Worked Solutions for ENGAA Past Papers

ENGAA S1 2017 - Question 1

1 Evaluate

$$\frac{\left(\sqrt{12}+\sqrt{3}\right)^2}{\left(\sqrt{12}-\sqrt{3}\right)^2}$$

A 1

B 3

C $\frac{5}{3}$

D $\frac{7}{3}$

E $3\sqrt{3}$

F 9

ENGAA 2017 - Question 1 - Worked Solution

$$\frac{\left(\sqrt{12}+\sqrt{3}\right)^2}{\left(\sqrt{12}+\sqrt{3}\right)^2} = \frac{\left(\sqrt{12}+\sqrt{3}\right)\left(\sqrt{12}+\sqrt{3}\right)}{\left(\sqrt{12}-\sqrt{3}\right)\left(\sqrt{12}-\sqrt{3}\right)}$$

$$= \frac{\left(\sqrt{12}\right)^2+\sqrt{3}\sqrt{12}+\sqrt{3}\sqrt{12}+\left(\sqrt{3}\right)^2}{\left(\sqrt{12}\right)^2-\sqrt{3}\sqrt{12}-\sqrt{3}\sqrt{12}+\left(\sqrt{3}\right)^2}$$

$$= \frac{12+3+2\sqrt{36}}{12+3-2\sqrt{36}}$$

$$= \frac{15+(2\times 6)}{15-(2\times 6)}$$

$$= \frac{27}{3}$$

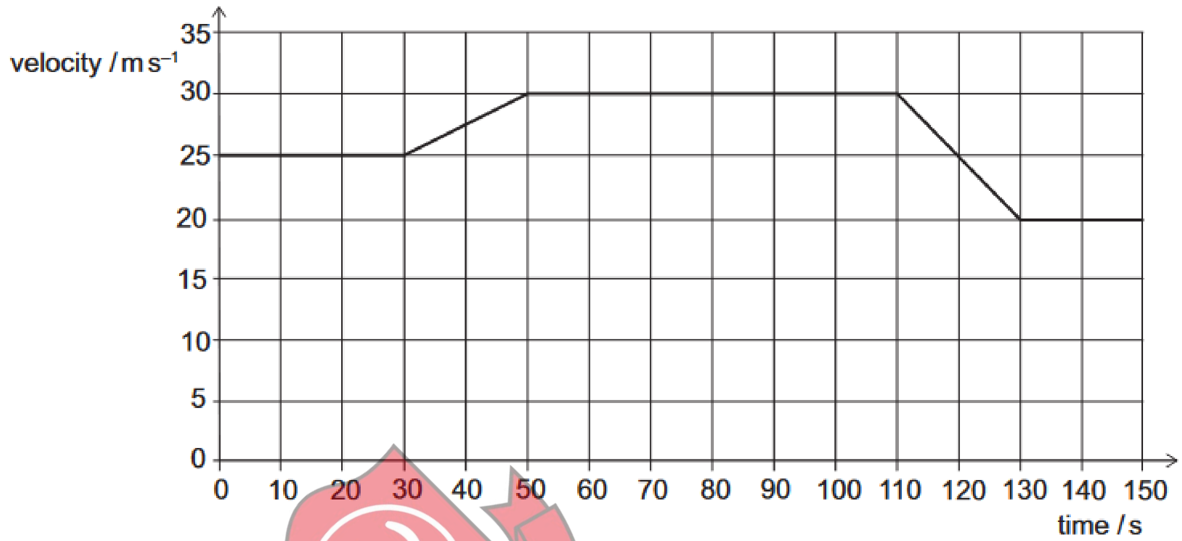
$$= 9$$

Answer is F.

ENGAA S1 2017 - Question 2

2 A car is travelling along a horizontal road in a straight line.

The graph is a velocity–time graph for part of the car's journey.



During this part of the journey, what is the total distance that the car travels while it is decelerating?

A 400 m

B 500 m

C 550 m

D 600 m

E 750 m

F 1400 m

G 1800 m

H 1900 m

ENGAA 2017 - Question 2 - Worked Solution

Velocity time graph, so area = displacement
Gradient = acceleration

Deceleration = negative gradient

Area = area at trapezium

$$= \frac{1}{2} \times (30 + 20) \times 10$$

$$= 500m$$

Answer is B

ENGAA S1 2017 - Question 3

3 Solve fully the inequality

$$2x^2 \geq 15 - x$$

A $x \leq -3$

B $x \geq 2.5$

C $x \leq -1.5$, $x \geq 5$

D $-1.5 \leq x \leq 5$

E $x \leq -3$, $x \geq 2.5$

F $-3 \leq x \leq 2.5$

ENGAA 2017 - Question 3 - Worked Solution

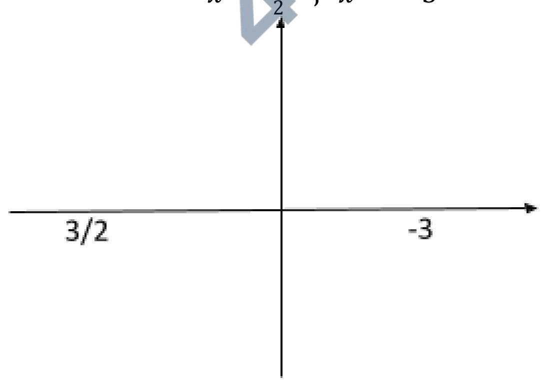
$$2x^2 \geq 15 - x$$

$$2x^2 + x - 15 \geq 0$$

$$(2x - 5)(x + 3) \geq 0$$

Critical values are then:

$$x = \frac{5}{2} \ , \ x = -3$$



$$x \leq -3 \ , \ x \geq \frac{5}{2}$$

Answer is F

ENGAA S1 2017 - Question 4

4 When a saucepan of water is heated from below, convection currents form and transfer heat through the liquid.

Here are three statements about the water as it is heated:

1 The mass of a fixed volume of the water increases.

2 The density of a fixed mass of the water decreases.

3 The volume of a fixed mass of the water increases.

Which of these statements help(s) to explain how convection currents are formed?

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

ENGAA 2017 - Question 4 - Worked Solution

$$\rho = \frac{m}{v}$$

- When water is heated , water molecules vibrate more
- So become more spaced out
- This means the value increases so ③ holds it mass is fixed
- So as $\rho = m/v$ and volume increases,, density decreases and ② holds , if mass is fixed
- If the volume was fixed however ρ would till decreases , so m would have to decrease , so ① is false.
- Convection currents are created when this hot, less dense water rises , and is replaced by the cooler, move dense water

Answer is G

ENGAA S1 2017 - Question 5

5 The equation gives y in terms of x:

$$y = 3\left(\frac{x}{2} - 1\right)^2 - 5$$

Which one of the following is a rearrangement for x in terms of y?

A $x = 2 \pm 2\sqrt{\frac{y-5}{3}}$

B $x = 2 \pm 2\sqrt{\frac{y+5}{3}}$

C $x = 2 \pm 3\sqrt{\frac{y+5}{3}}$

D $x = -2 \pm 2\sqrt{\frac{y+5}{3}}$

E $x = -2 \pm 3\sqrt{\frac{y+5}{2}}$

F $x = 2 + 2\left(\frac{y+5}{3}\right)^2$

G $x = -2 + 2\left(\frac{y+5}{3}\right)^2$

ENGAA 2017 - Question 5 - Worked Solution

$$y = 3\left[\frac{x}{2} - 1\right]^2 - 5$$

$$y + 5 = 3\left[\frac{x}{2} - 1\right]^2$$

$$\frac{y+5}{3} = \left[\frac{x}{2} - 1\right]^2$$

$$\pm\sqrt{\frac{y+5}{3}} = \frac{x}{2} - 1$$

$$1 \pm \sqrt{\frac{y+5}{3}} = \frac{x}{3}$$

$$2 \pm 2\sqrt{\frac{y+5}{3}} = x$$

Answer is B

ENGAA S1 2017 - Question 6

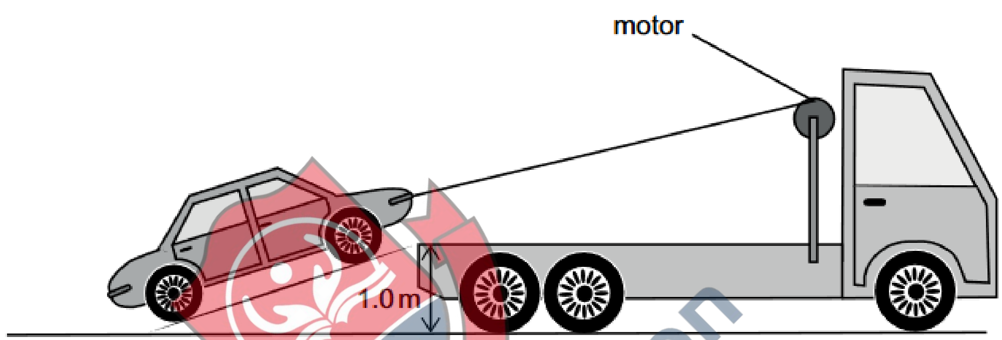
6 An electric motor is used to pull a broken-down car slowly from the road up a ramp on to the back of a breakdown truck.

The car has a mass of 1200 kg and is lifted through a vertical height of 1.0 m.

The total input energy to the motor is 28 kJ and it is 75% efficient.

In the process of lifting the car, energy is lost to the surroundings from the motor and from other causes.

What is the **total** energy lost to the surroundings?



(gravitational field strength = $10\,N\,kg^{-1}$)

A 7.0 kJ

B 9.0 kJ

C 12 kJ

D 16 kJ

E 21 kJ

F 33 kJ

ENGAA 2017 - Question 6 - Worked Solution

- Energy gained by car : (GPE)

mgh

$= 1200\times10\times1$

$12000\,J$

- Conservation at energy:
  Input Energy = Energy lost + Energy gained by car

$28000 = Energy\ lost + 12000$

$Energy\ lost = 16000J$

$Energy\ lost = 16kJ$ (include the energy lost due to the efficiency of the motor)
Answer is D

ENGAA S1 2017 - Question 7

7 A fruit stall sells apples costing £x each, and pears costing £y each.

Sam bought 2 apples and 5 pears, and the total cost of these was £P.

Lesley bought 3 apples and 2 pears, and the total cost of these was £Q.

Which of the following is an expression for the cost, in pounds (£), of a pear?

A $\frac{2Q-3P}{3}$

B $\frac{2Q-3P}{11}$

C $\frac{Q-P}{3}$

D $\frac{Q-P}{11}$

E $\frac{P-Q}{3}$

F $\frac{3P-2Q}{3}$

G $\frac{3P-2Q}{11}$

ENGAA 2017 - Question 7 - Worked Solution

$$Sam: 2x + 5y = P \quad --- \enclose{circle}{1}$$

$$Lesley: 3x + 2y = Q \quad --- \enclose{circle}{2}$$

Want to find value at y

$$3\times\enclose{circle}{1}: 6x + 15y = 3P$$

$$2\times\enclose{circle}{2}: 6x + 4y = 2Q$$

Subtract to eliminate x:

$$15y - 4y = 3P - 2Q$$

$$11y = 3P - 2Q$$

$$y = \frac{3P-2Q}{11}$$

Answer is G

ENGAA S1 2017 - Question 8

8 In one type of medical scanner a source is placed inside a patient's body. This source causes pairs of gamma-rays to be emitted simultaneously in opposite directions.

Detectors on each side of the patient are used to detect the gamma-rays. The distance between the two detectors is 3.0 m. When the source is at Q, half-way between the detectors, the two gamma-rays arrive at the same time.

In a particular scan the gamma-rays arrive at the two detectors with a time difference of 4.0×10^{-10} s.

Assume that, inside the patient, the gamma-rays travel at a speed of $3.0 \times 10^{8}\, m\, s^{-1}$.

How far from Q, half-way between the detectors, is the gamma-ray source?

A 6.0 mm

B 12 mm

C 24 mm

D 6.0 cm

E 12 cm

F 24 cm

ENGAA 2017 - Question 8 - Worked Solution

① $L - x = t_1 C$

② $x + L = t_2 C$

Subtract two equations

$$x - (-x) = t_2 C - t_1 C$$

$$2x = C(t_2 - t_1)$$

$$x = \frac{C(t_2 - t_1)}{2}$$

$$x = \frac{3.0\times10^{8}\times4.0\times10^{-10}}{2}$$

$$x = 0.06m$$

$$x = 6cm$$

Answer is D

ENGAA S1 2017 - Question 9

9 P is directly proportional to Q squared.

When P is 2, Q is 4.

Q is inversely proportional to R.

When Q is 2, R is 5.

What is P in terms of R?

A $P = \frac{5}{R}$

B $P = \frac{5}{4R}$

C $P = \frac{1}{800R^2}$

D $P = \frac{5}{4R^2}$

E $P = \frac{25}{2R^2}$

F $P = \frac{800}{R^2}$

G $P = \frac{R^2}{50}$

H $P = \frac{25R^2}{2}$

ENGAA 2017 - Question 9 - Worked Solution

① $P \propto Q^2$

$P = kQ^2$, $k = Constant$

② $P = 2$, $Q = 4$

So

$$2 = k(4)^2$$

$$2 = 16k$$

$$P = 4^2/8$$

$$③ \quad Q \propto \frac{1}{R}$$

$$Q = \frac{c}{R} \ , \quad c = another\ constant$$

$$④ \ Q = 2 \, , \, R = 5$$

So

$$2 = \frac{c}{5}$$

$$c = 5 \times 2$$

$$c = 10$$

$$Q = \frac{10}{R}$$

$$P = \frac{Q^2}{8}$$

$$P = \left(\frac{10}{R}\right)^2 \times \frac{1}{8}$$

$$P = \frac{100}{8R^2}$$

$$P = \frac{25}{2R^2}$$

Answer is E

ENGAA S1 2017 - Question 10

10 When a plutonium-239 nucleus absorbs a neutron it undergoes nuclear fission. One particular fission reaction results in the creation of xenon and zirconium as daughter nuclei. The nuclear equation for this reaction is shown but with some non-zero integers replaced by the letters w, x, y and z.

$$^{1}_{0}\text{n} + ^{239}_{94}\text{Pu} \rightarrow ^{w}_{54}\text{Xe} + ^{y}_{x}\text{Zr} + z\,^{1}_{0}\text{n}$$

Which equation is correct?

A $w + y = 240$

B $z = 240 - (w + y)$

C $x = 40 - z$

D $94 = 54 + x + 1$

E $240 = 54 + x$

F $94 = w + y + 1$

ENGAA 2017 - Question 10 - Worked Solution

Balance Nucleon Number

$$1 + 239 = w + g + z\times 1$$

$$240 = w + g + z$$

Balance Atomic Numbers

$$0 + 94 = 54 + x + 0$$

$$x = 94 - 54$$

$$x = 40$$

Answer is B

ENGAA S1 2017 - Question 11

11 Which one of the following is a simplification of

$$2-\frac{x^2(9x^2-4)}{x^3(2-3x)}$$

A $-1-\frac{2}{x}$

B $-1+\frac{2}{x}$

C $5-\frac{2}{x}$

D $5+\frac{2}{x}$

E $5-\frac{3}{x}$

F $5+\frac{3}{x}$

ENGAA 2017 - Question 11 - Worked Solution

$$2-\frac{x^2(9x^2-4)}{x^3(2-3x)}$$

$$=2-\frac{(9x^2-4)}{x(2-3x)}$$

$$=2-\frac{(3x-2)(3x-2)}{x(2-3x)}$$

$$=2+\frac{(3x-2)(3x-2)}{x(2-3x)}$$

$$=2+\frac{3x+2}{x}$$

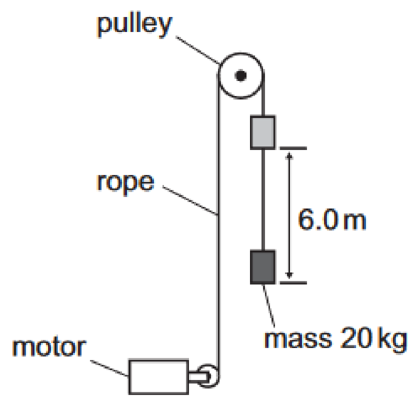
$$=2+3+\frac{2}{x}$$

$$=5+\frac{2}{x}$$

Answer is D

ENGAA S1 2017 - Question 12

12 An electric motor is connected to a constant 12 V d.c. supply. The motor is used to lift a mass of 20 kg by means of a rope and pulley. The mass is lifted vertically through a height of 6.0 m in a time of 5.0 s. The complete lifting system (motor, rope and pulley) is 80% efficient.



What is the current in the electric motor?

(gravitational field strength = 10 N kg^{-1})

A 1.6 A

B 2.0 A

C 2.5 A

D 16 A

E 20 A

F 25 A

ENGAA 2017 - Question 12- Worked Solution

$$\Delta CPE = mgh = 20 \times 10 \times 6 = 1200\,J$$

$$Efficiency = \frac{Useful\ Energy}{Total\ Energy}$$

$$rp = power$$

$$rp = 1V$$

$$V = 12V$$

$$0.8 = \frac{\Delta CPE}{p \times time}$$

$$0.8 = \frac{1200}{1 \times 12 \times 5}$$

$$0.8 = \frac{16}{I}$$

$$I = \frac{16}{0.8}$$

$$I = 20\ A$$

Answer is E

ENGAA S1 2017 - Question 13

13 What is the value of x that makes the following expression correct?

$$2^{3+2x}4^{x}8^{-x} = 4\sqrt{2}$$

A -2.25

B -1.75

C -1.5

D -0.5

E -0.25

ENGAA 2017 - Question 13 - Worked Solution

$$2^{3+2x}4^{x}8^{-x} = 4\sqrt{2}$$

$$2^{3+2x}\left(2^{2}\right)^{x}\left(2^{3}\right)^{-x} = 2^{2}2^{1/2}$$

$$2^{3+2x}2^{2x}2^{-3x} = 2^{2}2^{1/2}$$

$$2^{3+2x+2x-3x} = 2^{2+1/2}$$

$$2^{x+3} = 2^{2+1/2}$$

$$x + 3 = 2 + 0.5$$

$$x = -0.5$$

Answer is D

ENGAA S1 2017 - Question 14

14 The nuclide $^{P}_{Q}X$ decays to the stable nuclide Y. During this process four particles are emitted: an α-particle and three β^{-} particles.

Which of the following is **not** a nuclide that could be formed at any stage during this process?

nuclide	*atomic mass*	*atomic number*
A	P	Q − 1
B	P	Q + 1
C	P	Q + 2
D	P	Q + 3
E	P − 4	Q − 2
F	P − 4	Q − 1
G	P − 4	Q
H	P − 4	Q + 1

ENGAA 2017 - Question 14 - Worked Solution

− α particles:

$$\alpha^{4}_{2}$$

β^{-} particle
An electron

$$\beta^{0}_{-1}$$

- Atomic and mass number need to always balanced
- A is not possible as:
  Atomic mass is P , so would describe a nuclei atom pre α emission
- But $\alpha - 1$ is not possible as a β^{-} decay would causes atomic number of nuclide to increase

Answer is A

ENGAA S1 2017 - Question 15

15 There are 100 students in Year 10.

Each student studies exactly one of French, German, and Spanish.

X girls study French and there are $3X$ girls in total.

$2Y$ boys study German.

There are 35 students studying Spanish of which Y are boys.

Which of the following is an expression for the total number of students studying German?

A $X + 2Y$

B $X + Y + 35$

C $X + 3Y - 35$

D $2X + 2Y$

E $2X + Y - 35$

F $2X + 3Y - 35$

G $2X + Y + 35$

ENGAA 2017 - Question 15 - Worked Solution

Draw table and fill in using information from question

	French	German	Spanish	Total
G	X			3Y
B		2Y	Y	
Total		?	35	100

Fill in table using "total" column/row

	French	German	Spanish	Total
G	X			3X
B	100-3X-3Y	2Y	Y	100-3X

Total	100-2X-3Y	2X+3Y-35	35	100

So number of German students is :

$$2X + 3Y - 35$$

Answer is F.

ENGAA S1 2017 - Question 16

16 The radius of an iron-56 atom is 3.0×10^4 times greater than the radius of an iron-56 nucleus.

What is the value of $\frac{\text{density of an iron atom}}{\text{density of an iron nucleus}}$?

A $(3.0 \times 10^4)^{-3}$

B $(3.0 \times 10^4)^{-2}$

C $(3.0 \times 10^4)^{-1}$

D $(3.0 \times 10^4)^{1}$

E $(3.0 \times 10^4)^{2}$

F $(3.0 \times 10^4)^{3}$

ENGAA 2017 - Question 16 - Worked Solution

$$R_a = 3.0 \times 10^4 R_n$$

$$\frac{\rho_a}{\rho_n} = \frac{M_a}{M_n} \times \frac{V_n}{V_a}$$

$$= \frac{V_n}{V_A}$$

$$= \frac{R_n^3}{R_A^3}$$

$$= \left(3.0 \times 10^4\right)^{-3}$$

Answer is A

ENGAA S1 2017 - Question 17

17 An exterior angle of a regular polygon with n sides is 4° larger than an exterior angle of a regular polygon with $(n+3)$ sides.

What is the value of n?

A 10

B 12

C 15

D 18

E 21

F 24

G 27

ENGAA 2017 - Question 17 - Worked Solution

External angles all add up to 360^0

$one\ external\ angle = \frac{360}{n}$

$$\frac{360}{n} = 4 + \frac{360}{(n+3)}$$

$$360(n + 3) = 4n(n + 3) + 360n$$

$$1080 = 4n^2 + 12n$$

$$n^2 + 3 - 270 = 0$$

$$(n + 18)(n - 15) = 0$$

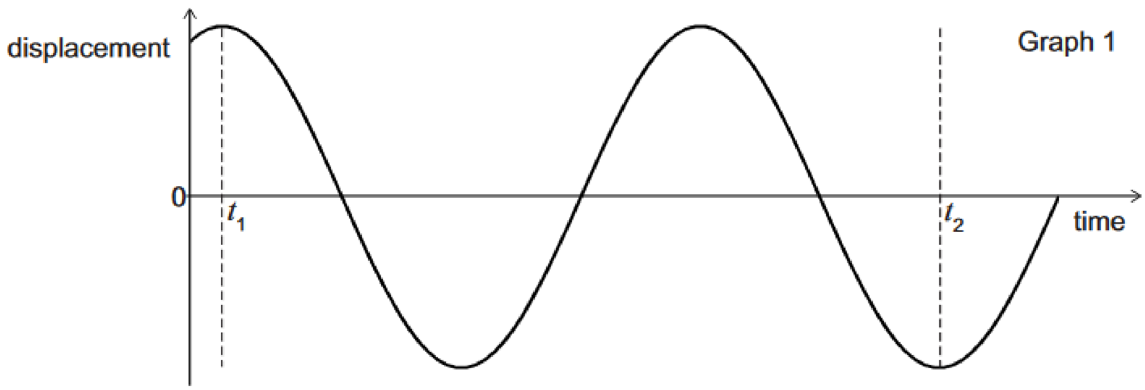
$$n = -18 \quad or \quad 15 \qquad (n > 0)$$

$$n = 15$$

Answer is C

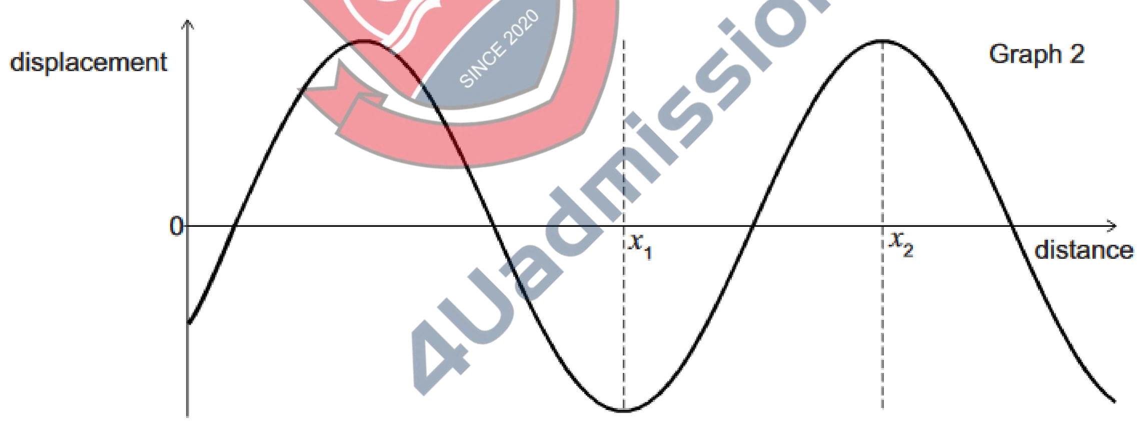
ENGAA S1 2017 - Question 18

18 Graph 1 shows how the displacement of one of the particles of a medium varies with time in seconds as a wave travels through the medium.



Graph 2 shows how the displacement varies with distance in metres at one time for the same wave.

Graph 2 shows how the displacement varies with distance in metres at one time for the same wave.



Which expression gives the speed in m s^{-1} of the wave?

A $\frac{4(x_2 - x_1)}{3(t_2 - t_1)}$

B $\frac{3(x_2 - x_1)}{2(t_2 - t_1)}$

C $\frac{2(x_2 - x_1)}{t_2 - t_1}$

D $\frac{8(x_2 - x_1)}{3(t_2 - t_1)}$

E $\frac{3(x_2 - x_1)}{t_2 - t_1}$

F $\frac{6(x_2 - x_1)}{t_2 - t_1}$

ENGAA 2017 - Question 18 - Worked Solution

$$\frac{3T}{2} = (t_2 - t_1)$$

$$T = \frac{2(t_2 - t_1)}{3}$$

$$\frac{\lambda}{2} = x_2 - x_1$$

$$\lambda = 2(x_2 - x_1)$$

$$v = f\lambda$$

$$v = \frac{\lambda}{T}$$

$$V = \frac{3(x_2 - x_1)}{(t_2 - t_1)}$$

Answer is E

ENGAA S1 2017 - Question 19

19 The bearing of a ship *R* from a lighthouse *L* is 220°

A canoe *C* is due North of *R*.

C is the same distance from the ship and the lighthouse.

What is the bearing of *L* from *C*?

A 070°

B 080°

C 090°

D 100°

E 140°

ENGAA 2017 - Question 19 - Worked Solution

- Alternative angles rule:

$CRL = 40°$

- Isosceles

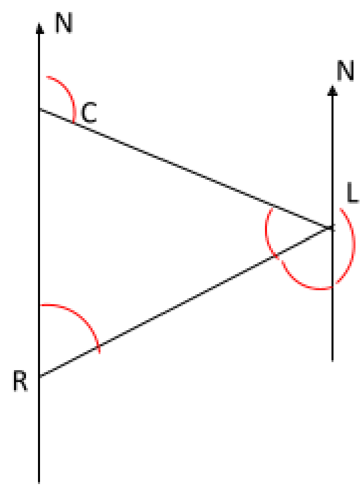
$CLR = 40°$

- In triangle all angles = 180°

$\Rightarrow (180 - \theta) + 40 + 40 = 180$

$\Rightarrow \theta = 80°$

Answer is B



ENGAA S1 2017 - Question 20

20 A kettle is designed to work from a car's power socket. The kettle has a power rating of 150 W when a constant voltage of 12.0 V d.c. is applied across its element.

How much charge passes through the element of this kettle when the voltage of 12.0 V is applied across it for 20 minutes?

A 96 C

B 250 C

C 15 000 C

D 36 000 C

E 900 000 C

F 2 160 000 C

ENGAA 2017 - Question 20 - Worked Solution

$$P = IV$$

$$\Rightarrow I = \frac{P}{V} = \frac{150}{12} = 12.5A$$

$$Q = It$$

$$= 12.5 \times 20 \times 60$$

$$= 15000\ C$$

Answer is C

ENGAA S1 2017 - Question 21

21 The hands of a 12-hour analogue clock move continuously. When the time on the clock is 4:00, the angle between the minute hand and the hour hand is 120°.

What is the angle between the two hands at 4:40 ?

A 80°

B 100°

C 110°

D 120°

E 140°

ENGAA 2017 - Question 21 - Worked Solution

- In 1hr, hour hand moves

$\frac{360°}{12} = 30°$

- So in 40 min, moves

$\frac{40}{60} \times 30 = 20°$

- Minute hand moves

$\frac{40}{60} \times 360 = 240°$

$\Rightarrow \theta = 240 - (120 + 20) = 100°$

Answer is B

ENGAA S1 2017 - Question 22

22 A freight train travelling on a straight horizontal track at $2.0\,m\,s^{-1}$ collides with a passenger train travelling at $5.0\,m\,s^{-1}$ in the opposite direction. Both trains immediately come to a complete stop on the track.

The freight train has three locomotives of 130 tonnes each and seven container wagons of 30 tonnes each. The passenger train has two locomotives of 70 tonnes each and a number of passenger carriages of 10 tonnes each.

How many passenger carriages does the passenger train have?

A 7

B 9

C 10

D 24

E 46

ENGAA 2017 - Question 22 - Worked Solution

$$M_t = 3\times130 + 7\times30 = 600$$

$$M_p = 2\times70 + 10N = 140 + 10N$$

Conserve momentum:

$$2M_t - 5M_p = 0$$

$$\Rightarrow M_p = \frac{2M_t}{5}$$

$$\Rightarrow 140 + 10N = \frac{2}{5}\times600 = 240$$

$$\Rightarrow 10N = 100$$

$$\Rightarrow N = 10$$

Answer is C

ENGAA S1 2017 - Question 23

23 A pet shop has 4 female rabbits and x male rabbits for sale.

A customer buys 2 of the rabbits, chosen at random, and each rabbit is equally likely to be chosen.

The probability that both the chosen rabbits are male is $\frac{1}{3}$.

What is the value of x?

A 2

B 4

C 6

D 8

E 9

F 11

G 12

ENGAA S1 2017 - Question 23 - Worked Solution

$$p(male) = \frac{x}{4+x} \times \frac{x-1}{4+x-1} = \frac{1}{3}$$

$$\Rightarrow 3x(x-1) = (4+x)(3+x)$$

$$\Rightarrow 3x^2 - 3x = 12 + 7x + x^2$$

$$\Rightarrow 2x^2 - 10x - 12 = 0$$

$$\Rightarrow x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$\Rightarrow x = 6 \quad or \quad -1 \qquad (x \geq 0)$$

$$x = 6$$

Answer is C

ENGAA S1 2017 - Question 24

24 Consider the following three statements about a parachutist of mass 72 kg falling vertically at a constant velocity of 5.0 m s^{-1} after the parachute has opened:

1 The parachutist has a constant kinetic energy of 1800 J.

2 The parachutist is losing gravitational potential energy at a rate of 3600 J s^{-1}.

3 Air resistance and the force of gravity acting on the parachutist are a Newton's third law pair of forces.

Which of the statements is/are correct?

(gravitational field strength = 10 N kg^{-1})

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

ENGAA S1 2017 - Question 24 - Worked Solution

1)

$$Ex = \frac{1}{2}mv^2$$

$$Ex = \frac{1}{2}\times 72\times 5^2 = 900J$$

So False

2)

$$\frac{dE}{dt} = \frac{d}{dt}(mgh)$$

$$= mgh\frac{dh}{dt}$$

$$= mgv$$

$$= 72 \times 10 \times 5$$

$$3600\ JS^{-1}$$

So True

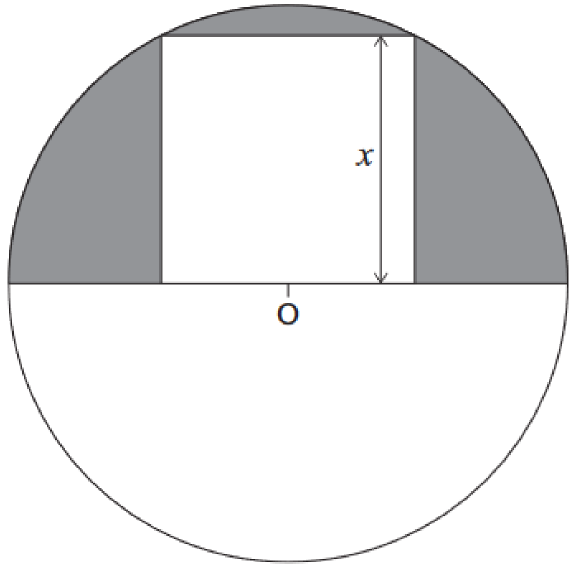
3)

The two forces act on the same body (parachutist) , so false.

Answer is C

ENGAA S1 2017 - Question 25

25



The diagram shows a square with side of length x cm. A circle is drawn with centre O which lies at the mid-point of one of the sides of the square. This side forms part of a diameter of the circle. The circle passes through two corners of the square as shown.

What is the area, in cm^2, of the shaded part of the semi-circle?

A $(\pi-1)x^2$

B $\left(\frac{\pi-2}{2}\right)x^2$

C $\left(\frac{3\pi-2}{2}\right)x^2$

D $\left(\frac{3\pi-4}{4}\right)x^2$

E $\left(\frac{5\pi-4}{4}\right)x^2$

F $\left(\frac{5\pi-8}{8}\right)x^2$

ENGAA S1 2017 - Question 25 - Worked Solution

Using Pythagoras

$$r = \sqrt{x^2 + \left(\frac{x}{2}\right)^2}$$

$$r = \sqrt{\frac{5}{4}x^2}$$

$$r = \frac{\sqrt{5}x}{2}$$

$$A = \frac{1}{2}\times circle - square$$

$$A = \frac{1}{2}\pi r^2 - x^2$$

$$A = \frac{1}{2} \times \frac{5}{4}x^2\pi - x^2$$

$$A = \left(\frac{5}{8}\pi - 1\right)x^2$$

$$A = \left(\frac{5\pi - 8}{8}\right)x^2$$

Answer is F

ENGAA S1 2017 - Question 26

26 Two radioactive sources X and Y have half-lives of 3.0 hours and 2.0 hours respectively. The product of the decay of both of the sources is a stable isotope of the element Z.

Six hours ago a mixture contained the same number of atoms of both X and Y, and no other atoms.

What fraction of the mixture is now made up of atoms of Z?

A $\frac{10}{16}$

B $\frac{11}{16}$

C $\frac{12}{16}$

D $\frac{13}{16}$

E $\frac{14}{16}$

F $\frac{15}{16}$

ENGAA S1 2017 - Question 26 - Worked Solution

- Initially $N_x = N$, $N_Y = N$
- In 6 hours:

Two half lives of X:

$$N_X = \frac{N}{4}$$

Three half lives of Y:

$$N_Y = \frac{N}{8}$$

$$\Rightarrow N_Z = \left(N - N_X\right) + \left(N - N_Y\right)$$

$$= 2N - \frac{N}{4} - \frac{N}{8}$$

$$= \frac{13}{8}N$$

$$\Rightarrow \frac{N_Z}{N_Z + N_X + N_Y} = \frac{\frac{13}{8}}{\frac{13}{8} + \frac{1}{4} + \frac{1}{8}}$$

$$= \frac{13}{8 \times 2}$$

$$= \frac{13}{16}$$

Answer is D

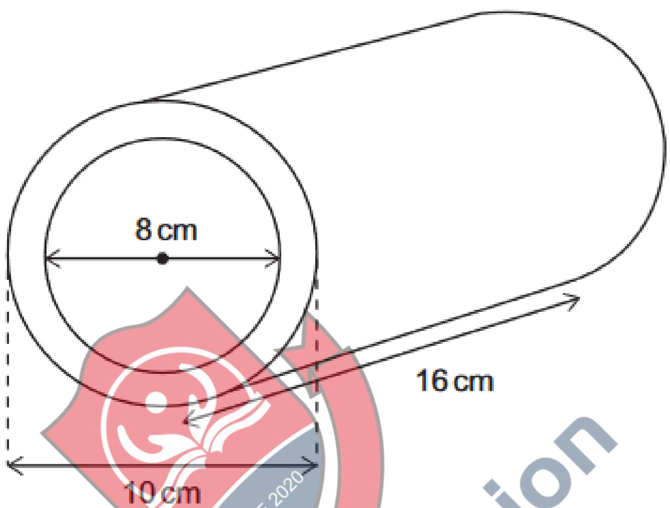
ENGAA S1 2017 - Question 27

27 A cylindrical hollow metal pipe is 16 cm long.

It has an external diameter of 10 cm and an internal diameter of 8 cm.

The density of the metal from which the pipe is made is 8 grams per cm^3.

[diagram not to scale]



What is the mass of the pipe in grams?

A 8π

B 16π

C 18π

D 72π

E 128π

F 512π

G 1152π

H 4608π

ENGAA S1 2017 - Question 27 - Worked Solution

mass = density x volume

volume = volume of outer cylinder – volume of inner cylinder

$= (5^2\pi \times 16) - (4^2\pi \times 16)$

$= 144\pi\ cm^3$

$mass\ =\ 8 \times 144\pi$

$mass\ =\ 1152\ \pi g$

Answer is G

ENGAA S1 2017 - Question 28

28 Car X passes car Y on a motorway.

Car X is travelling at 1.5 times the speed of car Y.

The mass of car X is $\frac{4}{5}$ of the mass of car Y.

How do the kinetic energies of the two cars compare?

A kinetic energy of car X = 0.90 × kinetic energy of car Y

B kinetic energy of car X = 0.96 × kinetic energy of car Y

C kinetic energy of car X = 1.20 × kinetic energy of car Y

D kinetic energy of car X = 1.44 × kinetic energy of car Y

E kinetic energy of car X = 1.80 × kinetic energy of car Y

ENGAA S1 2017 - Question 28 - Worked Solution

$$V_X = 1.5V_Y$$

$$M_X = \frac{4}{5}M_Y$$

$$E_Y = \frac{1}{2}M_Y V_Y$$

$$E_X = \frac{1}{2}M_X {V_X}^2$$

$$= \frac{1}{2} \times \left(\frac{4}{5}M_Y\right) \times \left(\frac{3}{2}V_X\right)^2$$

$$= 1.8 \times \frac{1}{2}M_X M_Y$$

$$= 1.8E_Y$$

Answer is E

ENGAA S1 2017 - Question 29

29 Which one of the following is a simplification of

$$1-\left(\frac{3+\sqrt{3}}{6-2\sqrt{3}}\right)^2$$

A $-\frac{3}{4}$

B $\frac{3}{4}$

C $-\frac{3}{4}-\frac{\sqrt{3}}{7}$

D $\frac{3}{4}-\frac{\sqrt{3}}{7}$

E $-\frac{3}{4}-\sqrt{3}$

F $\frac{3}{4}-\sqrt{3}$

G $-\frac{\sqrt{3}}{2}$

H $\frac{\sqrt{3}}{2}$

ENGAA S1 2017 - Question 29 - Worked Solution

$$1-\left(\frac{3+\sqrt{3}}{6-2\sqrt{3}}\right)^2$$

$$=1-\frac{(3+\sqrt{3})(3+\sqrt{3})}{(6-2\sqrt{3})(6-2\sqrt{3})}$$

$$=1-\frac{9+3+3\sqrt{3}+3\sqrt{3}}{36+(4\times 3)-(6\times 2\sqrt{3})-(6\times 2\sqrt{3})}$$

$$=1-\frac{12+6\sqrt{3}}{48-24\sqrt{3}}$$

$$=1-\frac{(2+\sqrt{3})}{4(2-\sqrt{3})}$$

$$= \frac{6-5\sqrt{3}}{4(2-\sqrt{3})}$$

$$= \frac{(6-5\sqrt{3})(2+\sqrt{3})}{4(2-\sqrt{3})(2+\sqrt{3})}$$

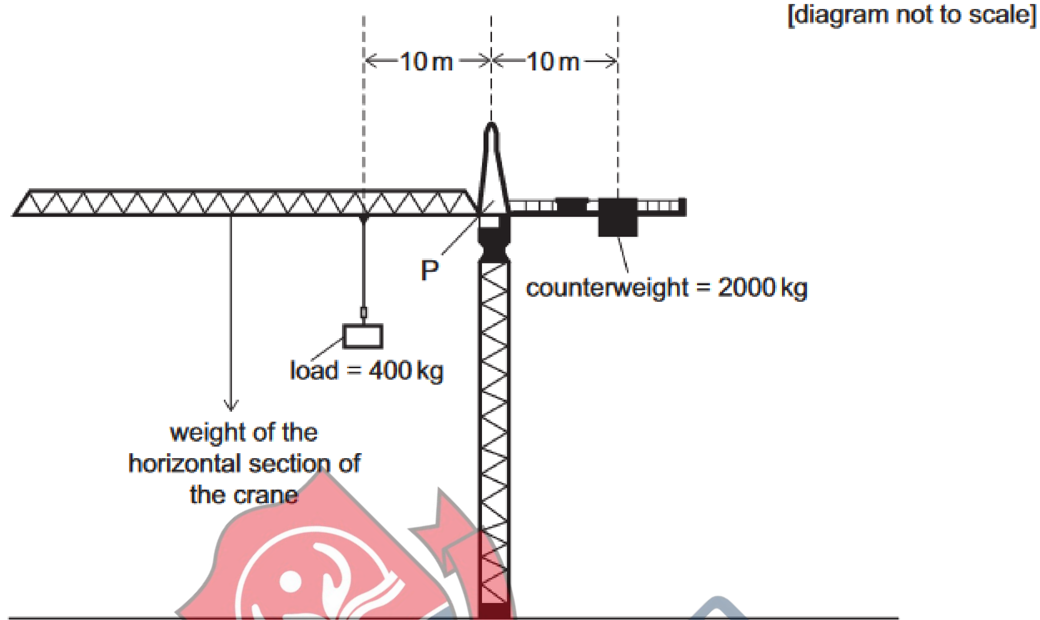
$$= \frac{-3-4\sqrt{3}}{4\times1}$$

$$= \frac{3}{4} - \sqrt{3}$$

Answer is E

ENGAA S1 2017 - Question 30

30 The diagram shows a crane being used on a building site. The crane is perfectly balanced about P.



The load is now moved to the left by 5.0 m.

To keep the crane perfectly balanced about P, how far does the counterweight have to move, and in which direction?

(gravitational field strength = $10\,N\,kg^{-1}$)

A 1.0 m to the left

B 1.0 m to the right

C 3.0 m to the left

D 3.0 m to the right

E 4.0 m to the left

F 4.0 m to the right

ENGAA S1 2017 - Question 30 - Worked Solution

- External torque

$T = 400g \times 5$

$= 400 \times 10 \times 5$

$= 20000\ Nm$

- So need an clockwise torque to balance so need to move counterweight by Δx to the right

$T = 2000\ ug \times \Delta x$

$20000 = 2000 \times 10 \times \Delta x$

$\Delta x = 1m$

Answer is B

ENGAA S1 2017 - Question 31

31 k is the smallest positive value of x which is a solution to **both** the equations $2\sin x + 1 = 0$ **and** $2\cos 2x = 1$

How many values of x in the range $0 \le x \le k$ are solutions to at least one of these equations?

A 0

B 2

C 3

D 4

E 8

ENGAA S1 2017 - Question 31 - Worked Solution

$$2 \sin sin\, x + 1 = 0$$

$$\sin sin\, x = -\frac{1}{2}$$

$$x = 210°, 330°, ...$$

$$2 \cos cos\, (2x) = 1$$

$$\cos cos\, (2x) = \frac{1}{2}$$

$$2x = 60°, 300°, 420°, 660°$$

$$x = 30°, 150°, 210°, 330°, ...$$

So for a solution to exist:

$$k = 210°$$

Number of solutions : 3 (210 , 30 , 50 each solve at least one)

Answer is C

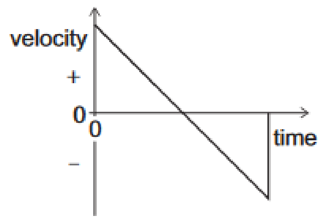
ENGAA S1 2017 - Question 32

32 A ball is thrown vertically upwards in still air and is then caught at the same height when it comes back down.

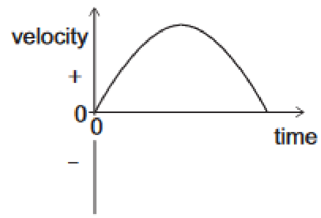
Which velocity–time graph shows this complete motion?

(Take upwards as positive, and ignore air resistance.)

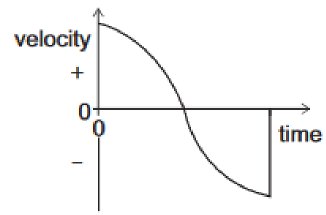
A



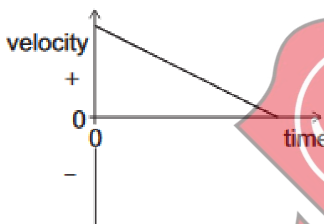
B



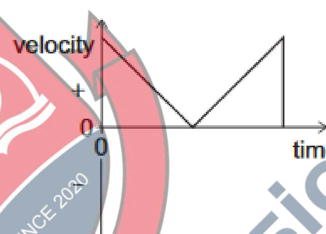
C



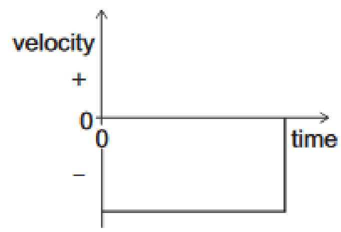
D



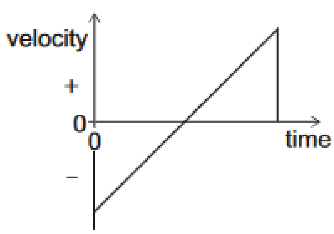
E



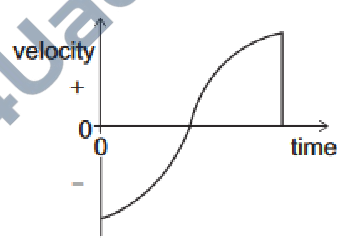
F



G



H



ENGAA S1 2017 - Question 32 - Worked Solution

- a = g, and constant ⇒ gradient is constant
- when ball falls back down, velocity is negative
- ball is thrown up, so initially velocity is positive

Answer is A

ENGAA S1 2017 - Question 33

33 Which of the following is a solution to the equation $3^{(2x+1)} - 6(3^x) = 0$?

A $\log_2 3$

B $\log_3 2$

C 2

D $\log_{10} 2$

E $\frac{2}{3}$

ENGAA S1 2017 - Question 33 - Worked Solution

$$3^{(2x+1)} - 2\times3\times3^{x} = 0$$

$$3^{(2x+1)} = 2\times3^{(x+1)}$$

$$\frac{3^{(2x+1)}}{3^{(x+1)}} = 2$$

$$3^{x} = 2$$

$$x = 2$$

Answer is B

ENGAA S1 2017 - Question 34

34 An aircraft is climbing at constant speed in a straight line at an angle of 10° to the horizontal.

Which statement about the resultant force on the aircraft is correct?

A It is parallel to its motion.

B It is perpendicular to its motion.

C It is zero.

D It is equal to its weight.

E It is equal to the drag acting on the aircraft.

ENGAA S1 2017 - Question 34 - Worked Solution

- Speed is constant
- So acceleration is 0
- $F = ma$, so resultant force is 0

Answer is C

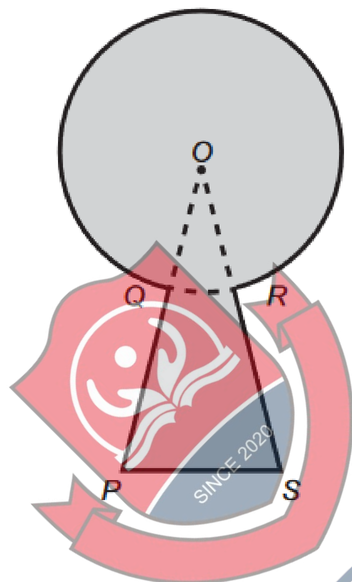
ENGAA S1 2017 - Question 35

35 The diagram shows the outline of a keyhole consisting of three straight sides and an arc from a circle.

The sides *PQ* and *RS* are both 18 mm in length and when extended meet at the centre of the circle *O* forming an angle of $\frac{\pi}{6}$ radians.

The longer arc from *Q* to *R* has length 22π mm.

[diagram not to scale]



A $121\pi + \frac{841}{4}$

B $121\pi + \frac{841\sqrt{3}}{4}$

C $132\pi + 225$

D $132\pi + 225\sqrt{3}$

E $144\pi + 225$

F $144\pi + 225\sqrt{3}$

ENGAA S1 2017 - Question 35 - Worked Solution

$$22\pi = r\theta$$

$$= r\left(2\pi - \frac{\pi}{6}\right)$$

$$= \frac{11}{6}\pi r$$

$$r = 12$$

A = circle + triangle – QOR sector

$$= \pi r^2 + \frac{1}{2}PQ\times QS\times \sin sin\left(\frac{\pi}{6}\right) - \frac{1}{2}r^2\left(\frac{\pi}{6}\right)$$

$$= \pi\times 12^2 + \frac{1}{2}\times(18 + 12)^2\times\frac{1}{2} - \frac{1}{2}\times(12)^2\times\left(\frac{\pi}{6}\right)$$

$$= 144\pi - 12\pi + 225$$

$$= 132\pi + 225$$

Answer is C

ENGAA S1 2017 - Question 36

36 A horizontal, uniform bar of mass 60 kg is 4.0 m long and is pivoted at one end. The bar is held in equilibrium by a force *F* at the other end of the bar, acting at an angle of 60° to the horizontal.

[diagram not to scale]

Which expression gives the magnitude of *F* in newtons?

(gravitational field strength = $10\,\text{N kg}^{-1}$)

A $\dfrac{30}{\sin 60°}$

B $\dfrac{30}{\cos 60°}$

C $\dfrac{60}{\sin 60°}$

D $\dfrac{60}{\cos 60°}$

E $\dfrac{300}{\sin 60°}$

F $\dfrac{300}{\cos 60°}$

G $\dfrac{600}{\sin 60°}$

H $\dfrac{600}{\cos 60°}$

ENGAA S1 2017 - Question 36 - Worked Solution

- Moment = Perpendicular x distance
- Resolve moments about pivot

$2 \times 60\ g\ =\ F \sin sin\ (60) \times 4$

$$F = \frac{2\times60\times10}{\sin sin\,(60)\times4}$$

$$= \frac{300}{\sin sin\,(60)}$$

Answer is E

ENGAA S1 2017 - Question 37

37 It is given that $y = 8^p$ and $z = \left(\frac{1}{2}\right)^{2q}$ where p and q are real numbers.

Which of the following expressions is a simplification of $\log_2\left(\frac{y^3}{z^2}\right)$?

A $6p - 4q$

B $6p + 4q$

C $6p - 8q$

D $6p + 8q$

E $9p - 4q$

F $9p + 4q$

G $9p - 8q$

H $9p + 8q$

ENGAA S1 2017 - Question 37 - Worked Solution

$$\left(\frac{y^3}{z^2}\right) = \left(y^3\right) - \left(z^2\right)$$

$$= \left(8^{3p}\right) - \left(\left[\frac{1}{2}\right]^{4q}\right)$$

$$= 3p(8) - 4q\left(\frac{1}{2}\right)$$

$$= 3p\left(2^3\right) - 4q \log log\ (2\ -\ 1)$$

$$= 3p\times 3\ -\ 4q\times\ -\ 1$$

$$= 9p\ +\ 4q$$

Answer is F

ENGAA S1 2017 - Question 38

38 A ball starts at a speed of $40.0\,m\,s^{-1}$. The ball is subject to a constant deceleration of $14.4\,m\,s^{-2}$ as it travels a distance of $20.0\,m$ in a straight line.

What is the final speed of the ball?

A $16.0\,m\,s^{-1}$

B $20.0\,m\,s^{-1}$

C $25.6\,m\,s^{-1}$

D $32.0\,m\,s^{-1}$

E $36.2\,m\,s^{-1}$

F $39.3\,m\,s^{-1}$

ENGAA S1 2017 - Question 38 - Worked Solution

s = 20

u = 40

v = v

A = - 14.4

$$v^2 = u^2 + 2as$$

$$v = \sqrt{u^2 + 2as}$$

$$= \sqrt{(40)^2 - 2\times14.4\times20}$$

$$= \sqrt{1024}$$

$$= 32\ ms^{-1}$$

Answer is D

ENGAA S1 2017 - Question 39

39 The graph of the function $y = x^3 + px^2 + qx + 6$, where p and q are real constants, has a local maximum when $x = 2$ and a local minimum when $x = 4$.

What are the values of p and q?

A $p = -3$ and $q = -8$

B $p = -3$ and $q = 8$

C $p = 3$ and $q = -8$

D $p = -9$ and $q = 24$

E $p = 9$ and $q = 24$

F $p = 9$ and $q = -24$

ENGAA S1 2017 - Question 39 - Worked Solution

$$\frac{dy}{dx} = 3x^2 + 2px + q$$

$$At\ maximum:\ \frac{dy}{dx} = 0$$

$$x = 2:$$

$$3(2)^2 + 2p \times 2 + q = 0$$

$$\Rightarrow 4p + q + 12 = 0 \quad ------ \ ①$$

$$x = 4:$$

$$3(4)^2 + 2p \times 4 + q = 0$$

$$\Rightarrow 48 + 8p + q = 0 \quad ---- \ ②$$

② - ①

$$4p + 36 = 0$$

$$\Rightarrow p = -\frac{36}{4} = -9$$

Sub p = -9 into ①

$$q = -12 - 4p$$

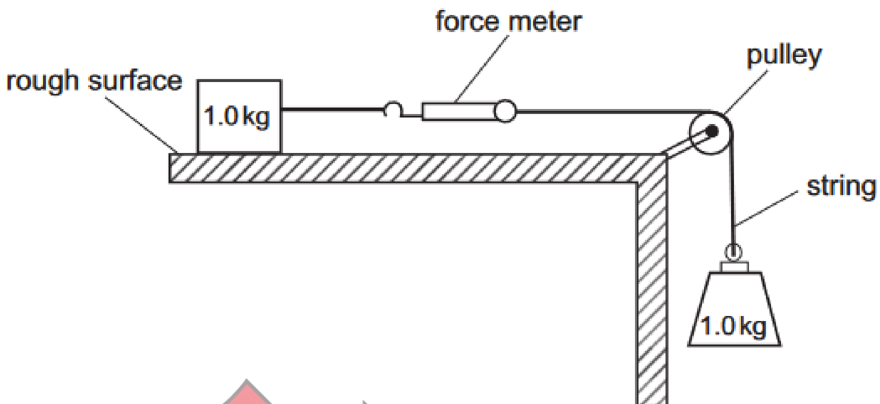
$$q = -12 + (4 \times 9)$$

$$q = 24$$

Answer is D

ENGAA S1 2017 - Question 40

40 A block of mass 1.0 kg is at rest on a rough horizontal surface. The block is attached by a light inextensible string to a force meter. The other end of the force meter is attached by another light inextensible string via a frictionless pulley to a load of mass 1.0 kg. The block remains stationary.



What is the reading on the force meter?

(gravitational field strength = 10 N kg^{-1})

A 0.0 N

B 0.5 N

C 1.0 N

D 2.0 N

E 5.0 N

F 10 N

G 20 N

ENGAA S1 2017 - Question 40 - Worked Solution

- String is inextensible and pulley is smooth
  ⇒tension is uniform

- Resolve forces on load:
  T = mg
  ⇒ T = 1 x 10 = 10N
- So reading on force meter is 10N

Answer is F

ENGAA S1 2017 - Question 41

41 In triangle PQR

$$PQ = 4x \text{ cm}$$

$$QR = (8 - 3x) \text{ cm}$$

$$\angle PQR = 60°$$

What is the maximum value of the area, in cm^2, of triangle PQR?

A $\frac{8\sqrt{3}}{3}$

B $\frac{16}{3}$

C $\frac{69\sqrt{3}}{16}$

D $\frac{16\sqrt{3}}{3}$

E $\frac{32}{3}$

F $\frac{32\sqrt{3}}{3}$

ENGAA S1 2017 - Question 41 - Worked Solution

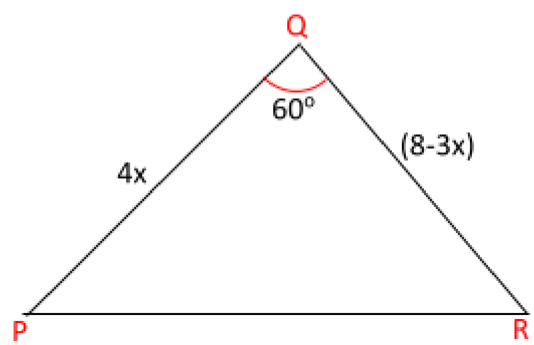
$$A = \frac{1}{2} PQ \times PR \sin sin\,(60)$$

$$= \frac{1}{2} \times 4x(8 - 3x) \times \frac{\sqrt{3}}{2}$$

$$= \sqrt{3}x(8 - 3x)$$

$$= -3\sqrt{3}\left[x^2 - \frac{8}{3}x\right]$$

$$= -3\sqrt{3}\left[\left(x - \frac{4}{3}\right)^2 - \frac{16}{9}\right]$$



$$= -3\sqrt{3}\left[\left(x-\frac{4}{3}\right)^2+\frac{16\sqrt{3}}{3}\right]$$

Answer is D

ENGAA S1 2017 - Question 42

42 An apple of mass 100 g, growing on a tree, falls vertically from a height of 4.0 m above the ground. It hits the ground with a speed of 8.0 m s^{-1}.

How much work does the apple do against resistive forces during its descent, before it hits the ground?

(gravitational field strength = 10 N kg^{-1})

A 0.80 J

B 3.6 J

C 4.0 J

D 7.2 J

E 8.0 J

ENGAA S1 2017 - Question 42 - Worked Solution

$$\Delta Ep = mgh$$

$$= 0.1\times10\times4$$

$$= 4\ J$$

$$\Delta Ex = \frac{1}{2}mv^2$$

$$= \frac{1}{2}\times0.1\times8^2$$

$$= 3.2\ J$$

Then work done against resistance is :

$$W = \Delta Ex - \Delta Ep$$

$$= 4 - 3.2$$

$$= 0.8\ J$$

Answer is A

ENGAA S1 2017 - Question 43

43 Given that $y=(2+3x)^6$, what is the coefficient of x^3 in $\frac{dy}{dx}$?

A 240

B 4320

C 4860

D 12960

E 19440

ENGAA S1 2017 - Question 43 - Worked Solution

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}, \text{ so look at } x^4 \text{ term in } y$$

$$x^4 \text{ term in } y = \binom{6}{2}.\,2^2.\,(3x)^4$$

$$= 4860x^4$$

$$Differentiate: 4\times 4860x^3$$

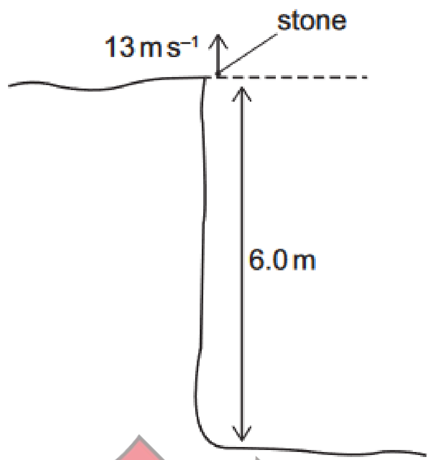
$$= 19440x^3$$

Answer is E

ENGAA S1 2017 - Question 44

44 A stone is fired vertically upwards at a speed of $13\,m\,s^{-1}$ on a still day from the top of a 6.0 m high cliff. It then falls down and lands at the bottom of the cliff.

[diagram not to scale]



From when the stone passes the top of the cliff on the way down, how long does it take to reach the ground at the bottom of the cliff?

(air resistance can be ignored; gravitational field strength = $10\,N\,kg^{-1}$)

A 0.40 s

B $\frac{6.0}{6.5}$ s

C 0.60 s

D $\sqrt{1.2}$ s

E 1.3 s

F 2.0 s

G 2.5 s

H 3.0 s

ENGAA S1 2017 - Question 44 - Worked Solution

Projectile motion is symmetric by conservation at energy, so will pass cliff with u = $-13ms^{-1}$

S = -6
u = -13
A = -10
T = t

$$S = ut + \frac{1}{2}at^2$$

$$-6 = -13t - \frac{10}{2}t^2$$

$$5t^2 + 13t + 6 = 0$$

$$(5t - 2)(t + 3) = 0$$

$$t = 0.4 \ or \ -3 \qquad (t > 0)$$

$$t = 0.4s$$

Answer is A

ENGAA S1 2017 - Question 45

45 A geometric progression has first term equal to 1 and common ratio $\frac{1}{2}\sin 2x$

The sum to infinity of the series is $\frac{4}{3}$

Find the possible values of x in the range $\pi \le x \le 2\pi$

A $\frac{13}{12}\pi, \frac{17}{12}\pi$

B $\frac{7}{6}\pi, \frac{4}{3}\pi$

C $\frac{7}{6}\pi, \frac{11}{6}\pi$

D $\frac{5}{4}\pi, \frac{7}{4}\pi$

E there are no values of x in this range

ENGAA S1 2017 - Question 45 - Worked Solution

$$U_n = ar^{n-1}$$

$$a = 1, r = \frac{1}{2}\sin sin(2x)$$

$$S_\infty = \frac{a}{1-r} = \frac{4}{3}$$

$$3 = 4(1 - r)$$

$$\frac{1}{4} = r$$

$$\frac{1}{2}\sin sin(2x) = \frac{1}{4}$$

$$\sin sin(2x) = \frac{1}{2}$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13}{6}\pi, \frac{17}{6}\pi$$

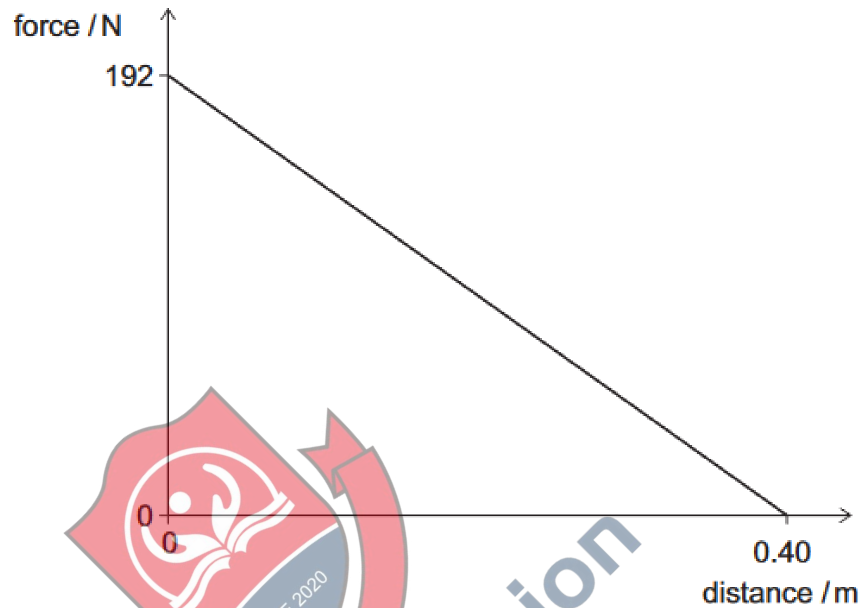
$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13}{12}\pi, \frac{17}{12}\pi$$

Answer is A

ENGAA S1 2017 - Question 46

46 An archer fires an arrow of mass 0.024 kg vertically upwards from a bow.

The graph shows how the force of the bowstring on the arrow varies with distance as the arrow moves upwards.



The work done by the force of the bowstring is given by the area under the force-distance graph.

When the arrow leaves the bow, what is the kinetic energy of the arrow, and what is the maximum height that it gains from this point?

(Air resistance can be ignored. The effect of gravity as the arrow is fired is negligible compared to the force of the bowstring. The gravitational field strength = $10\,N\,kg^{-1}$.)

	kinetic energy / J	*height* / m
A	38.4	16
B	38.4	160
C	38.4	1600
D	38.4	16 000
E	76.8	32
F	76.8	320
G	76.8	3200
H	76.8	32 000

ENGAA S1 2017 - Question 46 - Worked Solution

W = Area

$$= \frac{1}{2} \times 0.4 \times 192$$

$$= 38.4\,J$$

$$= Initial\ kinetic\ energy$$

At max height, 100w is at rest, so

$$\Delta Ep = \Delta Ex$$

$$mgh = \frac{1}{2}mv^2$$

$$h = \frac{38.4}{0.024 \times 10}$$

$$= 160m$$

Answer is B

ENGAA S1 2017 - Question 47

47 The sequence of numbers $u_1, u_2, u_3, \ldots, u_n, \ldots$ is given by

$$u_1 = 2$$
$$u_{n+1} = pu_n + 3$$

where p is an integer.

The fourth term, u_4, is equal to -7

What is the value of $u_1 + u_2 + u_3 + u_4$?

A -10

B -2

C -1

D 8

E 26

ENGAA S1 2017 - Question 47 - Worked Solution

$$u_1 = 2$$

$$u_2 = 2p + 3$$

$$u_3 = p(2p + 3) + 3$$

$$= 2p^2 + 3p + 3$$

$$u_4 = p\left(2p^2 + 3p + 3\right) + 3$$

$$= 2p^3 + 3p^2 + 3p + 3$$

$$= -7$$

$$2p^3 + 3p^2 + 3p + 10 = 0$$

Find real solution by trying p =1 , p = 2 …

Clearly p = -2 is a solution:

$$u_2 = -1$$

$$u_3 = 5$$

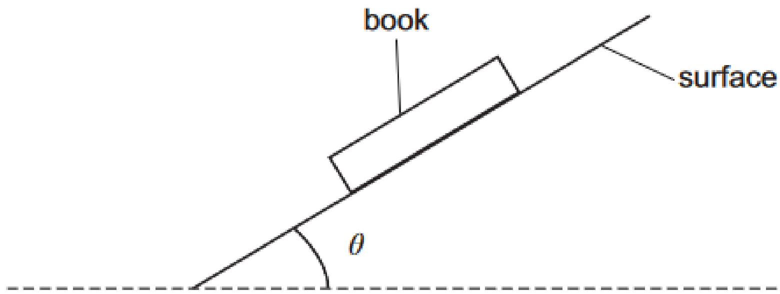
$$u_1 + u_2 + u_3 + u_4 = 2 - 1 + 5 - 7$$

$$= -1$$

Answer is C

ENGAA S1 2017 - Question 48

48 A book of mass m rests on a rough horizontal surface. The surface is now tilted as shown:



When the angle of tilt θ is 20°, the book slides down the slope at a constant speed.

What is the acceleration of the book down the slope when the angle of tilt is 25°?

(gravitational field strength = g)

A $g(\cos 20° - \sin 20° \tan 5°)$

B $g(\cos 20° - \sin 20° \tan 25°)$

C $g(\cos 25° - \sin 5° \tan 20°)$

D $g(\cos 25° - \sin 25° \tan 20°)$

E $g(\sin 20° - \cos 20° \tan 5°)$

F $g(\sin 20° - \cos 20° \tan 25°)$

G $g(\sin 25° - \cos 5° \tan 20°)$

H $g(\sin 25° - \cos 25° \tan 20°)$

ENGAA S1 2017 - Question 48 - Worked Solution

$$When\ \theta = 20°$$

Resolving I to slope:

$$N = mg\cos\cos(20)$$

Resolving ll to slope:

$$mg \sin sin\,(20) - F = 0$$

$$mg \sin sin\,(20) = \mu N$$

$$mg \sin sin\,(20) = \mu mg \cos cos\,(20)$$

$$\mu = \tan tan\,(20)$$

$$When\ \theta = 25°$$

Resolving ⊥ to slope:

$$N = mg \cos cos\,(25)$$

Resolving || to slope:

$$F = ma$$

$$mg \sin sin\,(25) - \mu(mg \cos cos\,25\,) = ma$$

$$g \sin sin\,(25) - g\mu \cos cos\,(25) = a$$

$$g[\sin sin\,(25) - \tan tan\,(20) \cos cos\,(25)\ ] = a$$

Answer is H

ENGAA S1 2017 - Question 49

49 Find the complete set of values of x for which

$$\frac{x^3 - 6x^2 + 9x - 4}{x} > 0$$

A $x < 0,\ x > 4$

B $0 < x < 4$

C $0 < x < 1,\ x > 4$

D $x < 0,\ 1 < x < 4$

E $x < 1,\ x > 4$

F $1 < x < 4$

ENGAA S1 2017 - Question 49 - Worked Solution

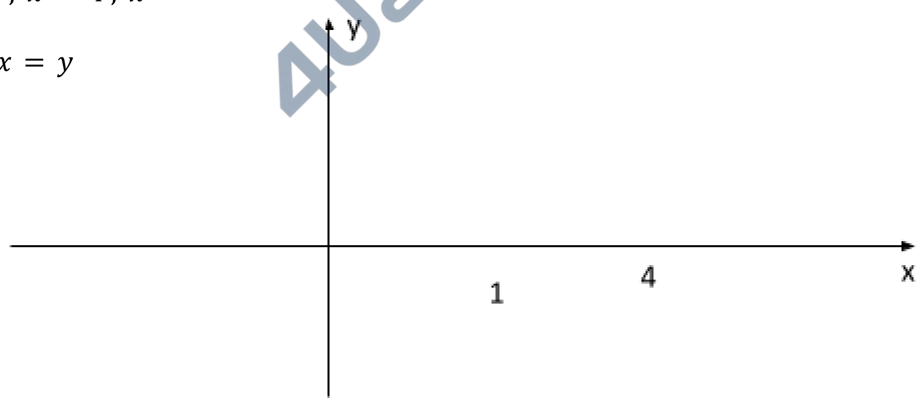
$(x^3 - 6x^2 + 9x - 4)x > 0$

$(x - 1)^2(x - 4)x > 0$

The cuitical points are

$x = 1\ (Repeated\ Roots)\ ,\ x = 4\ ,\ x$

$sketch\ (x - 1)^2(x - 4)x = y$



$x > 4\ \ ,\ x < 0$

Answer is A

ENGAA S1 2017 - Question 50

50 A suitcase of mass m is on a conveyor belt which moves upwards at a constant speed at an angle of θ to the horizontal. The coefficient of friction between the suitcase and the slope is μ. The suitcase does not slip, even if angle θ is made slightly larger.

Which expression gives the friction force between the suitcase and the belt?

(gravitational field strength $= g$)

A μmg

B $mg \sin \theta$

C $mg \cos \theta$

D $\mu mg \sin \theta$

E $\mu mg \cos \theta$

ENGAA S1 2017 - Question 50 - Worked Solution

Resolving ∥ to slope:

$$F - mg \sin sin \theta = ma = 0 \quad (as\ speed\ is\ constant)$$

$$F = mg \sin sin \theta$$

Can not use $F = \mu N$ as friction might not be limiting so $F \leq \mu N$

Answer is B

ENGAA S1 2017 - Question 51

51 The curve $y = \sin x$ is stretched by a scale factor of $\frac{1}{2}$ parallel to the x-axis and then translated by $\frac{\pi}{4}$ in the negative x-direction.

What is the equation of the new curve?

A $y = \sin\left(\frac{x}{2} - \frac{\pi}{4}\right)$

B $y = \sin\left(\frac{x}{2} + \frac{\pi}{4}\right)$

C $y = \sin\left(\frac{x}{2} - \frac{\pi}{8}\right)$

D $y = \sin\left(\frac{x}{2} + \frac{\pi}{8}\right)$

E $y = \sin\left(2x - \frac{\pi}{4}\right)$

F $y = \sin\left(2x + \frac{\pi}{4}\right)$

G $y = \sin\left(2x - \frac{\pi}{2}\right)$

H $y = \sin\left(2x + \frac{\pi}{2}\right)$

ENGAA S1 2017 - Question 51 - Worked Solution

Initially:

$$y = \sin sin\,(x)$$

Stretch in x-axis

$$x \to 2x$$

Translation :

$$x \to x + \frac{\pi}{4}$$

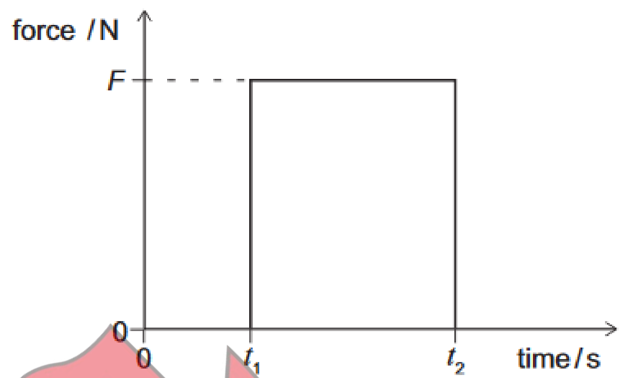
$$\Rightarrow y = \sin sin\left(2\left(x + \frac{\pi}{4}\right)\right)$$

$$y = \sin sin\left(2x + \frac{\pi}{2}\right)$$

Answer is H

ENGAA S1 2017 - Question 52

52 The graph shows how the horizontal force on a tennis ball of mass m varies during a shot in a tennis match. The ball is initially travelling horizontally toward the racket with speed u and leaves the racket horizontally travelling in the opposite direction with speed v.



Which expression gives the magnitude of the momentum of the ball as it leaves the racket?

A $F(t_2 - t_1)$

B $F(t_2 - t_1) - mu$

C $F(t_2 - t_1) + mu$

D $mv - mu$

E $Ft_2 - mu$

ENGAA S1 2017 - Question 52 - Worked Solution

$$F = \frac{dp}{dt} \Rightarrow \Delta p = \int F dt$$

$$= F\left(t_2 - t_1\right)$$

$$\Delta p = mv - m(-u) = F\left(t_2 - t_1\right)$$

$$mv = F\left(t_2 - t_1\right) - mu$$

Answer is B

ENGAA S1 2017 - Question 53

53 The equations of two straight lines are $y = 3 + (2p^2 - p)x$ and $y = 7 + (p - 2)x$, where p is a real constant.

For certain values of p, the two lines are perpendicular.

Which of the following numbers is closest to the greatest such value of p?

A 2.00

B 1.75

C 1.50

D 1.00

E -0.25

F -0.50

ENGAA S1 2017 - Question 53 - Worked Solution

$perpendicular \Rightarrow (2p^2 - p)(p - 2) = -1$

Product of gradients is -1

$\Rightarrow 2p^3 - 5p^2 + 2p + 1 = 0$ ------ ①

Try p =1

$LHS = 2 - 5 + 2 + 1 = 0 \Rightarrow p = 1$ *is a root*, $(p - 1)$ *is a factor*

Use this to factorize ①

$2p^3 - 5p^2 + 2p + 1 = (p - 1)(2p^2 - 3p - 1) = 0$

Now need to solve:

$$(p - 1)(2p^2 - 3p - 1) = 0$$

$$p = 1 \ , \ (2p^2 - 3p - 1) = 0$$

Solving quadratic:

$$p = \frac{3 \pm \sqrt{9-8}}{4}$$

$$= \frac{3}{4} \pm \frac{\sqrt{17}}{4}$$

$\sqrt{16} = 4 \Rightarrow \frac{\sqrt{17}}{4} > 1$, *so the greatest value at p is*

$p = \frac{3}{4} + \frac{\sqrt{17}}{4} \qquad \langle\sqrt{17}\approx 4\rangle$

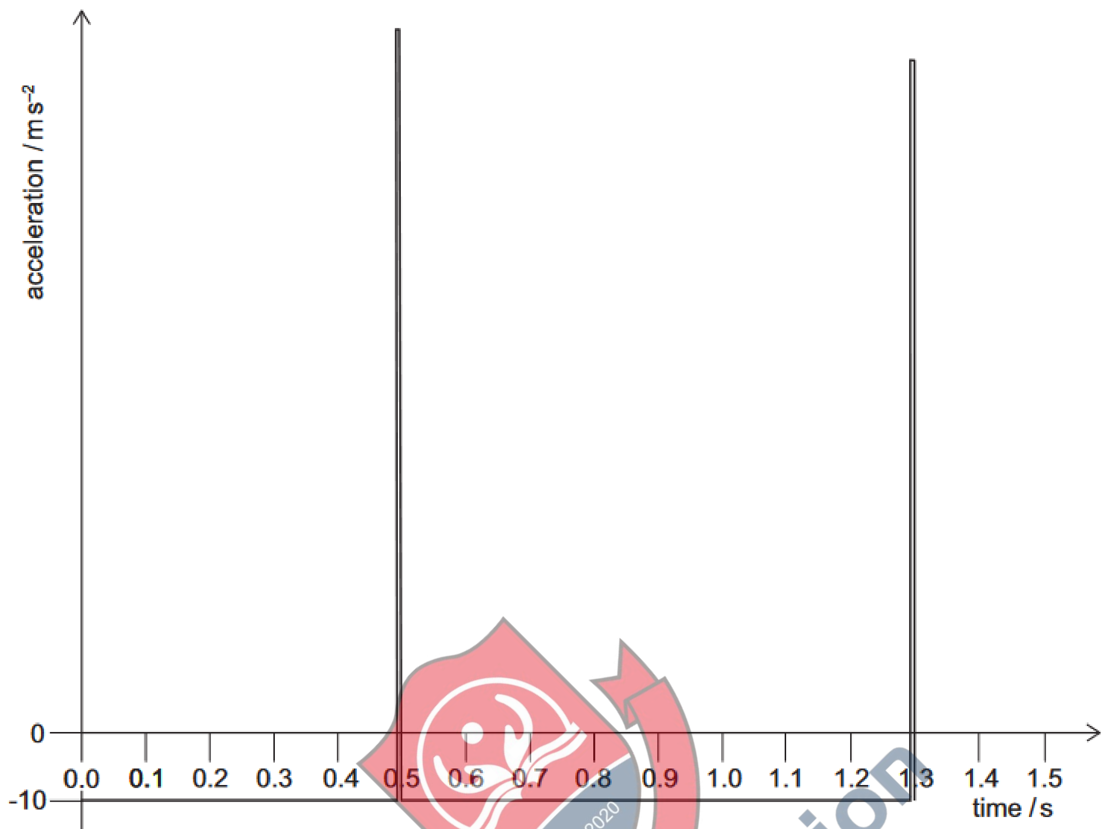
$\approx \frac{3}{4} + 1$

$= 1.75$

Answer is B.

ENGAA S1 2017 - Question 54

54 The acceleration versus time graph is for a ball dropped from rest, falling vertically and bouncing on the ground.



The time of contact with the ground can be ignored.

What is the speed of the ball immediately **after** hitting the ground for the first time, and what is the maximum height reached by the ball after the first bounce?

(gravitational field strength $= 10\,N\,kg^{-1}$)

	speed / $m\,s^{-1}$	*height* / m
A	4.00	0.80
B	4.00	1.25
C	5.00	0.80
D	5.00	1.25
E	8.00	3.20

ENGAA S1 2017 - Question 54 - Worked Solution

① Reaches max height at:

$$t = \frac{1.3-0.5}{2} = 0.4$$

At max height v = 0

S = ?
U = u
V = 0
A = -10
T = 0.4

$$v = utat$$

$$0 = u - 10\times0.4$$

$$u = 4ms^{-1}$$

②

S = s
U =
V = 0
A = -10
T = 0.4

$$S = vt - \frac{1}{2}at^2$$

$$= 0 - \left(\frac{1}{2}\right)(-10)(0.4)^2$$

$$= 0.8m$$

ENGAA S2 2017 - Question 1

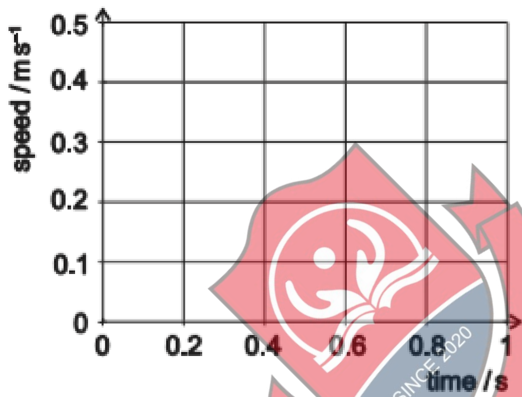
1 A ball of mass $m = 0.5$ kg is at rest a distance d above the flat floor of a spacecraft.

Installed in the floor is an artificial gravity generator which produces a field at right angles to the floor, directed towards the floor. There is no air in the spacecraft.

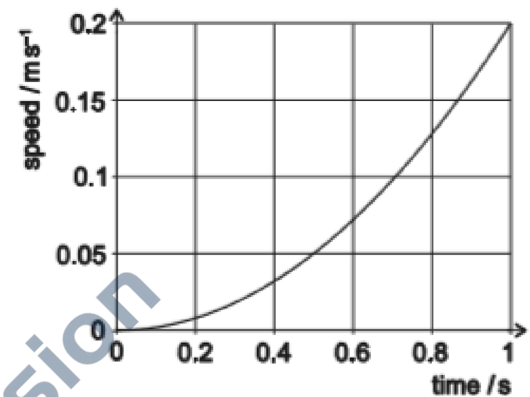
The generator is switched on at time $t = 0$ s and produces a field g that increases linearly with time, such that $g = 0.4t$ ms^{-2}. The artificial gravity is the only force experienced by the ball.

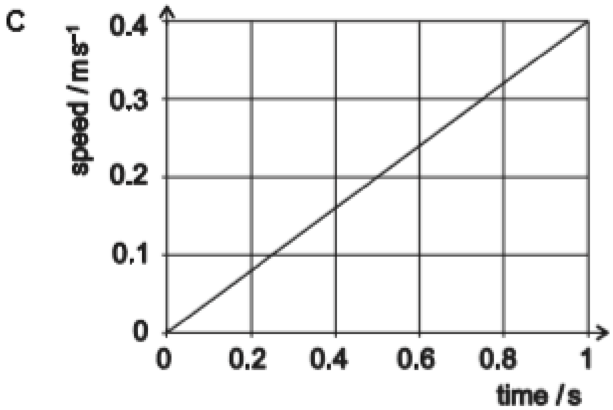
a) Assuming that the ball does not hit the floor within the first second of motion, which of these graphs represents the speed of the ball plotted against time? **[2 marks]**

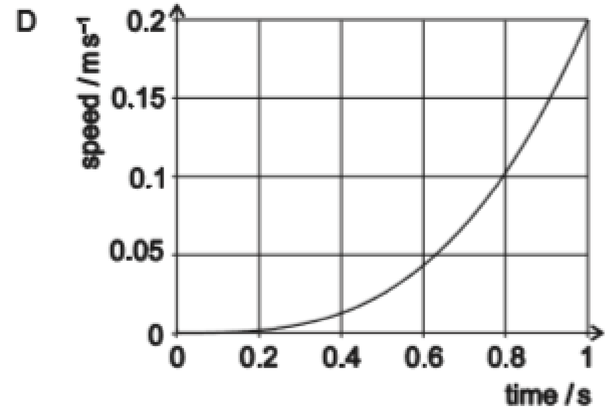
A

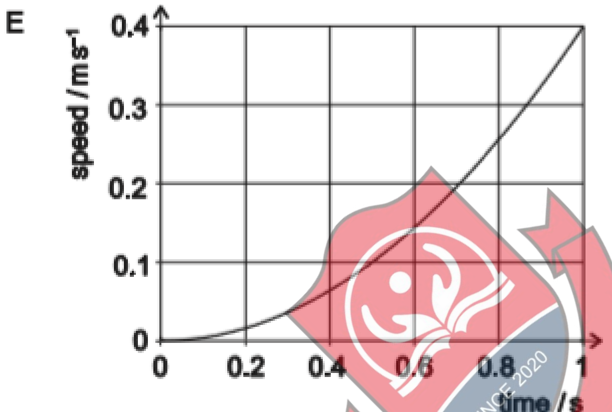


B









ENGAA S2 2017 - Question 1 - Worked Solution

a) Acceleration is not constant or 0, so not Graphs A or C

a = 0.4t

$\frac{dv}{dt} = 0.4t$

$v = \int_0^t 0.4t\, dt$

t = 0 , v = 0

$v = 0.2t^2$

So when t = 0.5 , v = 0.05 , so pick graph B

Answer is B

b)

b) Which of these expressions gives the time taken for the ball to first hit the floor? **[2 marks]**

A $(15d)^{\frac{1}{3}}$

B $(5d)^{\frac{1}{3}}$

C $(5d)^{\frac{1}{2}}$

D $\left(\frac{15d}{2}\right)^{\frac{1}{3}}$

E $\left(\frac{5d}{2}\right)^{\frac{1}{3}}$

T = time to hit floor

$v = \frac{ds}{dt} \quad (s = displacement)$

$total\ displacement = d = \int v\, dt$

$= d = \int_0^T 0.2t^2\, dt$

$= d = \frac{T^3}{15}$

$T = (15d)^{1/3}$

Answer is A

c)

A P only

B Q only

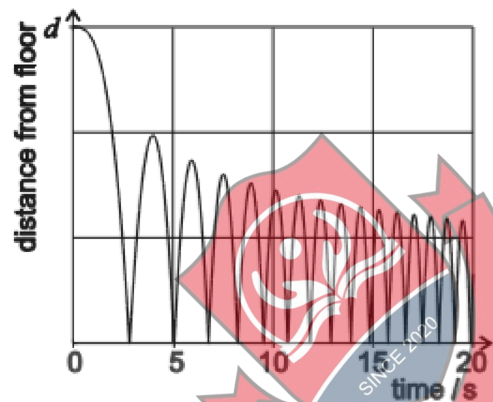
C R only

D P and Q only

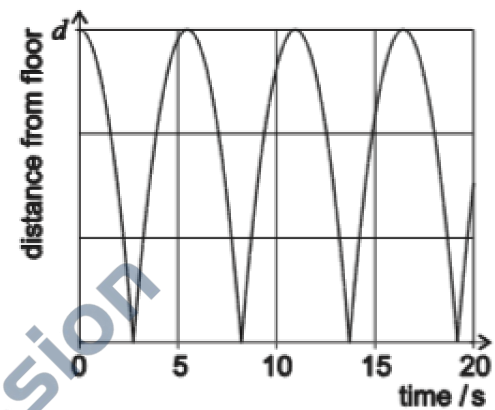
E Q and R only

c) The ball bounces and hits the floor repeatedly. Which of these graphs might represent the position of the ball plotted against time? **[3 marks]**

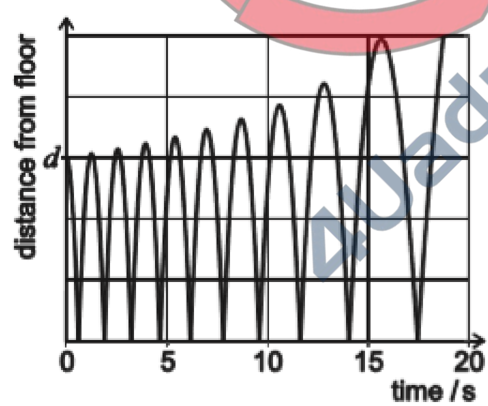
P



Q



R



- Q not possible as ball has downward acceleration
- Downward acceleration increases with time
- So the accelerating force on the ball before bounce is smaller than the decelerating force on it after bounce
- So max height reached after each bounce must decrease
- Graph P is correct

Answer is A

d)

d) Force is usually measured in Newtons (N). Given that $F = ma$, which of the following is an alternative unit for force? **[1 mark]**

A kg s m^{-2}

B $\text{kg}^{-1}\text{m}^{-1}\text{s}^{2}$

C kg m s^{-2}

D $\text{N kg}^{-1}\text{m}^{-1}\text{s}^{2}$

E $\text{N}^{-1}\text{kg m s}^{-2}$

F = ma

- m has unit kg
- $a = \frac{dv}{dt}$, so as v has units ms^{-1} , a has unis ms^{-2}
- So F has units $Kgms^{-2}$

Answer is C

e)

e) Air is now injected into the spacecraft, creating air resistance. The drag force D on the ball is given by

$$D = \frac{1}{2}X\rho v^2 A$$

where ρ is the air density, v is the ball's speed, A is its cross-sectional area and X is an unknown parameter.

What are the units of X? **[2 marks]**

A m s^{-2}

B m s^{-1}

C $\text{kg}^{-1}\text{m}^{-1}\text{s}^{2}$

D kg m s^{-2}

E X has no units

- D is a force so has units $kgms^{-2}$
- $\rho = \frac{m}{v}$, so has units $Kgms^{-3}$
- V has units ms^{-1}, so v^2 m^2s^{-2}
- A has units m^2

- $x = \frac{2D}{\rho V^2 A}$, so has units

$$\frac{kgms^{-2}}{kgm^{-3}m^{2}s^{-2}m^{2}}$$

$$= \frac{m^1}{m^1}$$

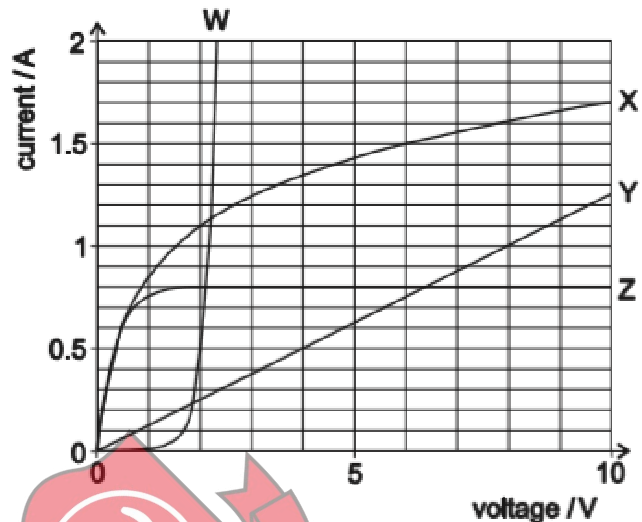
$$= 1$$

So x has no units

Answer is E

ENGAA S2 2017 - Question 2

2 The graph shows the current against voltage characteristics of four different electronic devices W, X, Y and Z. One of the devices is an 8Ω resistor and one is a filament lamp rated 9W at 6V. You may assume that the filament lamp does not 'blow' in the context of this question.



a) Which of the devices is the resistor?

A device W

B device X

C device Y

D device Z

ENGAA S2 2017 - Question 2 - Worked Solution

a)

- Resistor follows aim's law with constant R
  V = IR
- So $V \propto I$, and so resistor has a straight line I – V graph, through the origin
- So device V is a resistor

Answer is C

b)

b) Which of the devices is the filament lamp?

A device W

B device X

C device Y

D device Z

- As the current through a lamp increases, the lamp heats up, and so its resistance increases.
- So as the instantenous resistance of the lamp is R = V/I , the gradient of its I – V graph will decrease.
- This means x is the filament lamp.

Answer is B

c)

c) The filament lamp and the resistor are connected in parallel to a 6.0V power supply with negligible internal resistance.

Approximately what current is drawn from the supply? **[3 marks]**

A 0.75A

B 1.5A

C 1.83A

D 2.25A

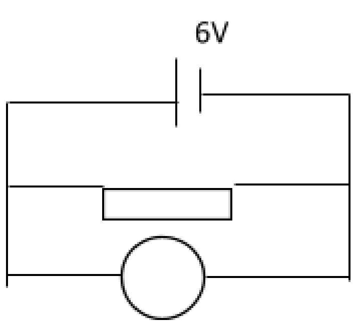
E 2.42A

Draw circuit and label currents

- Currents are parallel so voltage across resistor and lamp is 6V
- From question, at 6V, power dissipated by lamp is P = 9W
- $P = IV = 6I_2 = 9$

$I_2 = \frac{3}{2}A$

- For resistor



$V = I_1 R$

$6 = I_1 \times 8$

$I_1 = \frac{3}{2}A$

$I = I_1 + I_2$

$I = \frac{3}{4} \times \frac{3}{2}$

$I = 2.25A$

Answer is D

d)

d) The previous circuit is disconnected, and then devices W and Y are connected in series to the same 6.0V power supply.

Which one of the following statements about the new circuit must be correct? **[1 mark]**

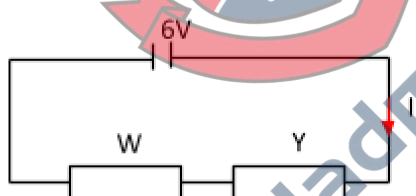
A Devices W and Y dissipate equal power.

B Devices W and Y have equal voltages across them.

C Equal currents flow through devices W and Y.

D The power supply delivers more power than it would if device W or device Y were connected alone.

E The power supply delivers more power than it would if devices W and Y were connected in parallel.



- C is correct as for a series circuit I is constant throughout.
  Why other statements are false
  a. P = IV, I is the same for W and Y, but V is not as they don't have equal resistant.
  b. Again W and X don't have equal resistance, so as p .d are shared in ratio of resistance , p.d across W and Y is not equal.
  c.
  d. $P = IV = \frac{V^2}{R}$, and if W and Y were connected alone, power supply would deliver more power.
  e. If connected in parallel, total R is lower, so similarly to d, power supply would deliver more power.
  Answer is C

e)

e) In the new circuit, approximately what power is dissipated by device W? **[3 marks]**

A 0.5W

B 1.0W

C 1.5W

D 2.0W

E 2.5W

- I is the same in W and Y
- Require $V_W + V_Y = 6V$
- So need to use I – V graphs to find such an I such that this holds
- At I = 0.5A

  $V_Y + V_W = 4 + 2 = 6V$

  $P_W = I\,V_W$

$$= 0.5 \times 2$$

$$= 1.0W$$

Answer is C

ENGAA S2 2017 - Question 3

3 Fig. 3(a) shows the results of an experiment in which a 0.5 m length of elastic cord has been extended by a force with a corresponding extension. The cord fails at point Q by fracture.

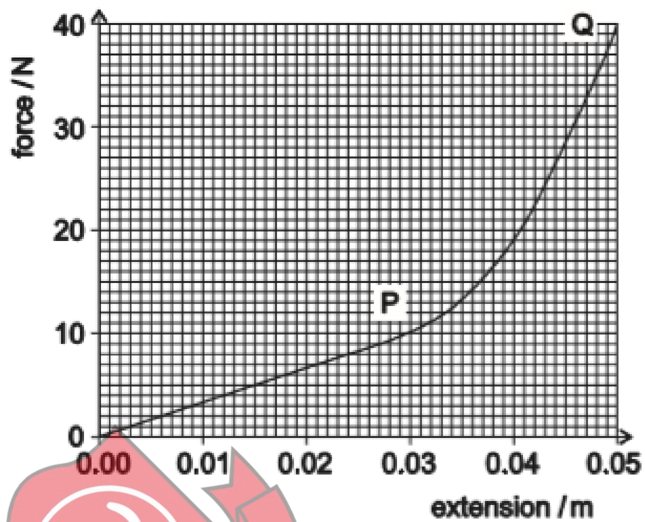


Fig. 3(a)

a) The elastic behaviour of a material can often be described by Hooke's law, which is given by the equation $F = kx$, where x is extension, F is force and k is an elastic constant which depends on the material studied.

Which of the following statements correctly describes the behaviour of the cord? **[2 marks]**

A no Hooke's law behaviour and fracture at a strain of 0.05

B Hooke's law behaviour up to P and fracture at a strain of 0.05

C Hooke's law behaviour up to Q and fracture at a strain of 0.05

D Hooke's law behaviour up to P and fracture at a strain of 0.1

E Hooke's law behaviour up to Q and fracture at a strain of 0.1

ENGAA S2 2017 - Question 3 - Worked Solution

a.

- $F \propto x$, so curd follows Hooke's law white graph is linear
- This is clearly the case up to point P
- The strain at fracture is.

$strain = \frac{x}{length}$

$= \frac{0.05m}{0.5m}$

$= 0.1$

Answer is D

b.

b) What is the work done U in stretching this 0.5 m length of elastic cord by 0.05 m (to 2 significant figures)? **[3 marks]**

A 0.15 J

B 0.30 J

C 0.60 J

D 2.0 J

E 6.0 J

Work done = area under graph

$U = \frac{1}{2} \times 0.03 \times 10 + 0.02 \times 10 + \frac{1}{2} \times 0.02 \times 30$

$U = 0.65$

$U = 0.6J$

Answer is C

c.

c) An unstretched 0.25 m length of the same type of cord is used in a catapult to propel a mass m, as illustrated in Fig. 3(b).

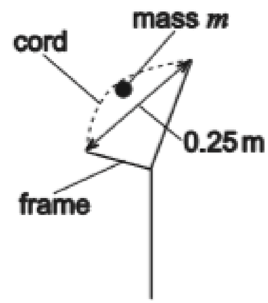


Fig. 3(b)

What is the maximum speed V_{max} at which the mass can be propelled (where U is the work done calculated in part **b)**)? **[3 marks]**

A $\sqrt{mU}$

B $\sqrt{\frac{U}{m}}$

C $\sqrt{\frac{2U}{m}}$

D $\sqrt{2mU}$

E $\sqrt{\frac{U}{2m}}$

- New cord has half the length of the old cord
- So the elastic potential energy the cord stores before snapping will also half
- So max e.p , e stored is U/2
- When catapulted , e.p e is transferred to kinetic energy

$$\frac{U}{2} = \frac{4}{2} m V_{max}^{2}$$

$$\frac{U}{m} = V_{max}^{2}$$

$$V_{max} = \sqrt{\frac{U}{m}}$$

Answer is B

d.

d) Two parallel 0.25 m lengths of the elastic cord are used in the catapult as shown in Fig. 3(c).

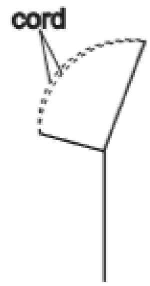


Fig. 3(c)

What is the maximum speed at which the mass can now be propelled? **[2 marks]**

A $\frac{1}{2}V_{max}$

B $\frac{1}{\sqrt{2}}V_{max}$

C V_{max}

D $\sqrt{2}V_{max}$

E $2V_{max}$

- Each cord stores $\frac{U}{2}$, so total EPE stored is U
- Now max speed is

$$U = \frac{1}{2}mV^2$$

$$V = \sqrt{\frac{2U}{m}}$$

$$V = \sqrt{2}\sqrt{\frac{U}{m}}$$

$$= \sqrt{2}V_{max}$$

Answer is D

ENGAA S2 2017 - Question 4

a) The pair of slits is illuminated by laser light of wavelength $\lambda = 600\,\text{nm}$.

Which of the following statements are correct (where n is an integer)? **[2 marks]**

1 Points of maximum brightness on the screen occur where the distances r_1 and r_2 differ by $n\lambda$.

2 Points of maximum brightness on the screen occur where the distances r_1 and r_2 differ by $\left(n + \frac{1}{2}\right)\lambda$.

3 Points of minimum brightness on the screen occur where the distances r_1 and r_2 differ by $\left(n + \frac{1}{2}\right)\lambda$.

4 For a diffraction pattern to appear, the light from the two slits must be coherent.

5 The maxima are all of equal brightness.

A 1 and 4 only

B 1, 3 and 4 only

C 1, 3 and 5 only

D 1, 4 and 5 only

E 2 and 4 only

ENGAA S2 2017 - Question 4 - Worked Solution

a.

1) Constructive interference occurs when path difference is $n\lambda$, so TRUE
2) FALSE , as when path difference is $\left(n + \frac{1}{2}\right)\lambda$, waves arrive in anti-phase, so interference is destructive, see minima
3) TRUE, as shown above
4) TRUE, as there will be no diffraction pattern if waves are not coherent (have no fixed phase difference)
5) FALSE as maxima further away from source are less bright due to intensity inverse square law$\left[I \propto (distance)^{-2}\right]$

Answer is B

b.

b) A thin piece of transparent material, thickness 300 nm and in which the speed of light is half that in air, is now placed immediately behind one of the two slits.

Which one of the following statements is correct? **[3 marks]**

A The diffraction pattern is unchanged.

B The diffraction pattern disappears because the light from the two slits is no longer coherent.

C The diffraction pattern disappears because the light from the two slits is no longer in phase.

D The complete diffraction pattern shifts in the y direction.

E Each maximum is replaced by two because the material alters the wavelength of the light coming from it.

$s = 300nm = \lambda/2$

$v = c/2$

As $\lambda = 600nm$

- $Period\ of\ wave:\ T = \lambda/C$
- $Time\ to\ transverse\ s\ in\ vaccum$:

$tv = \frac{s}{C} = \frac{\lambda}{2C} = \frac{1}{2}T$

- Time to transverse s in material

$tm = \frac{s}{C/2} = \frac{\lambda}{2C}\times 2 = T$

- So light which passes through material will be half a cycle out of phase with the light that didn't
- However, the sources will still have a constant path difference
- So d is true as now light from one slit will be in anti-phase (half a cycle out of phase) with light from the slit. So maxima will now occur when $r_1 - r_2 = \left(n + \frac{1}{2}\right)\lambda$, as now at these points the waves will be in phase and so form maxima
- So pattern will shift in y-direction

  Answer is D

c.

c) A radio transmitter transmits a signal at 600 MHz to a receiver 1 km away. In an attempt to double the strength of the signal at the receiver, a second antenna is added at the transmitter, 1 m away alongside the original one, and fed by the same signal. It is suggested that, instead of improving reception, diffraction effects might actually make reception much worse.

Which of the following statements is correct? **[3 marks]**

A Diffraction effects would not be a problem because light and radio are different types of wave.

B Diffraction effects would not be a problem because the waves are too low frequency to produce diffraction effects.

C Diffraction effects would not be a problem as the transmitting antennas are too far apart to produce diffraction effects.

D Diffraction effects will occur, but the maxima would be sufficiently close together that this would not be a problem.

E Diffraction effects could be a problem because the distance between the transmitting antennas is comparable to the wavelength.

- Wave length of the radio waves:

$$\lambda = \frac{c}{t} = \frac{3\times10^8}{600\times10^6}$$

$$= 0.5m$$

- λ is comparable to gap between sources (d = m) so diffraction occurs
- So D and E are true
- Diffraction creating formula

$$d \sin sin \theta = n\lambda$$

$$\sin sin \theta \approx \tan tan \theta \approx \frac{x}{D}, \ d = 1m$$

$$x = Dn\lambda$$

- So spacing between maxima is:

$$\Delta x = D\lambda$$

$$= 0.5\times1km$$

$$= 500m$$

- So spacing between maxima is large
- So D is wrong and E is true

  Answer is E