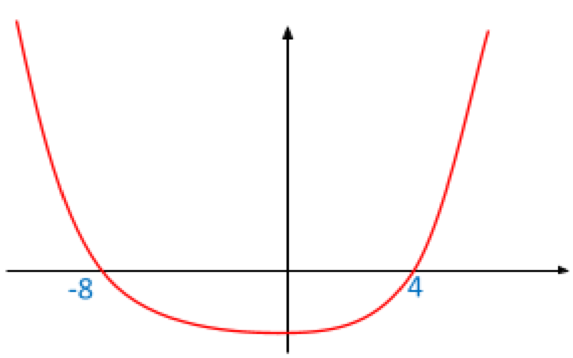
$$For\ \alpha x^2 + \beta x + \gamma = 0\ to\ have\ two\ real\ roots$$

$$\beta^2 - 4\alpha\gamma > 0$$

$$(a+2)^2 - 4(3)(3) > 0$$

$$a^2 + 4a + 4 - 36 > 0$$

$$a^2 + 4a - 32 > 0$$



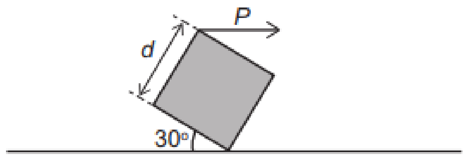
$$(a - 4)(a + 8) > 0$$

Answer is G

ENGAA S1 2016 - Question 40

40 The diagram shows a uniform, solid, heavy cube with side *d*. The cube rests with one of its edges in contact with a table that is perfectly level. A horizontal force *P* acts on another edge of the cube, and the cube is stationary.

[diagram not to scale]



Below are four statements about the forces on the cube.

1 It is possible that there is no frictional force between the cube and the table.

2 There must be a frictional force acting to the left between the cube and the table.

3 There must be a frictional force acting to the right between the cube and the table.

4 Force *P* has a clockwise moment about the edge in contact with the table equal to $P \times d$.

Which of the statements is/are correct?

A 1 only

B 2 only

C 3 only

D 1 and 4 only

E 2 and 4 only

F 3 and 4 only

ENGAA S1 2016 - Question 40 - Worked Solution

P acts to the right, therefore there must be a force to the left to counteract it as the cube
I > in
equilibrium.
The normal reaction force can only act straight upwards.
This force must therefore be friction
1 is false
2 is true
3 is false

4 is false as P is not perpendicular to any sides of the cube.

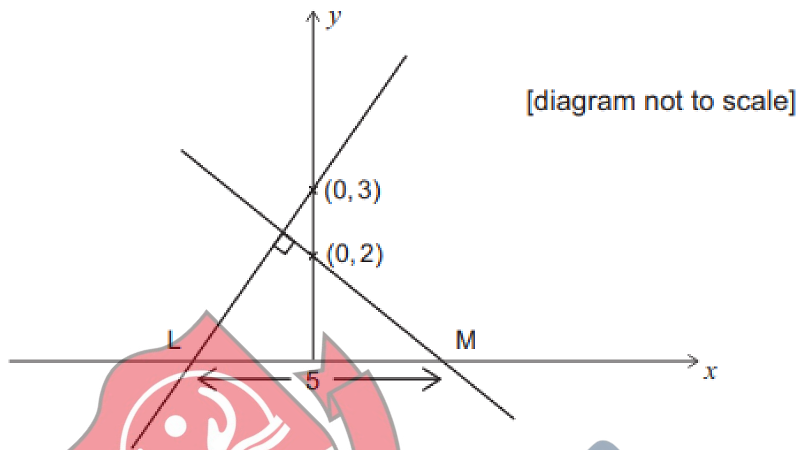
Answer is B

ENGAA S1 2016 - Question 41

41 The straight line with equation $y = mx + 3$, where $m > 0$, $m \neq 1$, is perpendicular to the line with equation $y = px + 2$

The lines cut the x-axis at the points L and M respectively. The length of LM is 5 units.

What is the value of $m + p$ given that $m > 1$?



A $-\frac{8}{3}$

B $-\frac{13}{6}$

C $-\frac{5}{6}$

D $\frac{5}{6}$

E $\frac{13}{6}$

F $\frac{8}{3}$

ENGAA S1 2016 - Question 41 - Worked Solution

Since the lines are perpendicular , the product of their gradients is -1.

$$mp = -1 \quad ---- \quad ①$$

To find the x intercepts, set y to 0.

$$0 = mx + 3 \;\; => \;\; x = -\frac{3}{m} => L \gg \left(-\frac{3}{m}, 0\right)$$

$$0 = px + 2 \;=>\; x =- \frac{2}{p} \;=>\; M => \left(-\frac{2}{p}, 0\right)$$

$$LM = 5$$

$$-\frac{2}{p} + \frac{3}{m} = 5$$

$$\frac{3p-2m}{mp} = 5$$

$$3p - 2m =- 5$$

$$3p + \frac{2}{p} =- 5$$

$$3p^2 + 5p = 2 = 0$$

$$(3p + 2)(p + 1) = 0$$

$$p =- \frac{2}{3} \;,\; p =- 1$$

$$=> \; m = \frac{3}{2} \;,\; m = 1 \; but \; m \neq 1$$

$$\therefore p =- \frac{2}{3} \;,\; m = \frac{3}{2}$$

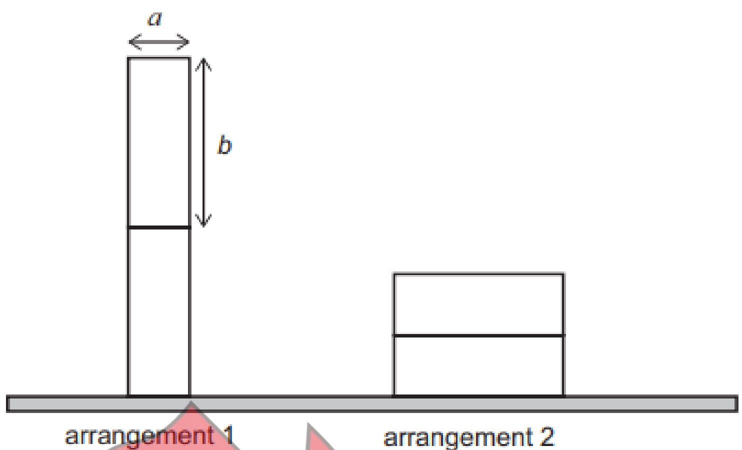
$$m + p = \frac{5}{6}$$

Answer is D

ENGAA S1 2016 - Question 42

42 The diagram shows two identical blocks, each of mass *m*, in two different arrangements.

[diagram not to scale]



Which expression gives:

[gravitational potential energy of arrangement 2] – [gravitational potential energy of arrangement 1] ?

(*g* is the gravitational field strength.)

A $2mg(a-b)$

B $2mg(b-a)$

C $-mg(b+a)$

D $mg(a+b)$

E $\frac{3}{2}mg(a-b)$

F $\frac{1}{2}mg(a-b)$

ENGAA S1 2016 - Question 42 - Worked Solution

Assuming the blocks are uniform:

$$GPE_1 = mg\left(\frac{b}{2} + \frac{3b}{3}\right) = 2mgb$$

$$GPE_2 = mg\left(\frac{a}{2} + \frac{3a}{2}\right) = 2mga$$

$$GPE_2 - GPE_1 = 2mg(a - b)$$

Answer is A

ENGAA S1 2016 - Question 43

43 $f(x) = x^3 - a^2x$ where a is a positive constant.

Find the complete set of values of x for which $f(x)$ is an increasing function.

A $x \le -a,\ x \ge a$

B $-a \le x \le a$

C $x \le -a,\ 0 \le x \le a$

D $-a \le x \le 0,\ x \ge a$

E $x \le -\frac{a}{3},\ x \ge \frac{a}{3}$

F $-\frac{a}{3} \le x \le \frac{a}{3}$

G $x \le -\frac{a}{\sqrt{3}},\ x \ge \frac{a}{\sqrt{3}}$

H $-\frac{a}{\sqrt{3}} \le x \le \frac{a}{\sqrt{3}}$

ENGAA S1 2016 - Question 43 - Worked Solution

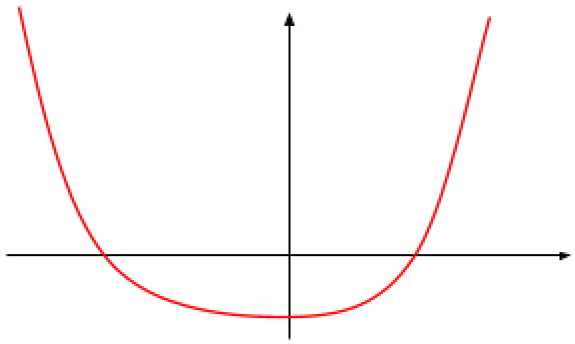
$$f(x) = x^3 - a^2x$$

$$f'x = 3x^2 - a^2$$

$$f'x = 3\left(x^2 - \left(\frac{a}{\sqrt{3}}\right)^2\right) > 0$$

$$3\left(x^2 - \left(\frac{a}{\sqrt{3}}\right)^2\right) > 0$$

$$3\left(x + \frac{a}{\sqrt{3}}\right)\left(x - \frac{a}{\sqrt{3}}\right) > 0$$



$$x \leq -\frac{a}{\sqrt{3}}, \; x \geq \frac{a}{\sqrt{3}}$$

Answer is G

ENGAA S1 2016 - Question 44

44 An object is fired vertically upwards from the ground at time $t = 0$ s in still air at a speed of $8.0\,\text{m s}^{-1}$.

On the way up, what is the height of the object above the ground when it has a speed of $2.0\,\text{m s}^{-1}$, and at what time does it reach this height on the way down?

(The gravitational field strength g is $10\,\text{N kg}^{-1}$. Air resistance can be ignored.)

	height / m	*time* / s
A	2.4	0.60
B	2.4	0.64
C	2.4	1.0
D	2.4	2.0
E	3.0	0.60
F	3.0	0.64
G	3.0	1.0
H	3.0	2.0

ENGAA S1 2016 - Question 44 - Worked Solution

Use suvats   Take as upward as +ve

$$v^2 = u^2 + 2as$$

$$a = -10ms^{-2}$$

$$u = 8.0ms^{-1}$$

$$2^2 = 8^2 - 2\times10\times h$$

$$20h = 8^2 - 2^2 = 60$$

$$h = 3.0m$$

$$s = ut + \frac{1}{2}at^2$$

$$3 = 8t - 5t^2$$

$$5t^2 - 8t + 3 = 0$$

$$t^2 - \frac{8}{5}t + \frac{3}{5} = 0$$

$$\left(t - \frac{4}{5}\right)^2 - \left(\frac{4}{5}\right)^2 + \frac{3}{5} = 0$$

$$\left(t - \frac{4}{5}\right)^2 = \frac{16}{25} - \frac{3}{5}$$

$$\left(t - \frac{4}{5}\right)^2 = \frac{1}{25}$$

$$t - \frac{4}{5} = \pm\frac{1}{5}$$

$$t = 1s,\ \ t = \frac{3}{5}s \quad \textit{The later time will be the way down}$$

Answer is G

ENGAA S1 2016 - Question 45

45 The curve $y = x^2$ is translated by the vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and then reflected in the line $y = -1$

Which one of the following is an equation of the resulting curve?

A $y = -3 - (x - 4)^2$

B $y = -3 + (x + 4)^2$

C $y = 3 - (x + 4)^2$

D $y = 3 + (x - 4)^2$

E $y = -5 - (x - 4)^2$

F $y = -5 + (x + 4)^2$

G $y = 5 - (x + 4)^2$

H $y = 5 + (x - 4)^2$

ENGAA S1 2016 - Question 45 - Worked Solution

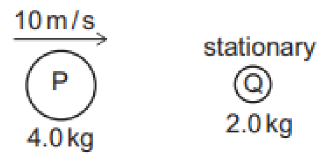
First the translation
This transforms the curve into
$y = (x - 4)^2 + 3$ As this is a shift to the right by 4 and up by 3.
Then the reflection, the bottom point of the curve is 4 units above the line y = - 1 and it is a positive quadratic.
After the reflection it will be 4 units below the line and a negative quadratic.

$$y = -(x - 4)^2 - 5$$

Answer is E

ENGAA S1 2016 - Question 46

46 The diagram shows a ball P, of mass 4.0 kg, moving to the right at 10 m s^{-1} directly towards a stationary ball Q, of mass 2.0 kg.



The balls collide but do not join together. Immediately after the collision ball Q moves at 10 m s^{-1} to the right.

What is the velocity of ball P immediately after the collision, and how much kinetic energy in total is lost during the collision?

	velocity of ball P after collision	*kinetic energy lost during collision / J*
A	0	50
B	0	150
C	10 m s^{-1} to the left	50
D	10 m s^{-1} to the left	150
E	5.0 m s^{-1} to the right	50
F	5.0 m s^{-1} to the right	150

ENGAA S1 2016 - Question 46 - Worked Solution

Conservation of momentum

Take going to the right as positive

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$4\times10 + 2\times0 = 4\times v_1 + 2\times10$$

$$v_1 = +\ 5.0ms^{-1}$$

$$Kinetic\ energy\ before = \frac{1}{2}\times4\times10^2 = 200J$$

$$Kinetic\ energy\ after = \frac{1}{2}\times4\times5^2 + \frac{1}{2}\times2\times10^2 = 150J$$

$$\Delta E = 50J$$

Answer is E

ENGAA S1 2016 - Question 47

47 The complete set of values of x for which $2x^4 - 9x^2 + 4 > 0$ is

A $x < \frac{1}{2},\ x > 4$

B $\frac{1}{2} < x < 4$

C $x < -2,\ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}},\ x > 2$

D $-2 < x < \frac{-1}{\sqrt{2}},\ \frac{1}{\sqrt{2}} < x < 2$

E $-2 < x < 2$

ENGAA S1 2016 - Question 47 - Worked Solution

$$2x^4 - 9x^2 + 4 > 0$$

$$Let\ u = x^2$$

$$2u^2 - 9u + 4 > 0$$

$$(2u - 1)(u - 4) > 0$$

$$u = \frac{1}{2}\ ,\ u = 4\ \ when\ the\ function\ equals\ 0$$

$$x^2 = \frac{1}{2}\ ,\ x^2 = 4$$

$$x = \frac{1}{\sqrt{2}}\ ,\ \ -\frac{1}{\sqrt{2}}\ ,\ 2\ ,\ \ -2$$

$$x <- 2\ ,\ \ -\frac{1}{\sqrt{2}}\ ,\ x > 2$$

Answer is C

ENGAA S1 2016 - Question 48

48 A point object of mass 2.0 kg is at rest on a level, horizontal surface. The coefficient of friction between the object and the surface is 0.25.

Two horizontal forces at right-angles to each other, with magnitudes 9.0 N and 12.0 N, are applied simultaneously to the object.

What is the magnitude of the acceleration of the object as it begins to move?

(The gravitational field strength g is $10\,N\,kg^{-1}$.)

A $5.0\,m\,s^{-2}$

B $7.25\,m\,s^{-2}$

C $7.5\,m\,s^{-2}$

D $8.0\,m\,s^{-2}$

E $10\,m\,s^{-2}$

F $10.5\,m\,s^{-2}$

ENGAA S1 2016 - Question 48 - Worked Solution

The magnitude of the applied horizontal forces is $\sqrt{9^2 + 12^2} = 15.0N$
The maximum frictional force $= \mu mg = 0.25 \times 2 \times 10 = 5N$
Therefore there is a resultant force of 10.0N 0n the object

$$\sum f = ma$$

$$a = 5ms^{-1}$$

Answer is A

ENGAA S1 2016 - Question 49

49 A cursor starts at the point (0, 10) and moves parallel to the x-axis in the negative direction.

What is the minimum distance parallel to the y-axis between the cursor and the graph of $y = 4x^3 - 12x^2 - 36x - 15$?

A 0

B 5

C 25

D 69

E 133

ENGAA S1 2016 - Question 49 - Worked Solution

The cursor is at the point , (x , 10) when $x \leq 0$

The distance parallel to the y axis from the curve to the cursor

$$s = 10 - \left(4x^3 - 12x^2 - 36x - 15\right)$$

$$s = -4x^3 + 12x^2 = 36x + 25$$

$$\frac{ds}{dx} = -12x^2 + 24x + 36$$

$$to\ find\ s_{min},\ set\ \frac{ds}{dx}\ to\ 0$$

$$-12x^2 + 24x + 36 = 0$$

$$-12\left(x^2 - 2x - 3\right) = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3\ ,\quad x = -1$$

$$x = 3\ can't\ be\ the\ solution\ as\ the\ cursor\ moves\ in\ negative\ x$$

Verify x = -1 is in fact a minimum by evaluating the second derivative

$$\frac{d^2s}{d^2x} = -24x + 24 = 48 > 0$$

$$s = -4(-1)^3 + 12(-1)^2 + 36(-1) + 25$$

$$s = 4 + 12 - 36 + 25 = 5$$

Answer is B

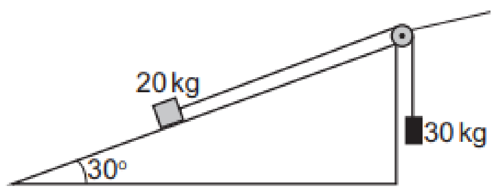
ENGAA S1 2016 - Question 50

50 An object of mass 20 kg is pulled up a rough plane inclined at 30° to the horizontal by a light, inextensible cable attached via a frictionless pulley to a freely-falling 30 kg mass. The acceleration of the object along the plane is $2.5\,\text{m s}^{-2}$.

$$\cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$$

[diagram not to scale]



What is the frictional force between the object and the plane?

(Air resistance and the mass of the pulley can be ignored. The gravitational field strength g is $10\,\text{N kg}^{-1}$.)

A 25 N

B 50 N

C 75 N

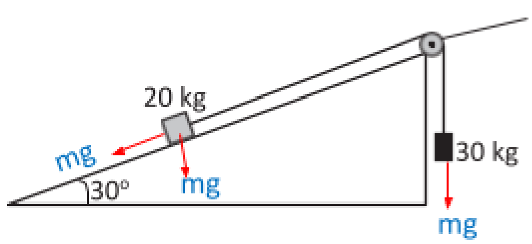
D 100 N

E 150 N

F 175 N

G 250 N

ENGAA S1 2016 - Question 50 - Worked Solution



$$\sum F = ma$$

Following mass :

$$mg - T = ma$$

$$30\times10 - T = 30\times2.5$$

$$T = 225N$$

Mass on slope :

$$T - F_R - mg \sin sin\,\theta = ma$$

$$225 - F_R - 20\times10\times\frac{1}{2} = 20\times2.5$$

$$225 - 100 - 50 = F_R$$

$$F_R = 75N$$

Answer is C

ENGAA S1 2016 - Question 51

51 What is the area enclosed by the line $x = 7$ and the curve $x = 3(y-1)^2 + 4$?

A 4

B 8

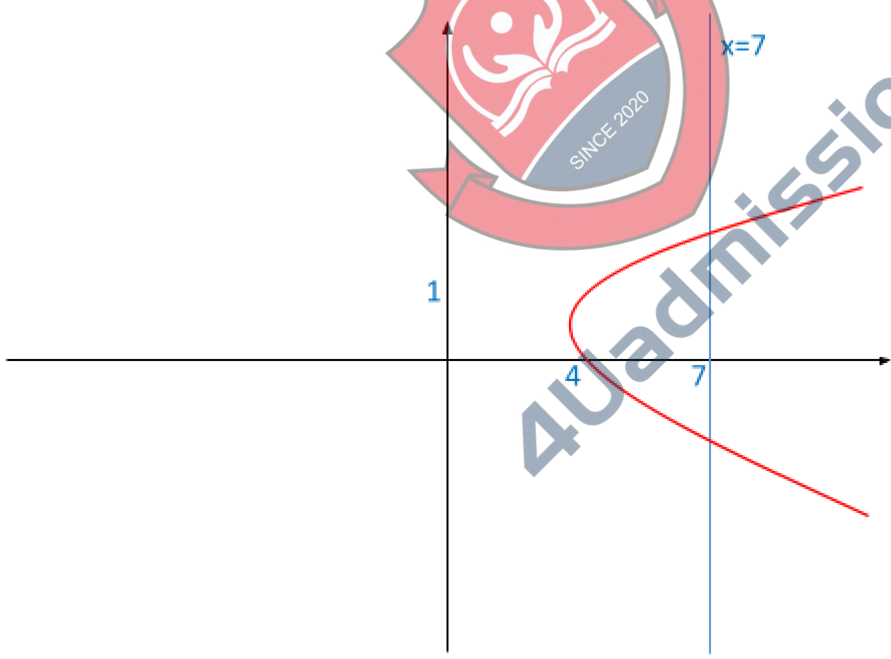
C 10

D 11

E 14

F 20

ENGAA S1 2016 - Question 51 - Worked Solution



$$3(y - 1)^2 + 4 = 7$$

$$3(y - 1)^2 = 3$$

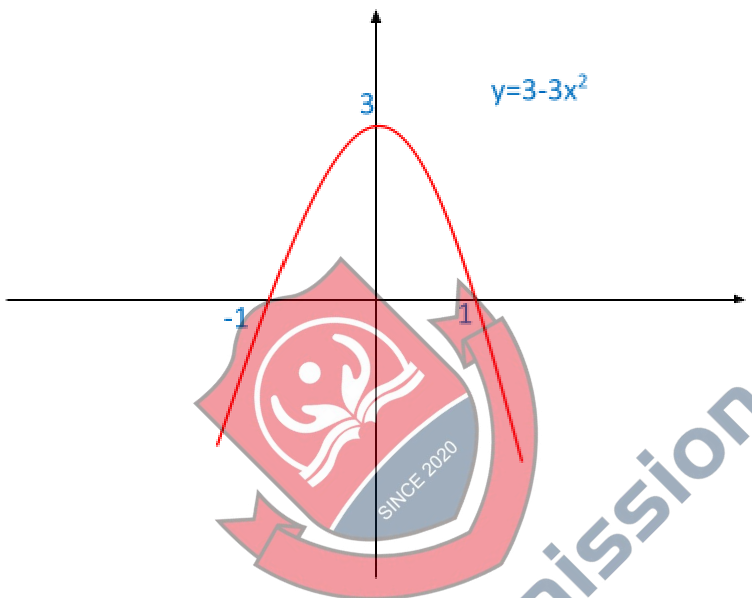
$$(y - 1)^2 = 1$$

$$y = 1 \pm 1$$

$$y = 0 \ , \ y = 2$$

Imagine the line x = 7 as the x axis.

The area enclosed by the parabola $x = 3(y - 1)^2 + 4$ and x = 7 is the same as the area enclosed by the x axis and the parabola $y = 3 - 3x^2$



$$A = \int_{-1}^{1} 3 - 3x^2 dx$$

$$A = \left[3x - x^3\right]_{-1}^{1}$$

$$A = (3 - 1) - (- 3 - (- 1))$$

$$A = 2 - (- 2)$$

$$A = 4$$

Answer is A

ENGAA S1 2016 - Question 52

52 A spacecraft of initial total mass 4000 kg is travelling relative to the Earth at a constant speed of 7425 m s^{-1}.

It ejects some fuel backwards in a sudden burst at a speed relative to the spacecraft of 1425 m s^{-1}. As a result of this, the speed of the spacecraft immediately after the fuel is ejected increases to 7500 m s^{-1}.

What is the mass of fuel ejected?

A 22 kg

B 34 kg

C 40 kg

D 50 kg

E 200 kg

F 210 kg

ENGAA S1 2016 - Question 52 - Worked Solution

Beware : the mass of fuel has a speed of 1425ms^{-1} relative to the rocket, which itself moving forward at 7425ms^{-1} when the fuel is ejected.
As a result , the fuel has a velocity of 6000ms^{-1} in the same direction as the rocket relative to an observer outside the rocket.

$(4000)(7425) = (\Delta M)(6000) + (4000 - \Delta M)(7500)$ *By conservation of P*

$$2.97\times10^{7} = 6000\Delta M + 4000\times7500 - 7500\Delta M$$

$$2.97\times10^{7} = 3\times10^{7} - 1500\Delta M$$

$$-3\times10^{-5} =- 7500\Delta M$$

$$\Delta M = 200\ kg$$

Answer is E

Alternatively consider the problem in the rockets frame of the references , in which the rocket is initially at rest.

$$v_1 = 7500 - 7425 = 75ms^{-1}$$

$$v_2 = 1425ms^{-1}$$

$$\Delta M v_2 = (M - \Delta M)v_1$$

$$1425\Delta M = 75(4000 - \Delta M)$$

$$1500\Delta M = 3\times10^5$$

$$\Delta M = 200\ kg$$

Note : This is called the center of mass frame

ENGAA S1 2016 - Question 53

53 A curve has equation $y = 3x^4 - 4x^3 - 12x^2 + 20$

What is the complete set of values of the constant k for which the equation

$$3x^4 - 4x^3 - 12x^2 + 20 = k$$

has exactly four distinct real roots?

A no values of k

B $-12 < k < 15$

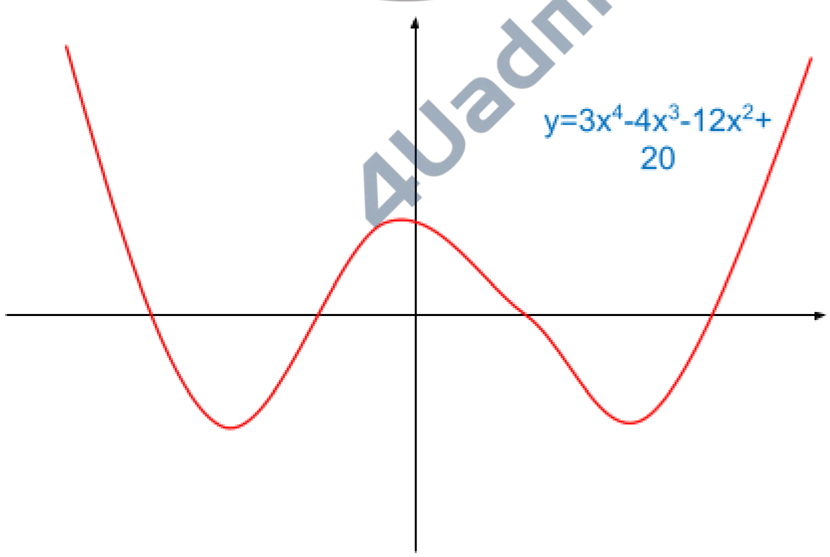
C $15 < k < 20$

D $k > 20$

E $7 < k < 20$

F all values of k

ENGAA S1 2016 - Question 53 - Worked Solution



$$y = 3x^4 - 4x^3 - 12x^2 + 20$$

As the curve is a quadratic it has this form with 2 minima and one maximum. The exact shapes does not matter only that it is positive.

For y = k to have 4 real roots , the line y = k must intersect $y = 3x^4 - 4x^3 - 12x^2 + 20$ 4 times. K must be higher than both of the minimum values and lower than the maximum value.

$$To\ find\ the\ extrema,\ set\ \frac{dy}{dx}\ to\ 0$$

$$\frac{dy}{dx} = 12x^3 - 12x^2 - 24x = 0$$

$$x\left(12x^2 - 12x - 24\right) = 0$$

$$12x(x + 1)(x - 2) = 0$$

$$x = 0 , x = - 1 , x = 2$$

The y values of these points are : 20 , 15 , and -12 respectively.

$$\therefore k > - 12 , k > 15 , k < 20$$

$$15 < k < 20$$

Answer is C

ENGAA S1 2016 - Question 54

54 An object of weight 40 N hangs from the end of a light inextensible string of length 0.35 m, which is attached to the ceiling. A constant horizontal force of 30 N is applied to the object, causing it to move to a new equilibrium position with the string no longer vertical.

By how much has the gravitational potential energy of the object increased as a result of its change of position?

A 2.1 J

B 2.8 J

C 3.5 J

D 4.2 J

E 4.9 J

F 5.6 J

ENGAA S1 2016 - Question 54 - Worked Solution

$$T \sin sin\, \theta \;=\; 40$$

$$T \cos cos\, \theta \;=\; 30$$

$$\tan tan\, \theta \;=\; \frac{4}{3}$$

$$h \;=\; 0.35 \sin sin\, \theta$$

$$\sin sin\, \theta \;=\; \frac{4}{5}$$

$$h \;=\; 0.28m$$

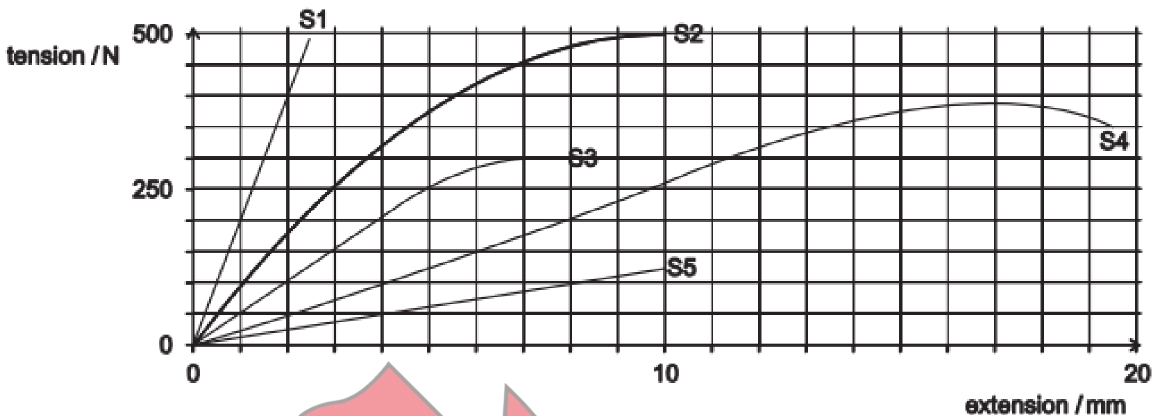
The object is 0.07m higher in the new equilibrium

$$\Delta GPE \;=\; mg\Delta h \;=\; 40 \times 0.07 \;=\; 2.8J$$

Answer is B

ENGAA S2 2016 - Question 1

1 A series of 100 mm long samples have a square cross-section of 5 mm × 5 mm. Each sample is tested in an apparatus which applies increasing tension to the ends of the sample until it breaks. The length of the sample is measured and a plot of tension vs extension obtained. The results for five samples S1-S5 are shown in the figure below.



a) Which of the samples is the stiffest compared to the other samples? **[1 mark]**

SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.

A S1

B S2

C S3

D S4

E S5

ENGAA S2 2016 - Question 1 - Worked Solution

a)

A stiffer sample will require more force to extend it to a given extension.
The sample with the highest Tension extension gradient 1 , S1
Answer is A

b)

b) Which sample does not obey Hooke's Law up to 2% strain? **[1 mark]**

SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.

A S1

B S2

C S3

D S4

E S5

Hooke's law states that force is proportional to extension.
Strain = extension / original length
2/strain => 2mm extension
52 is not a straight line until 2mm, thus it does not obey Hooke's law
Answer is B

c)

c) What is the value of the Young's Modulus of sample S3, assuming that changes in its cross-section are negligible? **[2 marks]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A 200 MPa

B 20 MPa

C 100 MPa

D 10 MPa

$$\sigma_1\ Stress\ =\ forcce \div cross\ -\ sectional\ area$$

$$\varepsilon_1\ strain\ =\ extension \div original\ length$$

Young's modulus , $E\ = \frac{\sigma}{\varepsilon}$

In the straight line portion of 53's curve, it reaches 250N tension at 5mm extension

$$\sigma\ =\ 250 \div \left(5 \times 10^{-3}\right)^{2}\ =\ 10^{7}\ Pa$$

$$\varepsilon\ =\ 5 \div 100\ =\ 0.05$$

$$E\ = \frac{\sigma}{\varepsilon} =\ 2 \times 10^{8}\ Pa\ =\ 200MPa$$

Answer is A

d)

d) The equation of the force-extension curve for sample S2 is given by $T = ax - bx^2$ where T is the tension in N, x the extension in m and a and b are constants. Sample S2 breaks when its extension is 10 mm. How much work does the apparatus do on sample S2 in breaking it?

[2 marks]

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A $\frac{a}{2} \times 10^{-4} - \frac{b}{3} \times 10^{-6}$ Nm

B $\frac{a}{2} \times 10^{-2} - \frac{b}{3} \times 10^{-3}$ Nm

C $\frac{a}{2} \times 10^{2} - \frac{b}{3} \times 10^{3}$ Nm

D $\frac{a^2}{2} \times 10^{2} - \frac{b^2}{3} \times 10^{3}$ Nm

E $\frac{a}{2} \times 10^{-2} - \frac{b}{2} \times 10^{-3}$ Nm

$$Work\ done,\ W = \int T\,dx$$

$$W = \int_{0}^{10^{-2}} ax - bx^2\,dx$$

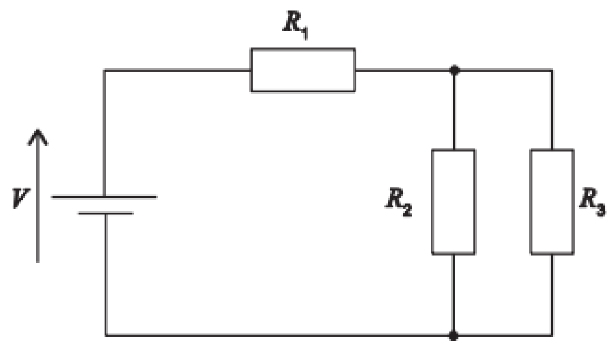
$$W = \left[\frac{1}{2}ax^2 - \frac{1}{3}bx^3\right]_0^{10^{-2}}$$

$$W = \frac{a}{2} \times 10^{-4} - \frac{b}{3} \times 10^{-6}\ Nm$$

Answer is A

ENGAA S2 2016 - Question 2

2 The figure below shows a network of three non-zero resistances R_1, R_2, R_3 connected to a voltage source V with zero internal resistance.



a) Which of the following statements must be correct? **[1 mark]**

NO WORKING NEEDS TO BE GIVEN FOR THIS PART OF THIS QUESTION.

A The currents through resistances R_1 and R_2 are the same.

B The currents through resistances R_1 and R_3 are the same.

C The currents through resistances R_2 and R_3 are the same.

D The voltages across resistances R_1 and R_2 are the same.

E The voltages across resistances R_1 and R_3 are the same.

F The voltages across resistances R_2 and R_3 are the same.

ENGAA S2 2016 - Question 2 - Worked Solution

a) by Kirchoff's voltage law

Answer is F

b)

b) Which of the following expressions gives the current through the voltage source? **[2 marks]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A $\frac{V(R_2 + R_3)}{R_1R_2 + R_1R_3 + R_2R_3}$

B $\frac{V(R_1R_2 + R_1R_3 + R_2R_3)}{R_2 + R_3}$

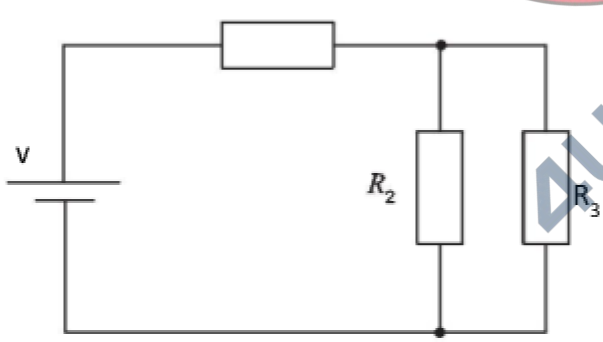
C $\frac{VR_2R_3}{R_2 + R_3 + R_1R_2R_3}$

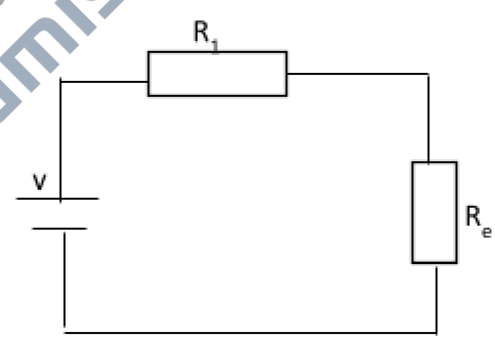
D $\frac{V}{R_1 + R_2 + R_3}$

E $\frac{V(R_1R_2 + R_1R_3 + R_2R_3)}{R_1R_2R_3}$

For resistance in parallel:

$$\frac{1}{R_e} = \sum_i \frac{1}{R_i}$$





$$\frac{1}{R_e} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_e = \frac{R_2R_3}{R_2+R_3}$$

For resistance in series:

$$R_T = \sum_i R_i$$

The total resistance of the circuit

$$R_T = R_1 + \frac{R_2R_3}{R_2+R_3}$$

$$\text{The current in the circuit} = \frac{V}{R_T}$$

The current is the same everywhere in a series circuit

$$I = \frac{V}{R_1+\frac{R_2R_3}{R_2+R_3}} = \frac{V(R_2+R_3)}{R_1R_2+R_1R_3+R_2R_3}$$

Answer is A

c)

c) R_1 and R_2 are now fixed such that $R_1 = R_2$. Which of the following expressions gives the power P that is dissipated by resistance R_3? **[3 marks]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A $\dfrac{V^2 R_3^3}{(R_1 + 2R_3)^2}$

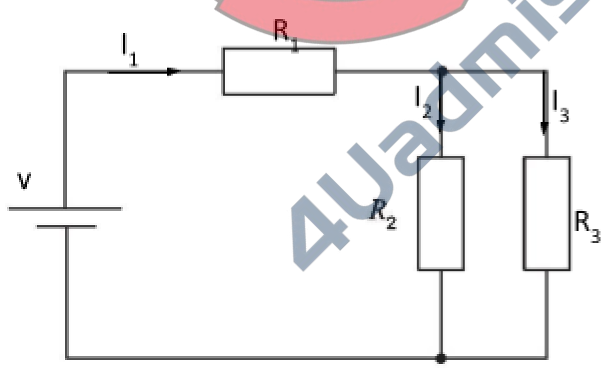
B $\dfrac{V^2}{2R_1 + R_3}$

C $\dfrac{V^2 (R_1 + 2R_3)^2}{R_3^3}$

D $\dfrac{V^2 R_3}{(R_1 + 2R_3)^2}$

E $\dfrac{V^2 (R_1 + R_3)^2}{(R_1 + 2R_3)^2 R_3}$

$R_1 = R_2$



$$I_1 = I_2 + I_3 \quad by\ Kirchoff's\ current\ law$$

$$I_1 R_1 + I_3 R_3 = V \quad by\ Kirchoff's\ voltage\ law$$

$$I_1 = \frac{V(R_1 + R_3)}{R_1(R_1 + 2R_3)}$$

$$I_1R_1 = \frac{V(R_1+R_3)}{(R_1+2R_3)}$$

$$I_3 = \frac{V}{R_3}\left(1 - \frac{R_1+R_3}{R_1+2R_3}\right)$$

$$I_3 = \frac{V}{R_3}\left(\frac{R_1+2R_3-R_1-R_3}{R_1+2R_3}\right)$$

$$I_3 = \frac{V}{R_1+2R_3}$$

$$P = I_3^{\ 2}R_3 = \frac{V^2R_3}{(R_1+2R_3)^2}$$

Answer is D

d)

d) For the case where $R_1 = R_2$, which of the following values of R_3 maximises its power dissipation? You may find it helpful to use the fact that any value of R_3 that maximises P also minimises $1/P$. **[3 marks]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A $R_3 = \frac{1}{4}R_1^2$

B $R_3 = \frac{1}{2}R_1$

C $R_3 = \exp\left(-\frac{4}{R_1^2}\right)$

D $R_3 = \frac{1}{\sqrt{2}}R_1$

E $R_3 = \frac{1}{4}R_1$

F Either $R_3 = \frac{1}{2}R_1$ or $R_3 = -\frac{1}{2}R_1$ would result in maximum power dissipation in R_3.

$$To\ find\ P_{max},\ set\ \frac{dP}{dR_3}\ to\ 0$$

$$\frac{dP}{dR_3} = \frac{1}{(R_1+2R_3)^4}\left[(R_1 + 2R_3)^2 - R_3\times 4(R_1 + 2R_3)\right]$$

$$\frac{dP}{dR_3} = \frac{R_1+2R_3-4R_3}{(R_1+2R_3)^3}$$

$$= \frac{R_1-2R_3}{(R_1+2R_3)^3} = 0$$

$$R_1 = 2R_3$$

$$R_3 = \frac{R_1}{2}$$

Answer is B

ENGAA S2 2016 - Question 3

3 The speed of light in vacuum and air can be taken to be $c = 3.0 \times 10^5 \, km \, s^{-1}$. The refractive index n of a material is the ratio of the speed of light c in vacuum to the speed of light in the material.

a) A lighthouse emits a beam of light. How far does this beam of light travel in 1.0 ns? **[1 mark]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A 0.30 mm

B 300 m

C 0.30 m

D 3.0×10^{-12} m

ENGAA S2 2016 - Question 3 - Worked Solution

a)

$$d = vt = 3.0 \times 10^8 \times 1.0 \times 10^{-9} = 0.3m$$

Answer is C

b)

b) The propagation time *T* is the time taken for a pulse of light to travel directly along an optical fibre. A straight optical fibre has a length of 9 km. Its refractive index is 1.5. What is *T* for this fibre? **[1 mark]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A 20 ms

B 20 μs

C 30 ms

D 30 μs

E 45 ms

F 45 μs

$$v^1 = \frac{c}{n} = \frac{3.0\times10^5}{1.5} = 2.0\times10^5 kms^{-1}$$

$$T = \frac{d}{v} = \frac{9}{2\times10^5} = 45\mu s$$

Answer is F

c)

c) An engineer has used a refractive index of $n = 1.5$ to estimate the nominal propagation time T_{nom} for an optical fibre. The actual refractive index of the fibre depends on the wavelength of the light. For red and blue light the refractive indices obey the inequality $n_{red} < n_{blue} < 1.5$. If T_{red} and T_{blue} are the propagation times for red and blue light, respectively, which of the following inequalities is correct? **[2 marks]**

SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.

A $T_{red} < T_{blue} < T_{nom}$

B $T_{blue} < T_{red} < T_{nom}$

C $T_{red} < T_{nom} < T_{blue}$

D $T_{blue} < T_{nom} < T_{red}$

E $T_{nom} < T_{red} < T_{blue}$

F $T_{nom} < T_{blue} < T_{red}$

$$T \propto \frac{1}{v} \propto \frac{1}{1/n} \propto n$$

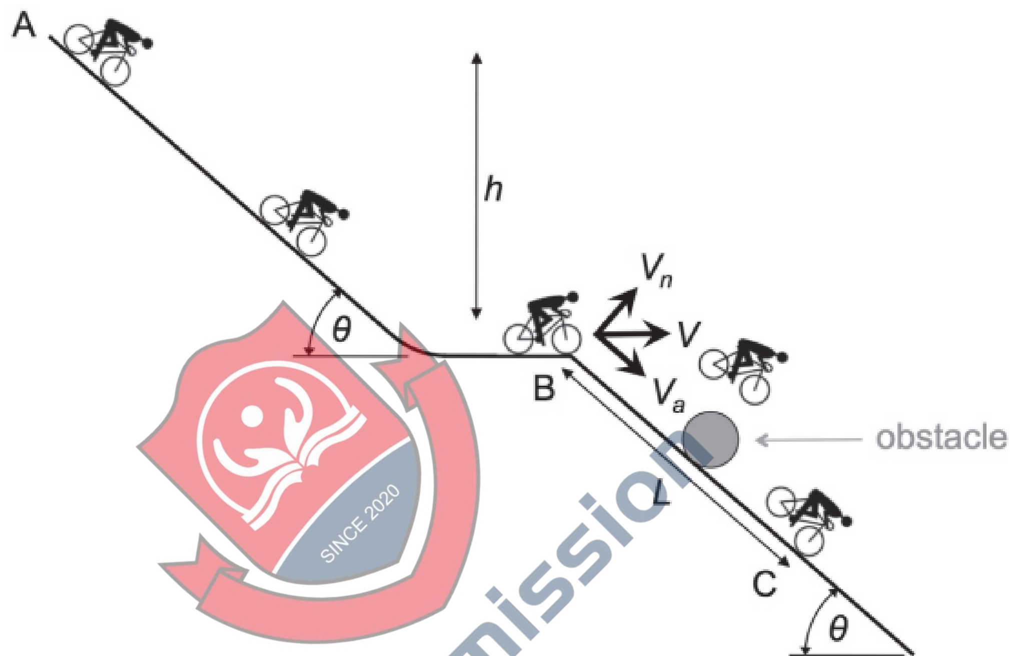
$$n_{red} < n_{blue} < 1.5$$

$$T_{red} < T_{blue} < T_{nom}$$

Answer is A

ENGAA S2 2016 - Question 4

4 A stunt cyclist is preparing a new trick. The track on which he will perform the trick is shown schematically in the figure below. As shown, most of the track is sloped at an angle θ to the horizontal.



The cyclist starts riding from rest at A. In riding down the slope from A to B he transfers an amount of energy E from his muscles to provide kinetic energy to the rider-bicycle system and descends through a vertical distance h. The cyclist leaves the track at B, travelling horizontally initially. He lands on the track at C, a distance L down the slope.

Assume that the rider-bicycle system can be modeled as a point mass of mass M, that frictional forces and air resistance can be neglected, and that the gravitational field strength is g.

a) What is V_a, the component of the velocity of the rider-bicycle system along (parallel to) the slope, immediately after the cyclist has left the track at B? **[2 marks]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A $\sqrt{\left[\frac{E}{M} + 2gh\right]} \cos\theta$

B $\sqrt{\frac{2}{M}[E + Mgh]} \sin\theta$

C $\sqrt{\frac{2}{M}[E + Mgh]} \frac{1}{\cos\theta}$

D $\sqrt{\left[\frac{E}{M} + 2gh\right]} \sin\theta$

E $\sqrt{2\left[\frac{E}{M} + gh\right]} \cos\theta$

ENGAA S2 2016 - Question 4 - Worked Solution

a) At B the cyclist has transferred an energy E and dropped a height h , therefore:

$$\frac{1}{2}MV^2 = Mgh + E$$

$$V^2 = 2gh + \frac{2E}{M}$$

$$V = \sqrt{2gh + \frac{2E}{M}}$$

At B all the V is horizontal due to the shape of the track.

$$\therefore V_a = V \cos \cos \theta$$

$$V_a = \sqrt{2\left(\frac{E}{M} + gh\right)} \cos \cos \theta$$

Answer is E

b)

b) The cyclist leaves the track at B at time $t = 0$ with an initial speed V. By considering motion parallel and/or perpendicular to the slope, or otherwise, find an expression for the time taken to land at C. **[2 marks]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A $2\frac{V}{g}$

B $2\frac{V}{g}\tan\theta$

C $\frac{V}{g}\sin\theta$

D $\frac{V}{g}\tan\theta$

E $2\frac{V}{g}\sin\theta$

The horizontal speed is constant at V

The horizontal distance from B to C = $L \cos cos\, \theta$

$$time = \frac{L coscos\, \theta}{V} \text{ ------ } ①$$

Consider motion perpendicular to the slope

$$s = 0$$

$$u = V_n = V \sin sin\, \theta$$

$$v = ?$$

$$a = -g \cos cos\, \theta$$

$$t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$0 = Vt \sin sin\, \theta - \frac{1}{2} g \cos cos\, \theta\, t^2$$

$$t = \frac{2V sinsin\, \theta}{g coscos\, \theta} = \frac{2V tantan\, \theta}{g} \quad \text{------ } ②$$

Answer is B

c)

c) How far along the slope will the cyclist land, i.e. what is the value of L? **[3 marks]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A $2\frac{V^2}{g}\sin\theta$

B $\frac{V^2}{2g}$

C $2\frac{V^2}{g}\frac{\sin\theta}{\cos^2\theta}$

D $2\frac{V^2}{g}\frac{\cos\theta}{\sin^2\theta}$

E $2\frac{V^2}{g}[\sin\theta + \tan^2\theta]$

From ① & ②

$$\frac{2V\mathrm{tan}tan\,\theta}{g} = \frac{L\mathrm{cos}cos\,\theta}{V}$$

$$L = \frac{2V^2}{g}\frac{\mathrm{sin}sin\,\theta}{cos^2\theta}$$

Answer is C

d)

d) As part of the trick, the cyclist wants to clear an obstacle placed on the slope between B and C. To give the cyclist the greatest chance of clearing the obstacle it should be placed at the point at which the cyclist's perpendicular distance from the track is greatest. At what distance from B should the obstacle be placed? **[3 marks]**

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

A $\frac{1}{2}L$

B $\frac{V^2}{2g}\frac{\sin^2\theta}{\cos\theta}$

C $\frac{1}{2}h$

D $\frac{V^2}{g}\sin\theta\left[1 + \frac{1}{2}\tan^2\theta\right]$

E $\frac{V^2}{2g}$

Distance perpendicular to the slope

$$S_n = Vt \sin sin\,\theta - \frac{1}{2}gt^2 \cos cos\,\theta$$

$$to\ find\ S_{max},\ set\ \frac{dS_n}{dt}\ to\ 0$$

$$\frac{dS_n}{dt} = V \sin sin\,\theta - gt \cos cos\,\theta = 0$$

$$gt \cos cos\,\theta = V \sin sin\,\theta$$

$$t = \frac{V}{g}\tan tan\,\theta$$

Distance parallel to the slope:

$$S_a = ut + \frac{1}{2}at^2$$

$$S = Vt \cos cos\,\theta + \frac{1}{2}gt^2 \sin sin\,\theta$$

$$t = \frac{V}{g}\tan tan\,\theta$$

$$S_a = \frac{V^2}{g}\tan tan\,\theta\ \cos cos\,\theta + \frac{1}{2}g \sin sin\,\theta\ \frac{V^2 tan^2\theta}{g^2}$$

$$S_a = \frac{V^2 \sin sin\,\theta}{g} + \frac{1}{2}\frac{V^2}{g}\sin sin\,\theta\, tan^2\theta$$

$$S_a = \frac{V^2}{g}\sin sin\,\theta\left(1 + \frac{1}{2}tan^2\theta\right)$$

Answer is D