Worked Solutions for ENGAA Past Papers

ENGAA S1 2018 - Question 1

1 A group of drivers, consisting of 200 women and 300 men, was asked if they passed their driving test at the first attempt.

Altogether 167 of the group said they passed at the first attempt.

Of the women, 143 said they did not pass at the first attempt.

How many of the men said they passed at the first attempt?

A 10

B 24

C 33

D 57

E 110

F 133

G 157

ENGAA S1 2018 - Question 1 - Worked Solution

Number of women passing in 1st attempt:

200 – 143 = 57

Number of men passing in 1st attempt:

167 – 57 = 110

Answer is E.

ENGAA S1 2018 - Question 2

2 An unstable nucleus X becomes a stable nucleus Y after a succession of decays, during which a total of 5 alpha particles and 2 beta (β^-) particles are emitted.

How many fewer protons does nucleus Y contain than nucleus X?

A 6

B 8

C 10

D 12

E 14

F 16

G 18

H 20

ENGAA S1 2018 - Question 2 - Worked Solution

$An\ \alpha\ particle$:

24α

$A\ \beta\ particle$:

$-\ 10\beta$

So total number of proton's lost;

$(5\times2) + (-\ 1\times2) = 8$

Answer is B.

ENGAA S1 2018 - Question 3

3 A cuboid has sides of length x, $\sqrt{2}x$ and $2x$, measured in cm.

The volume, in cm^3, of the cuboid is numerically equal to twice the total surface area, in cm^2, of the cuboid.

What is the value of x?

A 10

B $6 + 2\sqrt{2}$

C 5

D $3 + \sqrt{2}$

E $\frac{5}{2}$

F $\frac{3}{2} + \frac{1}{2}\sqrt{2}$

ENGAA S1 2018 - Question 3 - Worked Solution

$$V = 2A$$

$$V = 2\sqrt{2}x^3$$

$$A = 2\times\left[\sqrt{2}x^2 + 2\sqrt{2}x^2 + 2x^2\right]$$

$$= 2x^2\left[3\sqrt{2} + 2\right]$$

$$2\sqrt{2}x^3 = 4x^2\left[3\sqrt{2} + 2\right]$$

$$\sqrt{2}x = 2\left[3\sqrt{2} + 2\right]$$

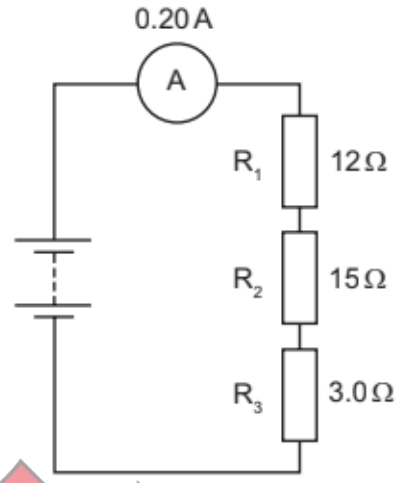
$$x = 2\left[3 + \frac{2}{\sqrt{2}}\right]$$

$$x = 6 + 2\sqrt{2}$$

Answer is B.

ENGAA S1 2018 - Question 4

4 The diagram shows three resistors R_1, R_2 and R_3 connected in series with a battery of constant voltage. The resistance of each resistor and the corresponding current are also shown.



Resistor R_3 is now removed and the circuit is reconnected.

What is the new current in the circuit?

A 0.20 A

B 0.22 A

C 0.33 A

D 0.40 A

E 0.50 A

F 2.0 A

G 6.0 A

ENGAA S1 2018 - Question 4 - Worked Solution

$R_T = R_1 + R_2 + R_3 = 30\Omega$

$V = IR$

$= 0.2 \times 30$

$= 6$

ENGAA S1 2018 - Question 5

5 The line joining the points with coordinates $(p, p-1)$ and $(1-p, 2p)$ is parallel to the line with equation $2x + 3y + 1 = 0$

What is the value of p?

A -1

B $-\frac{1}{7}$

C $\frac{1}{9}$

D $\frac{1}{8}$

E 1

F $\frac{5}{4}$

G 2

H 5

ENGAA S1 2018 - Question 5 - Worked Solution

Gradient ①

$$m = \frac{\Delta y}{\Delta x} = \frac{2p-(p-1)}{(1-p)-p}$$

$$= \frac{p+1}{-2p+1}$$

Gradient ②

$$2x + 3y + 1 = 0$$

$$y = -\frac{2}{3}x - \frac{1}{3}$$

$$m = -\frac{2}{3} = \frac{p+1}{-2p+1}$$

$$3p + 3 = -2 + 4p$$

$$p = 5$$

Answer is H.

ENGAA S1 2018 - Question 6

6 When travelling in a vacuum, visible light has a wavelength between 400 nm and 700 nm.

The speed of light in a vacuum is $3.0 \times 10^8\,\text{ms}^{-1}$.

What can be concluded about **ultraviolet** radiation from this information?

A It has a **maximum** frequency of 2.7×10^{14} Hz

B It has a **maximum** frequency of 4.3×10^{14} Hz

C It has a **maximum** frequency of 7.5×10^{14} Hz

D It has a **maximum** frequency of 1.0×10^{15} Hz

E It has a **minimum** frequency of 2.7×10^{14} Hz

F It has a **minimum** frequency of 4.3×10^{14} Hz

G It has a **minimum** frequency of 7.5×10^{14} Hz

H It has a **minimum** frequency of 1.0×10^{15} Hz

ENGAA S1 2018 - Question 6 - Worked Solution

Ultra light has max λ:

$$\lambda = 400m$$

$$c = \lambda f$$

$$f = \frac{c}{\lambda}$$

λ is a maximum so f will be a minimum

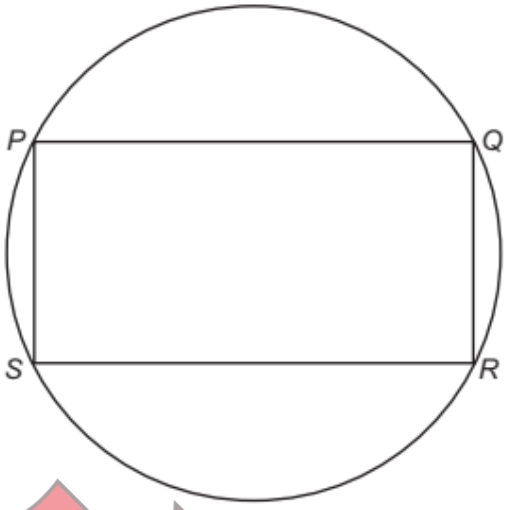
$$f = \frac{3.0\times10^{8}}{400\times10^{-9}}$$

$$= 7.5\times10^{14}Hz$$

Answer is G.

ENGAA S1 2018 - Question 7

7 A rectangle *PQRS* is drawn inside a circle, with its vertices on the circumference of the circle.



[diagram not to scale]

The ratio of the length of *PQ* to the length of *QR* is 2 : 1

The area of the rectangle *PQRS* is 96 cm^2.

What is the radius, in cm, of the circle?

A $\sqrt{6}$

B 3

C $3\sqrt{2}$

D $2\sqrt{15}$

E $4\sqrt{6}$

F 12

G $12\sqrt{2}$

H $8\sqrt{15}$

ENGAA S1 2018 - Question 7 - Worked Solution

Let QR = y

Then PQ = 2y

$$A = 2y^2 = 96$$

$$y = 4\sqrt{3}$$

$$PQ = 8\sqrt{3},\ QR = 4\sqrt{3}$$

Using Pythagoras:

$$\left(\frac{PQ}{2}\right)^2 + \left(\frac{QR}{2}\right)^2 = r^2$$

$$16\times3 + 4\times3 = r^2$$

$$r = \sqrt{60}$$

$$r = 2\sqrt{15}$$

Answer is D.

ENGAA S1 2018 - Question 8

8 A filament lamp working at its operating voltage converts electrical energy at a rate of 100 W.

The lamp has an efficiency of 5.0%.

How much energy is wasted by the lamp in 10 minutes?

A 50 J

B 950 J

C 1000 J

D 3000 J

E 57 000 J

F 60 000 J

ENGAA S1 2018 - Question 8 - Worked Solution

Lamp wastes 95% of energy

Energy transferred per second = Power

$P = 100\,W$

Energy wasted in 10 min:

$= Pt \times 95\%$

$t = 10 \times 60s$

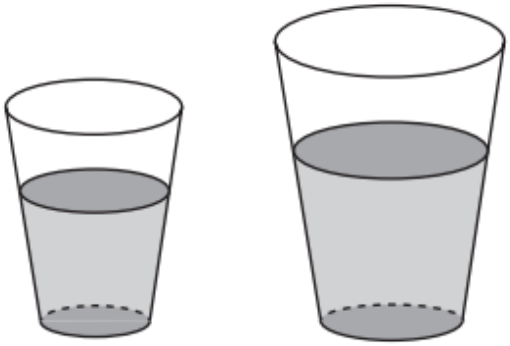
$= 100 \times 10 \times 60 \times 0.95$

$= 57000J$

Answer is E.

ENGAA S1 2018 - Question 9

9



[diagram not to scale]

At a cinema, drinks are sold in regular and large sizes.

The cups for these are mathematically similar.

The ratio of the heights of the cups and the ratio of the depths of the drinks are both 4 : 5

The volume of drink in a regular size cup is 320 cm^3.

What is the volume, in cm^3, of drink in a large size cup?

A 384

B 400

C 500

D 576

E 625

F 640

ENGAA S1 2018 - Question 9 - Worked Solution

If the ratio of the lengths (height and radius) is 4 : 5, then ratio of volumes is $4^3 : 5^3 = 64 : 125$
As volume scales as length^3

$$\frac{V_R}{V_L} = \frac{64}{125}$$

$$V_L = \frac{125}{64} \times 320$$

$$= 625 cm^3$$

Answer is E.

ENGAA S1 2018 - Question 10

10 The potential difference across the motor in an electric car is 400 V and the current in the motor is 1250 A.

The car accelerates along a horizontal road from rest for 4.0 s.

The efficiency of the overall system is 45%.

What is the kinetic energy of the car at the end of the 4.0 s?

(Ignore energy losses due to air resistance and due to friction between the tyres and the road.)

A 225 000 J

B 500 000 J

C 900 000 J

D 1 250 000 J

E 2 000 000 J

ENGAA S1 2018 - Question 10 - Worked Solution

Total energy from motor:

$$E = Pt$$

$$W = IVt$$

$$E = 400 \times 125 \times 4$$

$$E = 200000J$$

$$Useful\ energy = 0.45E$$

$$= 900000J$$

$$= KE$$

(As ignoring energy lost to air resistance and friction)

Answer is C.

ENGAA S1 2018 - Question 11

11 The straight lines

$$5x + 2y = 20$$

$$y = 3x - 23$$

$$x = 0$$

enclose a region with area K square units.

What is the value of K?

A 39

B 78

C 99

D 129

E 198

F 258

ENGAA S1 2018 - Question 11 - Worked Solution

$$① \ 5x + 2y = 20$$
$$② \ y = 3x - 23$$
$$③ \ x = 0$$
$$①: y = -\frac{5}{2}x + 10$$

Finding intersection : ② →①

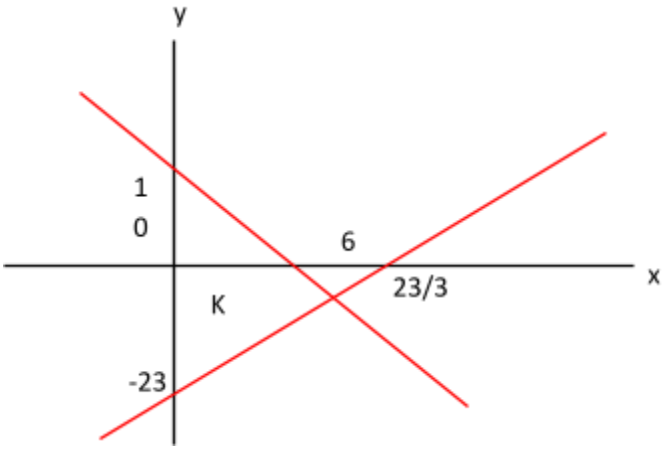
$$5x + 2(3x - 23) = 20$$

$$5x + 6x - 46 = 20$$

$$11x = 66$$

$$x = 6$$

Sketch lines:



$x = \frac{1}{2} \times 6 \times 33 = 99$

Answer is C.

ENGAA S1 2018 - Question 12

12 The momentum of a small object moving in a straight line is 24 kg m s^{-1} and its kinetic energy is 96 J.

What is the mass of the object?

A 3.0 kg

B 4.0 kg

C 6.0 kg

D 8.0 kg

E 12 kg

ENGAA S1 2018 - Question 12 - Worked Solution

$p = mv = 24$ ①

$k = \frac{1}{2}mv^2 = 96$ ②

①→②

$\frac{1}{2} \times 24 \times v = 96$

$v = 8$

Sub into ①

$m = \frac{24}{8} = 3\ kg$

Answer is A.

ENGAA S1 2018 - Question 13

13 A scale model of a cylindrical pillar is to be made.

The full-sized pillar has a volume of $12\pi\,\text{m}^3$.

The model will use a length scale of $1:40$

The model is to be a solid cylinder made of a plastic which has a density of $\frac{4}{3}\,\text{g cm}^{-3}$.

What is the mass of the model in grams?

A $\frac{9}{640}\pi$

B $\frac{1}{40}\pi$

C 40π

D $\frac{1125}{8}\pi$

E 250π

F $10\,000\pi$

G $225\,000\pi$

H $400\,000\pi$

ENGAA S1 2018 - Question 13 - Worked Solution

$Ratio\ of\ lengths = 1:40$

$Ratio\ of\ volumes = 1:40^3\ \ as$

$\frac{V_m}{V_f} = \frac{1}{40^3}$

$V_m = \frac{12\pi}{40^3}\ m^3$

$= 1.875\times10^{-4}\pi\ m^3$

$= 1.875\times10^{-4}\times100^3\ \pi cm^3$

$= \frac{375}{2}\pi\ cm^3$

$m = \rho v$

$= \frac{375}{2}\pi \times \frac{4}{3}$

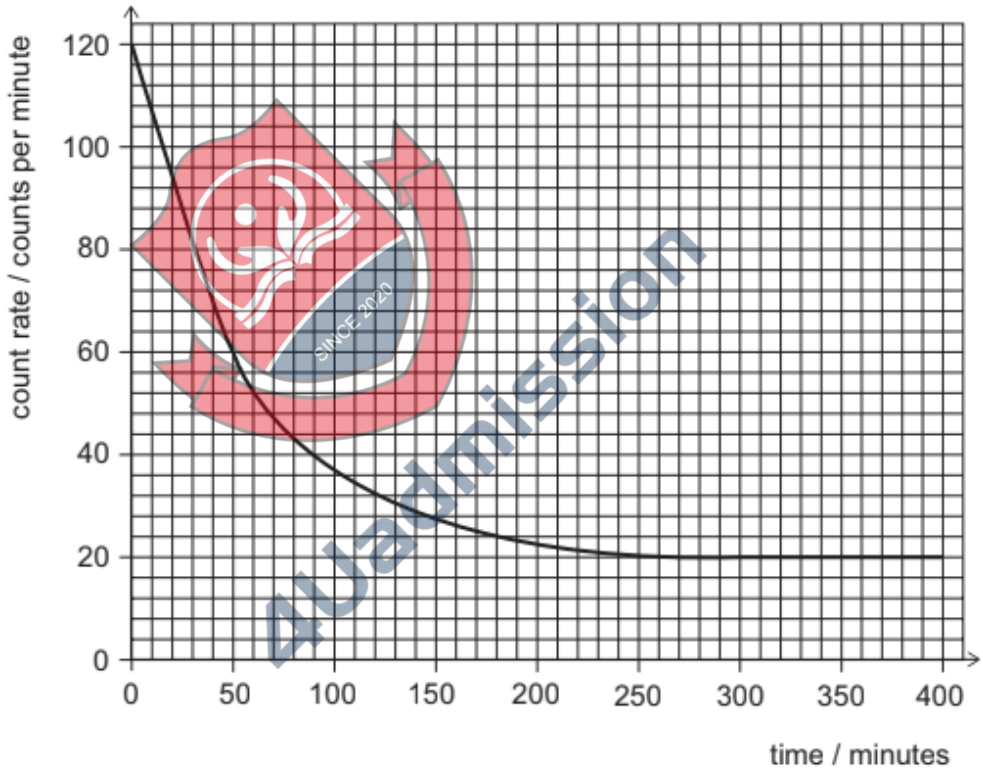
$= 250\,\pi g$

Answer is E.

ENGAA S1 2018 - Question 14

14 A radioactive isotope decays in a single step to a stable isotope.

A radiation detector is placed very near to a sample of the radioactive isotope in a laboratory. The count rate on the detector changes as time elapses. The graph shows how the measured count rate changes with time.



What is the background count rate and what is the half-life of the isotope?

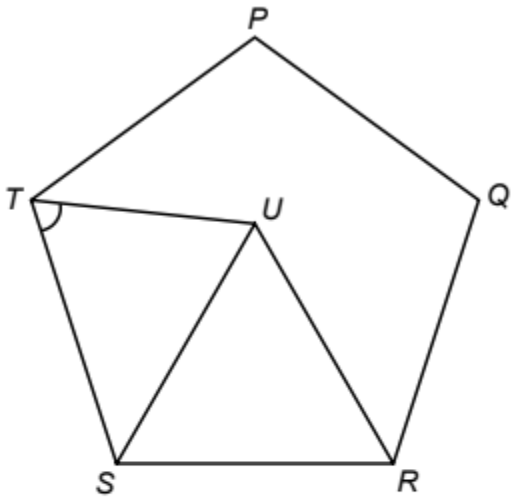
	background count rate / counts per minute	*half-life of isotope* / minutes
A	20	40
B	20	50
C	20	60
D	20	65
E	120	40
F	120	50
G	120	60
H	120	65

ENGAA S1 2018 - Question 14 - Worked Solution

- From graph
  Background counts 20 counts/min
- So initial sample a line has initial count:
  120 – 20 = 100
- At half life will have count : 50
- So total count will be:
  20 + 50 = 70
  From graph this is at : 40 min
  Answer is A.

ENGAA S1 2018 - Question 15

15



[diagram not to scale]

PQRST is a regular pentagon.

RSU is an equilateral triangle.

What is the size of angle *STU*?

A 48°

B 54°

C 60°

D 66°

E 84°

ENGAA S1 2018 - Question 15 - Worked Solution

Equilateral triangle:

$$SUR = \frac{180}{3} = 60°$$

Sum of internal angles of pentagon:

$$S = 180(n - 2) \quad (n = 5)$$

$$= 540$$

$$TSR = \frac{540}{5} = 108°$$

TS = SR = SU ⇒ TUS is isosceles

$$2x + TUS = 180°$$

$$2x + (180 - 60) = 180$$

$$2x = 132$$

$$x = 66°$$

Answer is D.

ENGAA S1 2018 - Question 16

16 A rock falling vertically experiences an air resistance force of 12 N at an instant when its acceleration is 2.0 m s^{-2} downwards.

What is the mass of the rock?

(gravitational field strength = 10 N kg^{-1})

A 1.0 kg

B 1.2 kg

C 1.5 kg

D 6.0 kg

E 10 kg

F 12 kg

G 15 kg

H 60 kg

ENGAA S1 2018 - Question 16 - Worked Solution

$F = ma$

$mg - 12 = m \times 2$

$10m - 12 = 2m$

$8m = 12$

$m = \frac{12}{8}$

$m = 1.5\ kg$

Answer is C.

ENGAA S1 2018 - Question 17

17 The original price of an item is p

The price is **increased by** 125%

The increased price is then **decreased by** 40% to q

The relationship between p and q can be expressed as $mp = q$

What is the value of m?

A $\frac{7}{20}$

B $\frac{17}{20}$

C $\frac{27}{20}$

D $\frac{33}{20}$

E $\frac{37}{20}$

ENGAA S1 2018 - Question 17 - Worked Solution

$$① \; New\ price = (1 + 1.25)p = 2.25p$$

$$② \; q = 2.25p \times (1 - 0.4)$$

$$= 2.25p \times 0.6$$

$$= 1.35p$$

$$m = 1.35$$

$$= \frac{135}{100}$$

$$= \frac{27}{20}$$

Answer is C.

ENGAA S1 2018 - Question 18

18 A transverse wave with an amplitude of 4.0 cm and a frequency of 10 Hz travels along a rope at a speed of 2.4 m s^{-1}.

What is the total distance travelled by a particle in the rope in a time of 20 s?

A 2.4 m

B 4.8 m

C 8.0 m

D 16 m

E 32 m

F 48 m

ENGAA S1 2018 - Question 18 - Worked Solution

$$f = 10Hz$$

$$T = \frac{1}{f} = 0.1s$$

$$\frac{20}{T} = \frac{20}{0.1} = 200$$

SO in 20s, 200 waves pass particle

In one wave particle travels $4\times4.0\ cm$

In 200 waves : distance $= 4\times4.0\times200$

$$= 3200cm$$

$$= 32m$$

Answer is E.

ENGAA S1 2018 - Question 19

19 Q is 5 km away from P on a bearing of 065°

R is 5 km away from Q on a bearing of 155°

What is the bearing of P from R?

A 070°

B 110°

C 225°

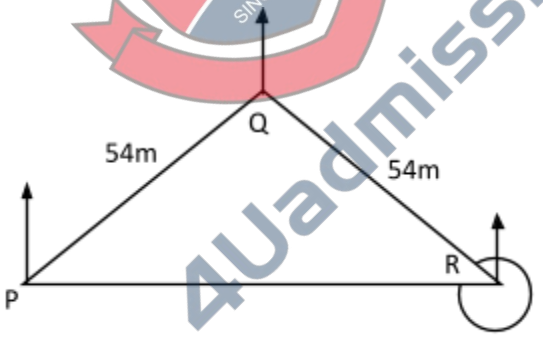
D 270°

E 290°

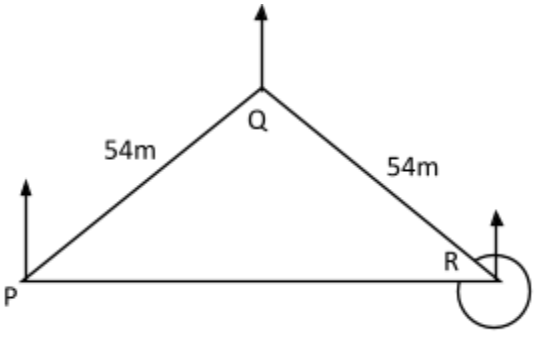
F 315°

G 335°

ENGAA S1 2018 - Question 19 - Worked Solution



- PQR is isosceles
- Using laws for corresponding and alternative angles and above point:



$$\theta = 360 - 45 - 25$$

$$\theta = 290°$$

Answer is E.

ENGAA S1 2018 - Question 20

20 A student places a measuring cylinder on a balance. She pours a volume *V* of water into the measuring cylinder, and finds that the mass of the measuring cylinder and water together is 290 g.

She then empties the measuring cylinder and dries it before putting it back on the balance.

She now pours the same volume *V* of olive oil into the measuring cylinder, and finds that the mass of the measuring cylinder and olive oil together is 270 g.

What is the mass of the measuring cylinder?

(densities: olive oil = 0.90 g cm^{-3}; water = 1.0 g cm^{-3})

A 18 g

B 20 g

C 90 g

D 180 g

E 200 g

ENGAA S1 2018 - Question 20 - Worked Solution

$$M_w + M_c = 290g$$

$$M_o + M_c = 270g$$

$$M = \rho V$$

$$M_w = 1 \times V = V$$

$$M_o = 0.9V$$

$$V = M_c = 290 \quad ①$$

$$0.9V + M_c = 270 \quad ②$$

① - ②

$$0.1V = 20$$

$$V = 200$$

Sub into ①

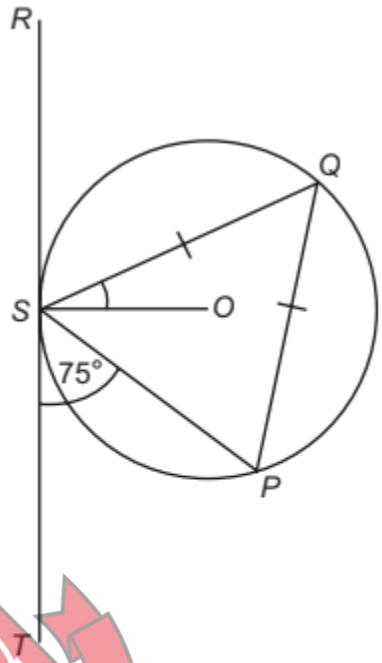
$$M_c = 290 - 200$$

$$= 90g$$

Answer is C.

ENGAA S1 2018 - Question 21

21



[diagram not to scale]

The line segment RT is a tangent at the point S to a circle with centre O

Q and P are points on the circumference of the circle such that $QS = QP$

Angle $PST = 75°$

What is the size of angle QSO?

A 15°

B 30°

C 37.5°

D 45°

E 52.5°

F 60°

G 67.5°

H 75°

ENGAA S1 2018 - Question 21 - Worked Solution

OS ⊥ ST as a tangent is always perpendicular to the radius.

$\text{OSP} = 90 - 75 = 15$

By alternate angle theorem:

PST = SQP

ΔSPQ is isosceles:

$$180 = 75 + 2(\theta + 15)$$

$$105 = 2(\theta + 15)$$

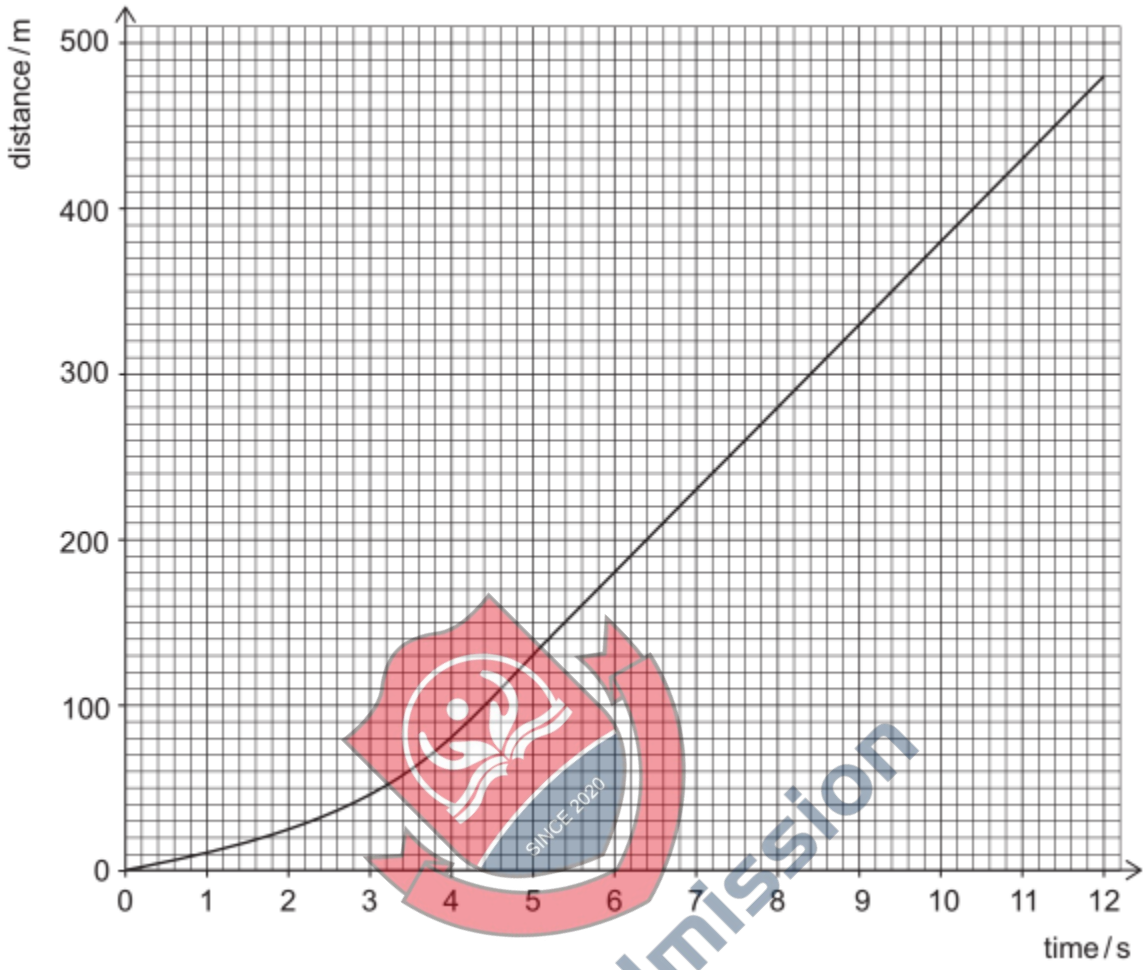
$$\theta = 37.5$$

Answer is C.

ENGAA S1 2018 - Question 22

22 A skydiver of weight 1000 N falls vertically.

The distance–time graph for the skydiver is shown below.



The air resistance F (in N) acting on the skydiver travelling at velocity v (in $m\,s^{-1}$) is given by the equation

$$F = kv^2$$

where k (in $N\,m^{-2}\,s^2$) is a constant.

What is the numerical value of k for the skydiver?

A 0.050

B 0.40

C 0.63

D 2.5

E 20

ENGAA S1 2018 - Question 22 - Worked Solution

When $w = kv^2$, skydiver reaches terminal velocity (constant)

$k = \frac{w}{v^2}$

From graph: v = gradient

$v = \frac{\Delta v}{\Delta t}$

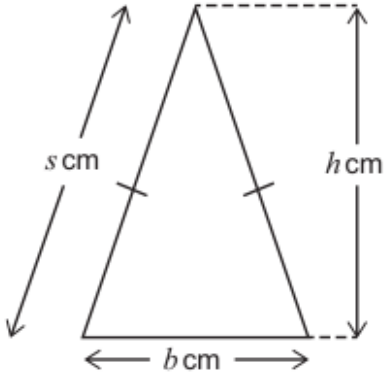
$v = \frac{50}{1} = 50\ ms^{-1}$

$k = \frac{w}{v^2} = \frac{1000}{50^2} = 0.4$

Answer is B.

ENGAA S1 2018 - Question 23

23



[diagram not to scale]

The vertical height h cm of an isosceles triangle is 3 cm longer than the base length of b cm.

The sloping side is of length s cm.

The area of the triangle is 14 cm^2.

There is one value of s which satisfies these conditions.

Within which range does this value of s lie?

A $5 < s < 6$

B $6 < s < 7$

C $7 < s < 8$

D $8 < s < 9$

E $9 < s < 10$

F $10 < s < 11$

ENGAA S1 2018 - Question 23 - Worked Solution

$$h = b + 3$$

$$A = 14$$

$$\frac{1}{2}bh = 14$$

$$(b + 3)b = 28$$

$$b^2 + 3b - 28 = 0$$

$$b = 4 \quad or \quad -7$$

$$b = 4 \quad as \quad b > 0$$

$$h = 7$$

using Pythagoras:

$$S^2 = \left(\frac{b}{2}\right)^2 + 4^2$$

$$S = \sqrt{4 + 49}$$

$$S = \sqrt{53}$$

$$\sqrt{49} = 7\,,\ \sqrt{64} = 8 \Rightarrow 7 < S < 8$$

Answer is C.

ENGAA S1 2018 - Question 24

24 A neutron is absorbed by a uranium-235 ($^{235}_{92}U$) nuclide.

The resulting nuclide undergoes fission to produce a bromine-88 ($^{88}_{35}Br$) nuclide, a lanthanum-145 nuclide and some neutrons.

The lanthanum-145 nuclide is radioactive and emits a beta (β^-) particle.

How many neutrons are emitted in the fission reaction and how many protons are there in the nuclide formed by the decay of lanthanum-145?

	neutrons	*protons*
A	2	55
B	2	56
C	2	57
D	2	58
E	3	55
F	3	56
G	3	57
H	3	58

ENGAA S1 2018 - Question 24 - Worked Solution

Write decay equation:

$$92235U + 01n \rightarrow 588Br + 57145La + x01n$$

$conserve\ mass\ number$:

$$236 = 88 + 145 + x$$

$$x = 3$$

3 Neutrons

Conserve mass number and proton number of 2nd decay:

$$57145La \rightarrow -10\beta + 58145X$$

58 Protons

Answer is H.

ENGAA S1 2018 - Question 25

25 The first five terms of a sequence in order are:

2 17 42 77 122

The n^{th} term of this sequence is $pn^2 + q$ where p and q are integers.

What is the value of $\frac{p-q}{p+q}$?

A $\frac{1}{4}$

B $\frac{1}{2}$

C 1

D $\frac{23}{17}$

E $\frac{13}{7}$

F 2

G 4

H 14

ENGAA S1 2018 - Question 25 - Worked Solution

$U_n = 5n^2 + cons^5$

U_n	2	17	42	77	122
S_n^2	5	20	45	80	125
	-3	-3	-3	-3	-3

$U_n = 5_n^2 - 3$

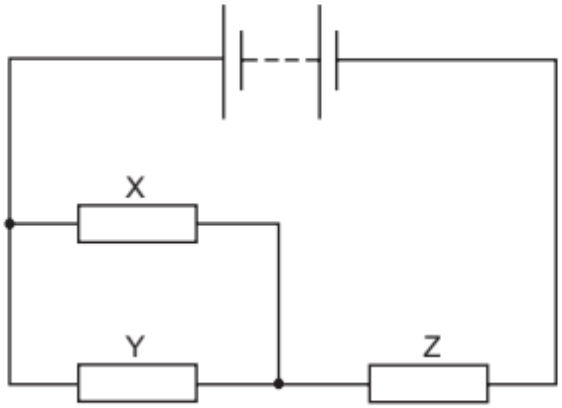
$p = 5\,,\ q = -3$

$\frac{p-q}{p+q} = \frac{8}{2} = 4$

Answer is G.

ENGAA S1 2018 - Question 26

26 The diagram shows a circuit containing a battery and three identical resistors X, Y and Z.



The total power supplied by the battery is 18 W.

What is the power dissipated as heat in resistor X?

A 1.5 W

B 2.0 W

C 3.0 W

D 4.5 W

E 6.0 W

F 8.0 W

G 12 W

ENGAA S1 2018 - Question 26 - Worked Solution

$$R_T = \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} + R$$

$$= \frac{R}{2} + R$$

$$= \frac{3R}{2}$$

$$P = I^2R$$

$$= \frac{I^2 \times 3R}{2} = 18$$

$$I^2R = 12$$

X and Y have same V and R

$$I_1 = I_2$$

$$I = I_1 + I_2$$

$$I_1 = \frac{I}{2}$$

So power dissipated in X is:

$$P_x = RI_1^{\ 2}$$

$$= \frac{I^2}{4}R$$

$$= \frac{12}{4}$$

$$= 3W$$

Answer is C.

ENGAA S1 2018 - Question 27

27 A bag contains 6 red and 6 green sweets. The sweets are identical apart from their colour.

A child takes a sweet at random from the bag.

If the sweet is red, the child stops taking sweets.

If the sweet is green, it is not replaced and the child takes another sweet.

This continues until a red sweet is taken at which point the child stops taking sweets.

What is the probability that the child takes **more** green sweets than red sweets?

A $\frac{3}{22}$

B $\frac{5}{22}$

C $\frac{3}{11}$

D $\frac{1}{2}$

E $\frac{8}{11}$

F $\frac{17}{22}$

ENGAA S1 2018 - Question 27 - Worked Solution

Child takes move green than red it takes at least two green in a row

$P(C > R) = P(at\ least\ two\ green\ in\ a\ row)$

$= 1 - (P(zero\ green) + P(1\ green))$

$= 1 - \left[\frac{6}{12} + \frac{6}{12} \times \frac{6}{11}\right]$

$= 1 - \frac{17}{22}$

$= \frac{5}{22}$

Answer is B.

ENGAA S1 2018 - Question 28

28 Three detectors X, Y and Z are separated by large distances.

Each of the detectors records a seismic wave from the same earthquake whose epicentre (source) is very close to the surface of the Earth.

The wave travels out from the epicentre at $4.0\,\text{km}\,\text{s}^{-1}$.

Detectors X and Y start to detect the wave at the same time, but detector Z starts to detect it one minute later.

Which of the following statements **must** be correct?

1 The epicentre is at the midpoint of the line XY.

2 Z is equidistant from X and Y.

3 Z is no more than 240 km away from X and from Y.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

ENGAA S1 2018 - Question 28 - Worked Solution

① Not necessary as:
② No, Z can be anywhere after X and Y.
③ No, as can be more than 240km
Answer is A.

ENGAA S1 2018 - Question 29

29 Curve *C* has equation $y = 9 - x^2$

Line *L* has equation $y = 5$

What is the area enclosed between *C* and *L*?

A $\frac{32}{3}$

B $\frac{62}{3}$

C $\frac{92}{3}$

D $\frac{122}{3}$

E $\frac{152}{3}$

ENGAA S1 2018 - Question 29 - Worked Solution

$$y = 9 - x^2 = (3 - x)(3 + x)$$

Intersect at:

$$5 = 9 - x^2$$

$$x^2 = 4$$

$$x = \pm 2$$

$$A = \int_{-2}^{2} 9 - x^2 dx - (4 \times 5)$$

$$= \left[9x - \frac{x^3}{3}\right]_{-2}^{2} - 20$$

$$= \frac{92}{3} - 20$$

$$= \frac{32}{3}$$

Answer is A.

ENGAA S1 2018 - Question 30

30 An aircraft moves from rest with uniform acceleration along a horizontal runway. After travelling 1600 m it reaches a speed of $80\,m\,s^{-1}$.

What is the acceleration of the aircraft?

A $0.025\,m\,s^{-2}$

B $0.050\,m\,s^{-2}$

C $0.10\,m\,s^{-2}$

D $0.50\,m\,s^{-2}$

E $2.0\,m\,s^{-2}$

F $4.0\,m\,s^{-2}$

G $10\,m\,s^{-2}$

H $20\,m\,s^{-2}$

ENGAA S1 2018 - Question 30 - Worked Solution

S = 1600

U = 0

V = 80

A = a

$$V^2 = U^2 + 2aS$$

$$a = \frac{V^2 - U^2}{2s}$$

$$= \frac{80^2 - 0^2}{3200}$$

$$= 2\ ms^{-2}$$

Answer is E.

ENGAA S1 2018 - Question 31

31 How many solutions of the equation $2\sin^3\theta = \sin\theta$ lie in the interval $-\frac{\pi}{2} \leq \theta \leq \pi$?

A 2

B 3

C 4

D 5

E 6

F 7

ENGAA S1 2018 - Question 31 - Worked Solution

$$\sin sin\, 6\left[2sin^2\theta - 1\right] = 0$$

$$\sin sin\, \theta = 0$$

$$\theta = 0, \pi$$

$$\sin sin\, \theta = \pm\frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}, \left(-\pi + \frac{\pi}{4}\right), -\frac{\pi}{4}, \left(-\pi + \frac{\pi}{4}\right)$$

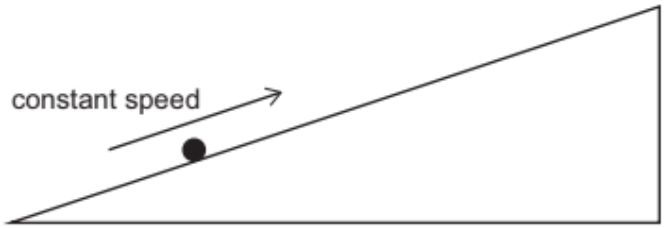
$$= \frac{4}{\pi}, \frac{3\pi}{4}, -\frac{4}{\pi}, -\frac{3\pi}{4}$$

5 Solutions

Answer is D.

ENGAA S1 2018 - Question 32

32 The diagram represents a mass that is moving in a straight line at constant speed up a slope of constant gradient.



Which statement about the forces acting on the mass **must** be correct?

A All the forces acting on the mass are equal in magnitude.

B Only three forces act on the mass.

C The force of friction on the mass is equal to the driving force.

D The weight of the mass acts in the opposite direction to the contact force.

E There is no air resistance acting on the mass.

F There is no resultant force acting on the mass.

ENGAA S1 2018 - Question 32 - Worked Solution

F = ma = 0 ⇒ resultant force is 0

B , C, D ⟶ could be true, but don't have to be

Answer is F.

ENGAA S1 2018 - Question 33

33 The line $y = x + k$, where k is a constant, is a tangent to the curve $y = 3x^2 - 2x + 1$

What is the value of k?

A -2

B -1

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{1}{2}$

F $\frac{3}{4}$

G 1

H 2

ENGAA S1 2018 - Question 33 - Worked Solution

$$① \; y = x + k$$

$$② \; y = 3x^2 - 2x + 1$$

$$y = 3x^2 - 2x + 1$$

$$\frac{dy}{dx} = 6x - 2$$

Gradient of y-axis is 1

So when tangent

$$6x - 2 = 1$$

$$x = \frac{1}{2}$$

$$y = \frac{3}{4} \quad \left(sub\ x = \frac{1}{2}\ into\ ②\right)$$

$$= \frac{1}{2} + k \quad \left(sub\ x = \frac{1}{2}\ into\ ①\right)$$

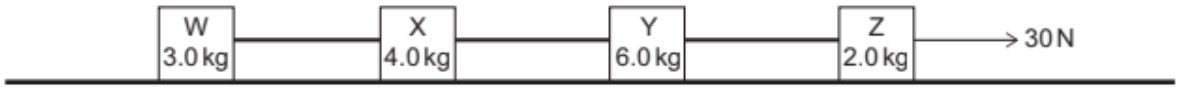
$$k = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$$

Answer is C.

ENGAA S1 2018 - Question 34

34 The diagram shows four objects W, X, Y and Z, of masses 3.0 kg, 4.0 kg, 6.0 kg and 2.0 kg respectively, connected by light, inextensible rods.

The objects are pulled along a smooth, horizontal surface by a constant force of 30 N in the direction indicated.



What is the tension in the rod connecting X and Y?

A 8.0 N

B 10 N

C 12 N

D 14 N

E 16 N

ENGAA S1 2018 - Question 34 - Worked Solution

Rod is inextensible ⇒ all blocks have same acceleration : a

F = ma

$$30 = 15a$$

$$a = 2ms^{-1}$$

Now for Z

$$30 - T_1 = ma = 2\times2 = 4$$

$$T_1 = 26N$$

For Y

$$T_1 - T_2 = ma$$

$$26 - T_2 = 6\times2$$

$$T_2 = 14N$$

Answer is D.

ENGAA S1 2018 - Question 35

35 A sector S of a circle has area $10\pi\,\text{cm}^2$.

The angle of sector S is increased by $\frac{\pi}{20}$ radians to form sector T.

The total area of sector T is $\frac{25}{2}\pi\,\text{cm}^2$.

What is the total arc length, in cm, of sector T?

A $\frac{9\sqrt{5}}{10}\pi$

B $\frac{5\sqrt{2}}{4}\pi$

C 2π

D $\frac{5}{2}\pi$

ENGAA S1 2018 - Question 35 - Worked Solution

r = radius of circle

$$A_S = \frac{1}{2}r^2\theta = 10\pi$$

$$A_T = \frac{1}{2}r^2\left(\theta + \frac{\pi}{20}\right) = \frac{25}{2}\pi \quad ①$$

$$A_S + r^2\frac{\pi}{40} = \frac{25}{2}\pi$$

$$10\pi + r^2\frac{\pi}{40} = \frac{25}{2}\pi$$

$$r^2\frac{\pi}{40} = \frac{5}{2}\pi$$

$$r^2 = 100$$

$$r = 10$$

$$S_T = r\left(\theta + \frac{\pi}{20}\right)$$

Sub into ①

$$\frac{1}{2}rS_t = \frac{25}{2}\pi$$

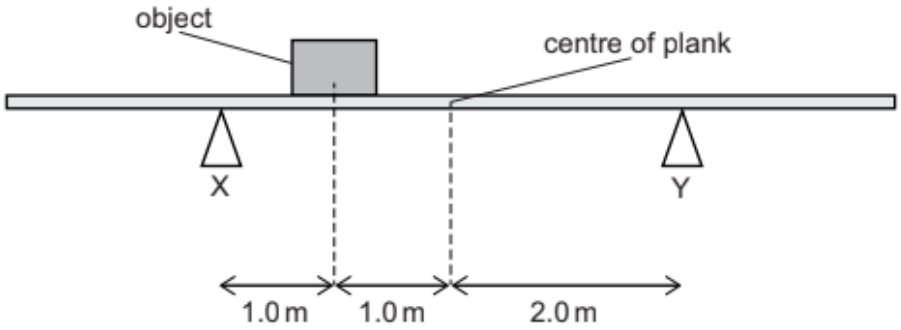
$$\frac{10}{2}S_T = \frac{25}{2}\pi$$

$$S_T = \frac{5}{2}\pi$$

Answer is D.

ENGAA S1 2018 - Question 36

36 An object of mass 40 kg is placed on a uniform, horizontal plank of mass 10 kg between two supports X and Y as shown in the diagram.



What is the contact force at X?

(gravitational field strength = 10 N kg^{-1})

A 15 N

B 35 N

C 150 N

D 250 N

E 300 N

F 350 N

G 375 N

ENGAA S1 2018 - Question 36 - Worked Solution

Reduce moments about Y:

$$R(1 + 1 + 2) - 40g(1 + 2) - 10g(2) = 0$$

$$4R - 140g = 0$$

$$R = 35g$$

$$R = 350\ N$$

Answer is F.

ENGAA S1 2018 - Question 37

37 In a particular arithmetic progression:

- the 13^{th} term is six times the 1^{st} term
- the 11^{th} term is 1 less than twice the 5^{th} term

What is the 3^{rd} term of the progression?

A -14.5

B -11

C $\frac{29}{19}$

D 3.5

E 11

F 14.5

ENGAA S1 2018 - Question 37 - Worked Solution

For arithmetic progression:

$$U_n = a + (n-1)d$$

$$U_{13} = a + 12d = 6U_1 = 6a$$

$$a + 12d = 6a$$

$$12d - 5a = 0 \quad ①$$

$$U_{11} = 2U_5 - 1$$

$$a + 10d = 2a + 8d - 1$$

$$2d - a = -1 \quad ②$$

① - 6②

$$-5a + 6a = 6$$

$$a = 6$$

$$2d - 6 = -1$$

$$d = \frac{5}{2}$$

$$U_3 = a + 2d$$

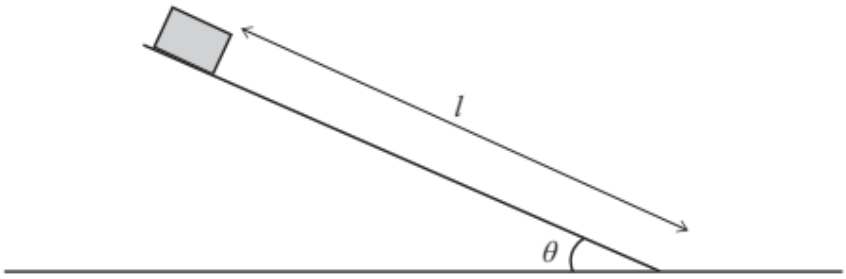
$$= 6 + 5$$

$$= 11$$

Answer is E.

ENGAA S1 2018 - Question 38

38 A block of mass m slides a distance l down a slope that is inclined at angle θ to the horizontal, as shown:



The block experiences a friction force of $kW\sin\theta$, where W is the weight of the block and k is a constant.

The block starts from rest at the top of the slope and slides down a distance l to the bottom, where its potential energy is zero.

What fraction of the initial potential energy at the top has become kinetic energy as the block reaches the bottom?

A k

B $1 - k$

C $k\sin\theta$

D $1 - k\sin\theta$

E $k\tan\theta$

F $1 - k\tan\theta$

ENGAA S1 2018 - Question 38 - Worked Solution

$$\Delta Ep = mgh$$

$$= WL\sin sin\,\theta$$

Work done by friction:

W=FL

$$= kW\sin sin\,\theta\, L$$

$$\Delta Ex = \Delta Ep - W = WL\sin sin\,\theta - kW\sin sin\,\theta$$

$$= (1 - k)WL \sin sin\, \theta$$

$$\frac{\Delta Ex}{\Delta Ep} = \frac{(1-k)WL\text{sinsin}\, \theta}{WL\text{sinsin}\, \theta} = 1 - k$$

Answer is B.

ENGAA S1 2018 - Question 39

39 The first three terms of a geometric progression, whose terms are all greater than zero, are $(p - 2)$, $(2p + 2)$ and $(5p + 14)$

What is the fifth term of the progression?

A 324

B 486

C 1250

D 1458

E 3888

ENGAA S1 2018 - Question 39 - Worked Solution

Geometric progression:

$$U_n = ar^{n-1}$$

$$U_1 = a = (p - 2)$$

$$\frac{U_2}{U_1} = \frac{U_3}{U_2} = r$$

$$\frac{2p+2}{p-2} = \frac{5p+14}{2p+2}$$

$$(2p + 2)^2 = (5p + 14)(p - 2)$$

$$4p^2 + 4 + 8p = 5p^2 + 14p - 10p - 28$$

$$p^2 - 4p - 32 = 0$$

$$p = 8 \, or \, p = -4$$

$$U_1 = p - 2$$

$$= 6 \ or \ -6 \quad (but \ U_n > 0)$$

$$p = 8$$

$$r = \frac{2p+12}{p-2}$$

$$r = 3$$

$$U_5 = ar^4$$

$$= (p - 2) \times 3^4$$

$$= 6 \times 3^4$$

$$= 486$$

Answer is B.

ENGAA S1 2018 - Question 40

40 An object X of mass 2.0 kg is initially moving at a speed of 4.5 m s^{-1} on a smooth, horizontal surface.

A 5.0 N force is applied to X in the direction of its motion for 3.0 seconds.

A short time later it collides head on with, and sticks to, a stationary object Y of mass 3.0 kg.

What is the speed of X and Y as they move off together after the collision?

A 1.8 m s^{-1}

B 3.0 m s^{-1}

C 3.6 m s^{-1}

D 4.8 m s^{-1}

E 5.4 m s^{-1}

ENGAA S1 2018 - Question 40 - Worked Solution

$$\Delta P = F\Delta t\ ,\ U = 4.5ms^{-1}$$

$$mv - mu = F\Delta t$$

$$2\left(V_x - 4.5\right) = 5\times3$$

$$V_x = 12ms^{-1}$$

Conservation of momentum:

$$M_x V_x = \left(M_x + M_t\right)V_{xt}$$

$$V_{xy} = \frac{2\times V_x}{(2+3)} = \frac{2}{5}\times12 = \times4.8ms^{-1}$$

Answer is D.

ENGAA S1 2018 - Question 41

41 Evaluate

$$\log_2\left(\frac{5}{4}\right) + \log_2\left(\frac{6}{5}\right) + \log_2\left(\frac{7}{6}\right) + \cdots + \log_2\left(\frac{64}{63}\right)$$

A -2

B 3

C 4

D 6

E $\log_2(3!)$

F $\log_2 60$

ENGAA S1 2018 - Question 41 - Worked Solution

$$= \left[\frac{5\times6\times7\times\ldots\times64}{4\times5\times6\times\ldots\times63}\right]$$

$$= \left[\frac{64}{4}\right]$$

$$= (16)$$

$$= \left(2^4\right)$$

$$= 4$$

Answer is C.

ENGAA S1 2018 - Question 42

42 A ball of mass 0.20 kg is thrown vertically downwards at an initial speed of 4.0 m s^{-1} and travels a distance of 0.45 m to the ground.

The ball hits the ground, and rebounds with an initial speed of 2.0 m s^{-1}.

How much energy does the ball lose in the bounce?

(gravitational field strength = 10 N kg^{-1}; air resistance can be ignored)

A 0.10 J

B 0.40 J

C 0.50 J

D 0.90 J

E 1.2 J

F 1.6 J

G 2.1 J

ENGAA S1 2018 - Question 42 - Worked Solution

$$\Delta Ep = mgh$$

$$= 0.2 \times 10 \times 0.45 = 0.95$$

$$= \Delta Wk \ \ by\ conservation\ of\ energy$$

$$initial\ \ Ek = \frac{1}{2} m \times 4^2 + 0.9$$

$$= \frac{1}{2} \times 0.2 \times 16 + 0.9 = 2.55$$

$$kinetic\ energy\ lost = 2.5 - final\ Ek$$

$$= 2.5 - \frac{1}{2} m \times 2^2$$

$$= 2.5 - 0.4$$

$$= 2.15$$

Answer is G.

ENGAA S1 2018 - Question 43

43 Circle C has equation $(x + 3)^2 + (y - 2)^2 = 5$

The length of the tangent from the circle C to the point P is $5\sqrt{3}$

What is the shortest distance from P to C?

A $5\sqrt{3}$

B $5\sqrt{3} + \sqrt{5}$

C $3\sqrt{5}$

D 5

E 10

ENGAA S1 2018 - Question 43 - Worked Solution

$Centre: (-3, 2)$

$Radius: \sqrt{5}$

$Radius \perp tangent$

Pythagoras:

$$(S + r)^2 = \left(5\sqrt{3}\right)^2 + r^2$$

$$\left(S + \sqrt{5}\right) = \sqrt{75 + 5}$$

$$S = \sqrt{80} - \sqrt{5}$$

$$= 4\sqrt{5} - \sqrt{5}$$

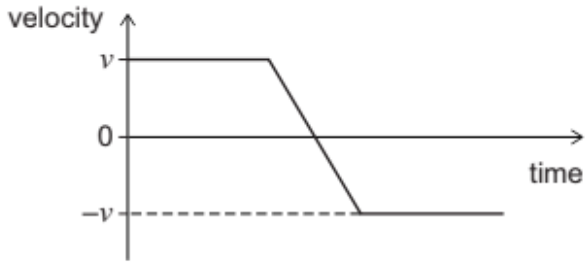
$$= 3\sqrt{5}$$

Answer is C.

ENGAA S1 2018 - Question 44

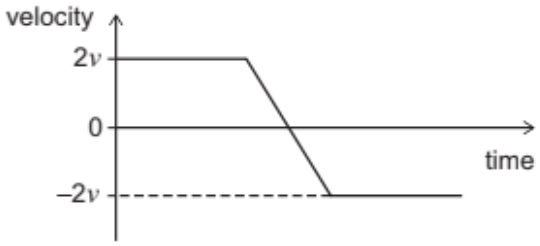
44 Two solid spheres X and Y have masses m and $2m$ respectively. They travel in opposite directions towards each other along the same line with speeds v and $2v$ respectively and collide head on.

The graph shows the variation of velocity with time for sphere X before, during, and after the collision.

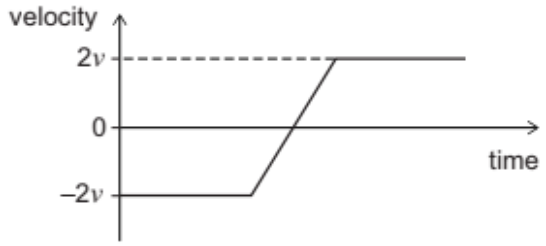


Which sketch shows the variation of velocity with time for sphere Y?

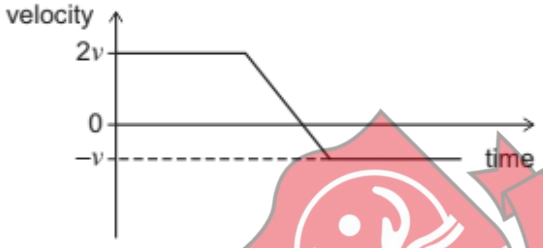
A



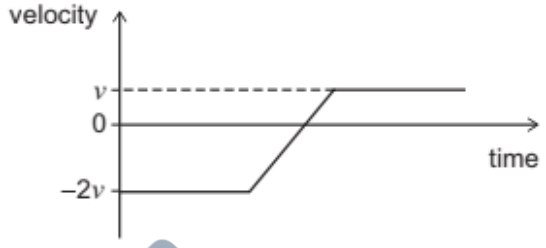
B



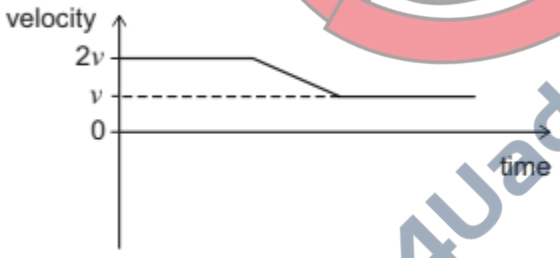
C



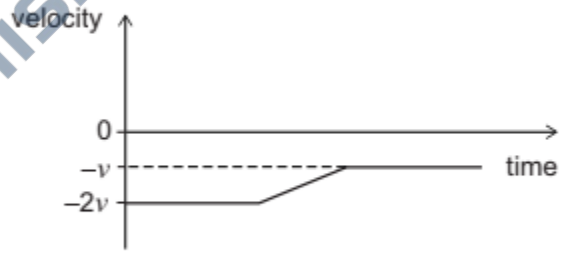
D



E



F



ENGAA S1 2018 - Question 44 - Worked Solution

So, at t = 0, Y has velocity -2v

After collision:

Vx = -v

(conserve momentum)

$$mv - 4vm = (-v)m + 2mv_t$$

$$2v_t = (1 - 4 + 1)v = -2v$$

$$v_t = -v$$

Answer is F.

ENGAA S1 2018 - Question 45

45 The points $A\ (-3, 2)$, $B\ (1, 3)$ and $C\ (-1, u)$ are such that the distances AC and AB are related by:

$$AC = 2AB$$

What are the possible values of u?

A 2 and −6

B −2 and 6

C 6 and −10

D −6 and 10

E $2+2\sqrt{13}$ and $2-2\sqrt{13}$

F $-3+2\sqrt{13}$ and $-3-2\sqrt{13}$

ENGAA S1 2018 - Question 45 - Worked Solution

$$AB = \sqrt{(1-(-3))^2 + (3-2)^2}$$

$$= \sqrt{17}$$

$$AC = \sqrt{(-1+3)^2 + (u-2)^2} = \sqrt{4 + (u-2)^2}$$

$$= 2\sqrt{17}$$

$$4 + (u-2)^2 = 4 \times 17$$

$$(u-2)^2 = 64$$

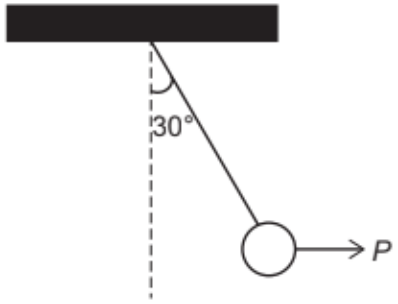
$$u - 2 = \pm 8$$

$$u = 10 \ or \ -6$$

Answer is D.

ENGAA S1 2018 - Question 46

46 A metal ball suspended from a steel cable is held at rest by a horizontal force P. The cable makes an angle of 30° to the vertical as shown in the diagram. The cable exerts a force T on the ball.



What is the magnitude of P?

A $\frac{T}{2}$

B T

C $2T$

D $\frac{T}{\sqrt{2}}$

E $\frac{T}{\sqrt{3}}$

F $\frac{2T}{\sqrt{3}}$

G $\frac{\sqrt{3}T}{2}$

ENGAA S1 2018 - Question 46 - Worked Solution

Resolve horizontally:

$$T \sin sin\, 30 = P$$

$$\frac{T}{2} = P$$

Answer is A.

ENGAA S1 2018 - Question 47

47 What is the coefficient of x^3 in the expansion of $(1 - 2x)^5(1 + 2x)^5$?

A −6400

B −640

C −80

D 0

E 80

F 800

G 960

ENGAA S1 2018 - Question 47 - Worked Solution

$$(1 - 2x)^5(1 + 2x)^5$$

$$= \left[1 - (2x)\times 5\times 1^3 \times (2x)^2\times 10\times 1^4 - (2x)^3\times 10\times 1^5 + \ldots\right]\left[1 + (2x)\times 5 + (2x^2)\times 10 + (2x)^3\times 10 + (2x)^3\times 10 + \ldots\right]$$

$$= \left[1 - 10x + 40x^2 - 80x^3 + \ldots\right]\left[1 + 10x + 40x^2 + 80x^3 + \ldots\right]$$

Find coefficient of x^3:

$$(1\times 80) - (10\times 4) + (40\times 10) - (80\times 1)$$

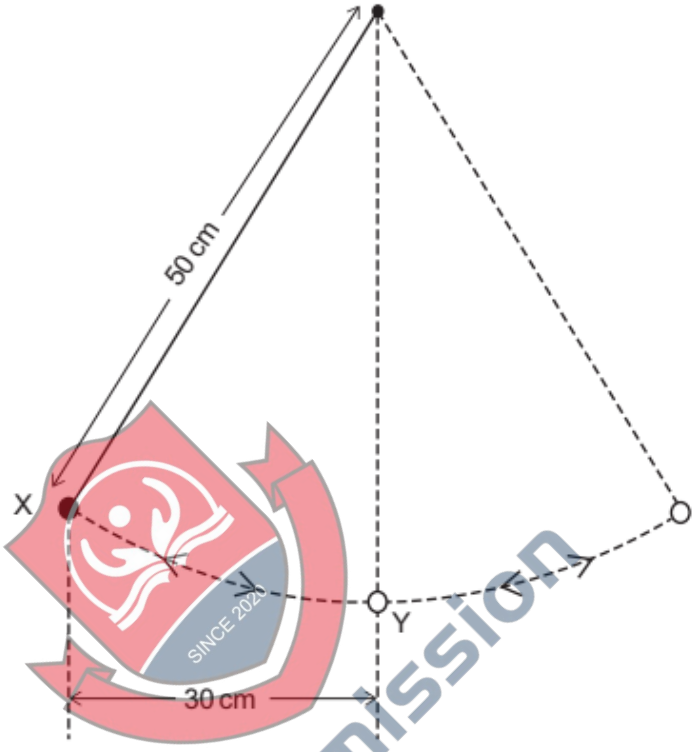
$$= 0$$

Answer is D.

ENGAA S1 2018 - Question 48

48 A pendulum bob of mass 10 g is suspended by a light, inextensible string of length 50 cm.

The bob is released from rest at position X.



What is the speed of the bob as it passes through position Y?

(gravitational field strength $g = 10\,\text{N kg}^{-1}$; assume that resistive forces are negligible)

A $\sqrt{2}\,\text{m s}^{-1}$

B $\sqrt{4}\,\text{m s}^{-1}$

C $\sqrt{6}\,\text{m s}^{-1}$

D $\sqrt{8}\,\text{m s}^{-1}$

E $\sqrt{10}\,\text{m s}^{-1}$

ENGAA S1 2018 - Question 48 - Worked Solution

$$L = \sqrt{50^2 - 30^2} = 40$$

$$h = 50 - L = 10cm = 0.1m$$

$$\Delta Ep = Ek\ at\ Y$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$= \sqrt{2 \times 10 \times 0.1}$$

$$= \sqrt{2}\ ms^{-1}$$

Answer is A.

ENGAA S1 2018 - Question 49

49 Given that

$$\int_0^2 x^m \mathrm{d}x = \frac{16\sqrt{2}}{7}$$

and

$$\int_0^2 x^{m+1} \mathrm{d}x = \frac{32\sqrt{2}}{9}$$

what is the value of m?

A $-\frac{11}{2}$

B $-\frac{9}{2}$

C $-\frac{22}{29}$

D $\frac{7}{22}$

E $\frac{5}{2}$

F $\frac{7}{2}$

ENGAA S1 2018 - Question 49 - Worked Solution

$$\int_0^2 x^m dx = \frac{16\sqrt{2}}{7}$$

$$\left[\frac{x^{m+1}}{m+1}\right]_0^2 = \frac{16\sqrt{2}}{7}$$

$$\frac{2^{m+1}}{m+1} = \frac{16\sqrt{2}}{7} \quad (1)$$

$$\int_0^2 x^{m+1} dx = \frac{32\sqrt{2}}{9}$$

$$\left[\frac{x^{m+2}}{m+2}\right]_0^2 = \frac{32\sqrt{2}}{9}$$

$$\frac{2^{m+2}}{m+2} = \frac{32\sqrt{2}}{9} \quad ②$$

② (①)

$$2\times\frac{m+1}{m+2} = \frac{32}{9} \times \frac{7}{16}$$

$$(m + 1) = \frac{1}{2} \times \frac{14}{9}(m + 2)$$

$$9m + 9 = 7m + 14$$

$$2m = 5$$

$$m = 5/2$$

Answer is E.

ENGAA S1 2018 - Question 50

50 An object of mass m is initially moving at constant speed u to the right. It collides with a stationary object of greater mass M and bounces back in the opposite direction at speed v.

What is the speed of the greater mass immediately after the collision?

A $\frac{mu}{M}$

B $\frac{Mu}{m}$

C $\frac{m(v-u)}{M}$

D $\frac{M(v-u)}{m}$

E $\frac{m(v+u)}{M}$

F $\frac{M(v+u)}{m}$

ENGAA S1 2018 - Question 50 - Worked Solution

Conserve momentum:

$$MU = -VM + MV_m$$

$$\frac{M(U+V)}{M} = V_M$$

Answer is E.

ENGAA S1 2018 - Question 51

51 The two functions f and g satisfy

$$f'(x) = ax + g(x)$$

where a is a constant.

Given that

$$\int_2^4 g(x)\,dx = 12$$

and

$$f(4) = 18 + f(2)$$

what is the value of a?

A 1

B 3

C 5

D 6

E 15

ENGAA S1 2018 - Question 51 - Worked Solution

$$f'(x) = ax + g(x)$$

$$\frac{df}{dx} = ax + g(x)$$

$$\int_2^4 g(x)dx = 12$$

$$\int_2^4 \frac{dt}{dx}dx = \left[\frac{ax^2}{2}\right]_2^4 + \int_2^4 g(x)dx$$

$$\int_2^4 dt = \frac{16a}{2} - \frac{4a}{2} + 12 \quad ①$$

$$f(4) - f(2) = 6a + 12$$

$$f(4) - f(2) = 18 \quad ②$$

$$8 = 6a + 12$$

$$6 = 6a$$

$$a = 1$$

Answer is A.

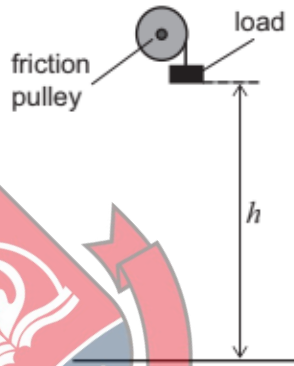
ENGAA S1 2018 - Question 52

52 A load drops from rest through a vertical height h to the ground.

A light cable attached to the load passes over a friction pulley that provides a braking force during the fall.

As the load falls through height h, 50% of the gravitational potential energy lost is transferred into thermal energy.

The load reaches a final speed of $10\,m\,s^{-1}$.



What is the vertical height h?

(gravitational field strength $g = 10\,N\,kg^{-1}$; air resistance can be ignored)

A 0.50 m

B 1.0 m

C 2.0 m

D 2.5 m

E 5.0 m

F 10 m

G 20 m

ENGAA S1 2018 - Question 52 - Worked Solution

$$\Delta Ep = mgh = 10mh$$

$$\Delta Ek = \frac{1}{2}mv^2 = \frac{1}{2}\times m\ 10^2$$

$$= 50m$$

From question:

$$\Delta Ek = \frac{1}{2}\Delta Ep$$

$$50m = \frac{1}{2}\times 10mh$$

$$50m = 5mh$$

$$10 = h$$

Answer is F.

ENGAA S1 2018 - Question 53

53 The dimensions of a solid cuboid, in cm, are x, $2x$ and y

The volume of the cuboid is 576 cm^3.

At this volume, the surface area of the cuboid has its maximum value.

What is the area, in cm^2, of the face that has the largest area?

A $2(288)^{\frac{2}{3}}$

B 72

C 96

D 432

E $4(144)^{\frac{2}{3}}$

ENGAA S1 2018 - Question 53 - Worked Solution

$$V = 2x^2y = 576$$

$$xy = \frac{288}{x} \quad ①$$

$$A = 2\left(2x^2 + xy + 2xy\right)$$

$$= 4x^2 + 6xy \quad ②$$

①→②

$$A = 4x^2 + \frac{1728}{x}$$

At maximum surface area

$$\frac{dA}{dx} = 0$$

$$8x - \frac{1728}{x^2} = 0$$

$$x^3 = 216$$

$$x = 6$$

$$②\Rightarrow 6y = \frac{288}{6} = 48$$

$$y = 8$$

Areas of faces:

$$2x^2 = 72$$

$$2xy = 96$$

$$xy = 48$$

So area of largest surface $= 96cm^2$

Answer is C.

ENGAA S1 2018 - Question 54

54 An object is thrown vertically upwards from ground level with an initial velocity of 40 m s^{-1}.

2.0 seconds later another object is released from a height above the ground and falls vertically from rest.

Both of the objects hit the ground at the same time.

From what height above the ground was the second object released?

(gravitational field strength g = 10 N kg^{-1}; air resistance can be ignored)

A 80 m

B 180 m

C 320 m

D 500 m

E 900 m

ENGAA S1 2018 - Question 54 - Worked Solution

From ①:

$$S = ut + \frac{1}{2}at^2 = 0$$

$$40t - \frac{1}{2}(10)t^2 = 0$$

$$40t - 5t^2 = 0$$

$$t = 0 \ or \ t = 8$$

So ② hits ground in 6s

For second mass:

S = -h

U = 0

V =

A = -10

T = 6

$$S = ut + \frac{1}{2}at^2$$

$$-h = -\frac{10}{2}6^2$$

$$h = 180m$$

Answer is B

ENGAA S2 2018 - Question 1

1 A man is cycling along a straight horizontal road at a constant speed of $9.00\,m\,s^{-1}$.

He passes a boy who is cycling at $5.00\,m\,s^{-1}$ in the same direction.

When the man is level with the boy, the boy begins to accelerate at a constant rate of $0.800\,m\,s^{-2}$.

The boy maintains this constant acceleration and the man continues at constant speed until the boy passes the man.

What is the time interval between the two instances when the man and the boy are level?

A 5.00 s

B 10.0 s

C 22.5 s

D 35.0 s

E 90.0 s

ENGAA S2 2018 - Question 1 - Worked Solution

S_M = distance travelled by man

S_B = distance travelled by boy

Meet when $S_M = S_B$

At t = 0 , B passes M initially

At t = t , B passes M again

$S_M = 9t$

Using SUVAT:

$$S_B = 5t + \frac{1}{2} \times 0.8t^2$$

$$= 5t + 0.4t^2$$

$S_M = S_B$

$$9t = 5t + 0.4t^2$$

$$0 = 4t(0.1t - 1)$$

$$t = 0 \ or \ t = \frac{1}{0.1} = 10$$

t = 10 when passes again

Answer is B.

ENGAA S2 2018 - Question 2

2 Two liquids P and Q can be mixed together in any proportion.

The density of liquid P is ρ_P and the density of liquid Q is ρ_Q.

A volume V_P of liquid P and a volume V_Q of liquid Q are mixed together to produce a volume that is equal to $V_P + V_Q$.

What is the density of the mixture?

A $\dfrac{\rho_P + \rho_Q}{2}$

B $\dfrac{\rho_P V_P + \rho_Q V_Q}{V_P + V_Q}$

C $\left(\dfrac{\rho_P}{V_P} + \dfrac{\rho_Q}{V_Q}\right)(V_P + V_Q)$

D $\dfrac{\rho_P V_Q + \rho_Q V_P}{V_P + V_Q}$

E $\dfrac{\left(\dfrac{\rho_P}{V_P} + \dfrac{\rho_Q}{V_Q}\right)}{V_P + V_Q}$

F $\left(\dfrac{\rho_P}{V_Q} + \dfrac{\rho_Q}{V_P}\right)(V_P + V_Q)$

ENGAA S2 2018 - Question 2 - Worked Solution

$$\rho = \frac{M}{V}$$

$$total\ mass = m$$

$$= M_Q + M_P$$

$$= \rho_P V_P + \rho_Q V_Q$$

$$total\ volume = v$$

$$= V_P + V_Q$$

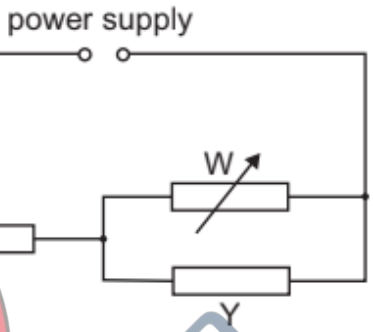
$$total\ density = \rho$$

$$= \frac{\rho_P V_P + \rho_Q V_Q}{V_P + V_Q}$$

Answer is B.

ENGAA S2 2018 - Question 3

3 A circuit contains two fixed resistors, X and Y, and a variable resistor W. The power supply has no internal resistance.



The resistance of W increases.

What happens to the power dissipated in X and in Y?

	power dissipated in X	*power dissipated in Y*
A	decreases	decreases
B	decreases	stays constant
C	decreases	increases
D	increases	decreases
E	increases	stays constant
F	increases	increases

ENGAA S2 2018 - Question 3 - Worked Solution

Total resistance for a parallel combination:

$$R_T = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

So as W increases, total resistance of combination of W and Y increases.
P.d across then increases (potential divider →potential is split in the same ratio as the resistance).
So P.d across X decreases.
$P = \frac{V^2}{R}$, so power dissipation across X decreases
As the potential across Y increases, power dissipation in Y increases, by same argument as above.
Answer is C.

ENGAA S2 2018 - Question 4

4 The diagram shows two bar magnets, X and Y, held at rest a short distance apart from one another on a smooth horizontal surface. They are aligned as shown in the diagram and both are released at the same time. They move towards each other, collide and coalesce. Just before they collide X has a velocity v to the right.

The mass of X is double the mass of Y.

Which row in the table gives the magnitude and direction of the velocity of the two magnets after the collision?

	magnitude of velocity	*direction of velocity*
A	0	not applicable
B	$\frac{1}{3}v$	to the left
C	$\frac{2}{3}v$	to the left
D	$\frac{4}{3}v$	to the left
E	$\frac{1}{3}v$	to the right
F	$\frac{2}{3}v$	to the right
G	$\frac{4}{3}v$	to the right

ENGAA S2 2018 - Question 4 - Worked Solution

Let Y has mass m, so X has mass 2m

Equal but opposite force acts on both magnets, so momentum is conserved, as no external forces act.

Conserve momentum:

$3mV_{final} = 2m\times0 + m\times0$

$V_{final} = 0$

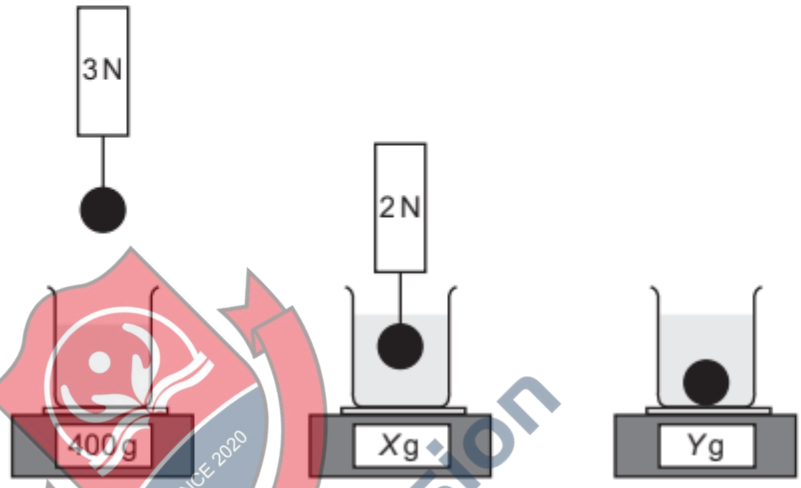
Answer is A.

ENGAA S2 2018 - Question 5

5 A stone is suspended from a newtonmeter and the meter reads 3 N. A beaker of water is placed on a top pan balance and the top pan balance reads 400 g.

The stone is lowered into the water so that it is at rest and fully submerged, but not touching the bottom of the beaker. The reading on the newtonmeter is now 2 N and the top pan balance reads *X* g.

The stone is detached from the newtonmeter and allowed to rest under water on the base of the beaker. The top pan balance now reads *Y* g.



What are the values of *X* and *Y*?

(gravitational field strength = 10 N kg^{-1})

	X	*Y*
A	400	500
B	400	600
C	400	700
D	500	500
E	500	600
F	500	700

ENGAA S2 2018 - Question 5 - Worked Solution

Weight at ball doesn't change ⇒Buogancy = 1N

$$X = 400g + \frac{1}{10}kg$$

$$= 500g$$

$$Y = 400g + mass\ of\ ball$$

$$= 400g + \frac{3}{10}$$

$$= 700g$$

Answer is F.

ENGAA S2 2018 - Question 6

6 Diagram 1 represents a stationary wave produced by sound in an open-ended tube of length 0.50 m containing a liquid. The speed of the wave in the liquid is 1000 m s^{-1}.

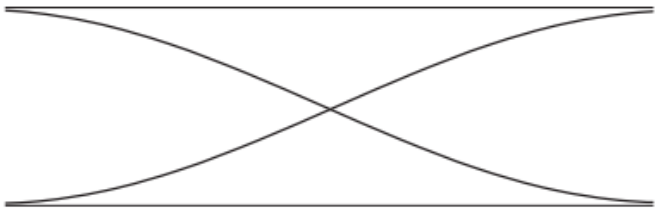


diagram 1

Diagram 2 is a displacement–time graph representing a progressive sound wave with the same frequency in the same liquid.

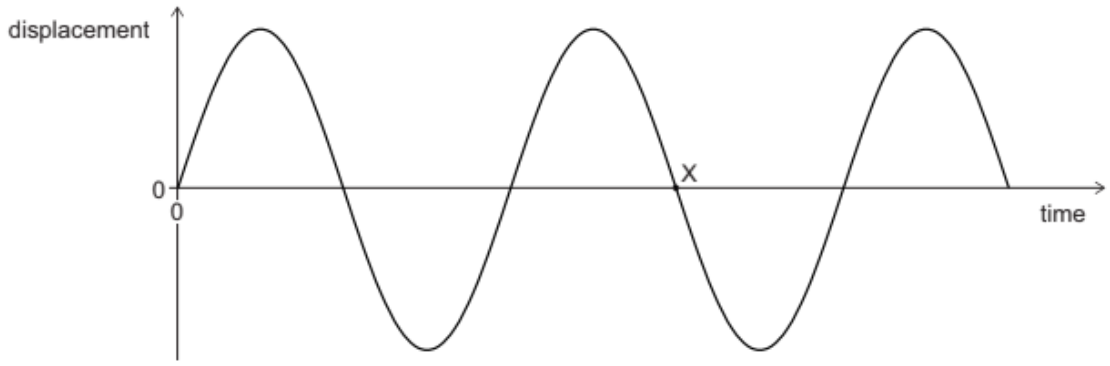


diagram 2

What is the value of the time at point X on the graph?

(End effects of the stationary wave can be ignored.)

A 2.5×10^{-4} s

B 3.75×10^{-4} s

C 5.0×10^{-4} s

D 7.5×10^{-4} s

E 1.0×10^{-3} s

F 1.5×10^{-3} s

ENGAA S2 2018 - Question 6 - Worked Solution

Shows half a cycle of the wave, so

$$\frac{\lambda}{2} = 0.5m$$

$$\lambda = 1m$$

Wave equation:

$$V = \lambda f$$

$$f = \frac{V}{\lambda} = \frac{1000}{1} = 1000Hz$$

$$T = \frac{1}{f} = 1\times10^{-3}$$

$$X\ is\ at\quad 1.5T \Rightarrow t = 1.5\times10^{-3}$$

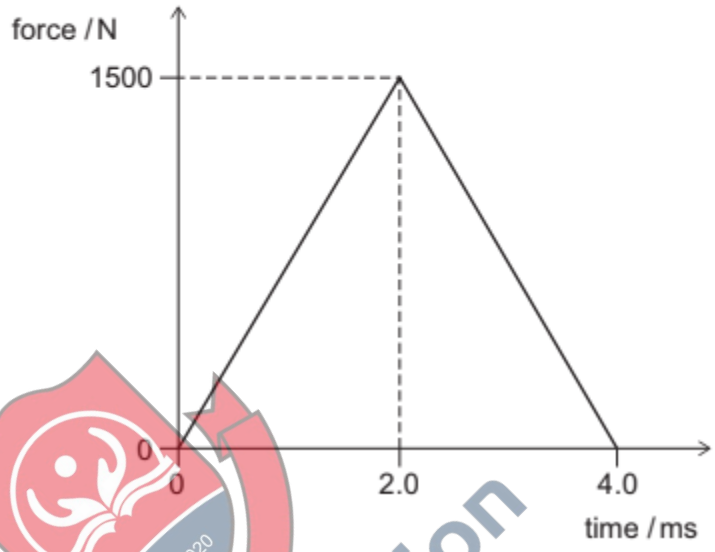
$$= 1.5\times10^{-3}s$$

Answer is F.

ENGAA S2 2018 - Question 7

7 A tennis ball travelling at a speed of $30\,m\,s^{-1}$ hits a racket elastically with a kinetic energy of 27 J.

The racket applies a variable force to the tennis ball for a time of 4.0 ms as shown.



The ball moves away in the opposite direction to its initial motion.

With what speed does the ball leave the racket?

(Assume that resistive forces are negligible.)

A $20\,m\,s^{-1}$

B $28\,m\,s^{-1}$

C $32\,m\,s^{-1}$

D $50\,m\,s^{-1}$

E $70\,m\,s^{-1}$

F $80\,m\,s^{-1}$

G $100\,m\,s^{-1}$

H $130\,m\,s^{-1}$

ENGAA S2 2018 - Question 7 - Worked Solution

Find mass of ball:

$$\frac{1}{2}mv^2 = 27J$$

$$m(30)^2 = 54$$

$$m = 0.6kg$$

$$F = \frac{dP}{dE}$$

$$\Delta P = \int F dt = area\ under\ graph$$

$$M(-V - U) = -\frac{1}{2} \times 4 \times 10^{-3} \times 1500$$

$$0.06(-V - 30) = -3$$

$$V = -20ms^{-1}$$

Answer is A.

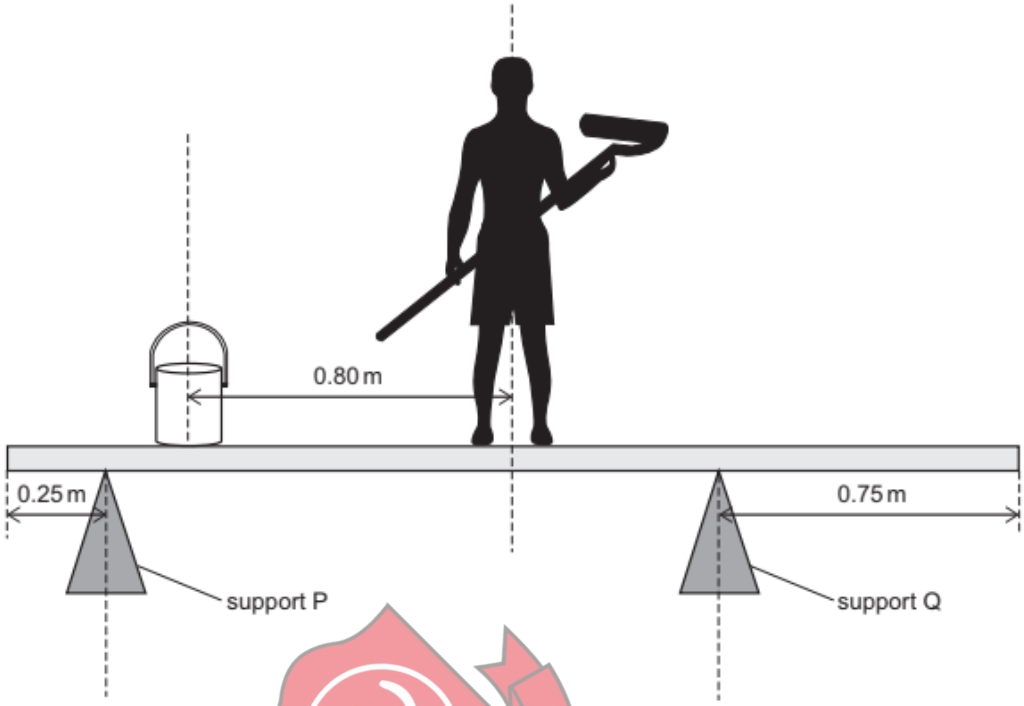
ENGAA S2 2018 - Question 8

8 A painter of mass 74 kg stands on a uniform wooden plank of length 2.5 m and of mass 24 kg.

The painter stands at the middle of the plank. The plank rests on two supports.

Support P is 0.25 m from one end of the plank and support Q is 0.75 m from the other end.

A pot of paint of mass 5.0 kg is 0.80 m from the centre of mass of the painter.



What is the force exerted by support Q on the plank?

(gravitational field strength = 10 N kg^{-1})

A 50 N

B 51.5 N

C 66 N

D 177 N

E 500 N

F 515 N

G 660 N

H 1770 N

ENGAA S2 2018 - Question 8 - Worked Solution

Label forces

Plank is uniform so centre of mass is in the middle

Resolve moments about P:

$[5g \times 0.2] + [24g \times (0.2 + 0.8)] + [74g \times (0.2 + 0.8)] = FQ \times (2.5 - 0.75 - 0.25)$

$99g = 1.5FQ$

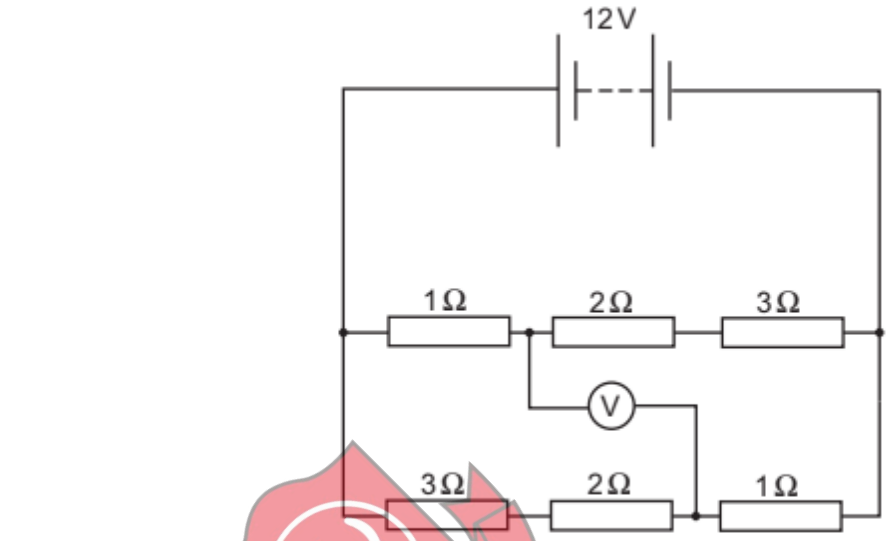
$FQ = \frac{99 \times 10}{1.5}$

$= 660N$

Answer is G.

ENGAA S2 2018 - Question 9

9 The circuit shown in the diagram contains six resistors and an ideal digital voltmeter.



What is the reading on the voltmeter?

A 0V

B 2V

C 4V

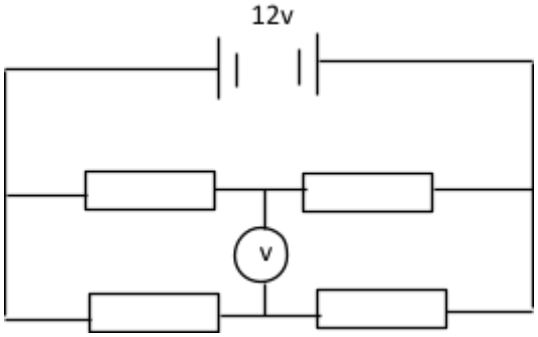
D 6V

E 8V

F 10V

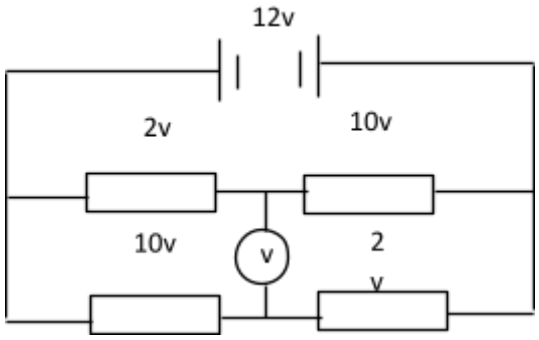
G 12V

ENGAA S2 2018 - Question 9 - Worked Solution



Redraw circuit:

Potential is split in ratio of resistance:

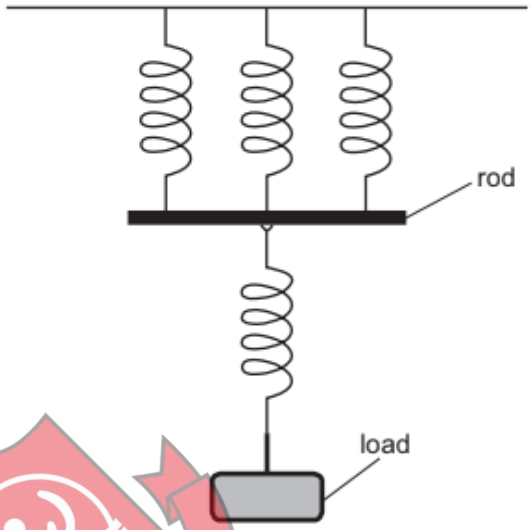


So, reading on voltmeter is 10v – 2v = 8v

Answer is E.

ENGAA S2 2018 - Question 10

10 Four identical light springs are connected together using a light rod. A load of mass m is suspended from the system so that the rod is horizontal, as shown in the diagram.



The spring constant of each spring is k and all four springs obey Hooke's law throughout.

Which of the following expressions gives the elastic potential energy stored in the system?

(gravitational field strength = g)

A $\frac{(mg)^2}{6k}$

B $\frac{(mg)^2}{2k}$

C $\frac{5(mg)^2}{9k}$

D $\frac{2(mg)^2}{3k}$

E $\frac{(mg)^2}{k}$

F $\frac{2(mg)^2}{k}$

ENGAA S2 2018 - Question 10 - Worked Solution

For spring in parallel:

K +k +k = 3k

Total spring constant for the springs in series:

$$K_T = \left(\frac{1}{3k} + \frac{1}{k}\right)^{-1}$$

$$= \frac{3k}{4}$$

Equal forces:

$$mg = \frac{3k}{4}x$$

$$x = \frac{4mg}{3k}$$

$$E = \frac{1}{2}kx^2$$

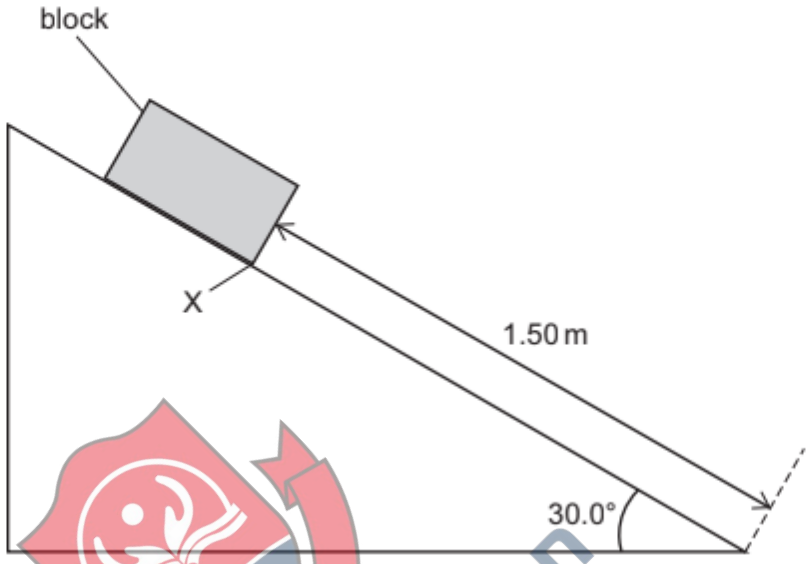
$$= \frac{1}{2} \times \frac{3k}{4} \times \left(\frac{4mg}{3k}\right)^2$$

$$= \frac{2(mg)^2}{3k}$$

Answer is D.

ENGAA S2 2018 - Question 11

11 A block of mass 3.60 kg is held stationary on a rough slope inclined at 30.0° to the horizontal. The edge X of the block is 1.50 m from the bottom of the slope.



The block is released and it accelerates uniformly down the slope. When X reaches the bottom of the slope, the speed of the block is 2.00 m s^{-1}.

What is the average rate at which work is done against resistive forces?

(gravitational field strength = 10 N kg^{-1})

A 4.8 W

B 13.2 W

C 15.6 W

D 18.0 W

E 26.4 W

ENGAA S2 2018 - Question 11 - Worked Solution

By conservation of energy:

$$\Delta Ep = \Delta E_k + W_R$$

$$-\frac{1}{2}\times(3.6)\times 2^{2}+mgh=W_{R}$$

$$-7.2+(3.6)\times 10\times 1.5\sin sin(30)=W_{R}$$

$$W_{R}=19.85$$

Time taken to slide down:

$$S=\frac{1}{2}\times 2\times t$$

$$t=1.5s$$

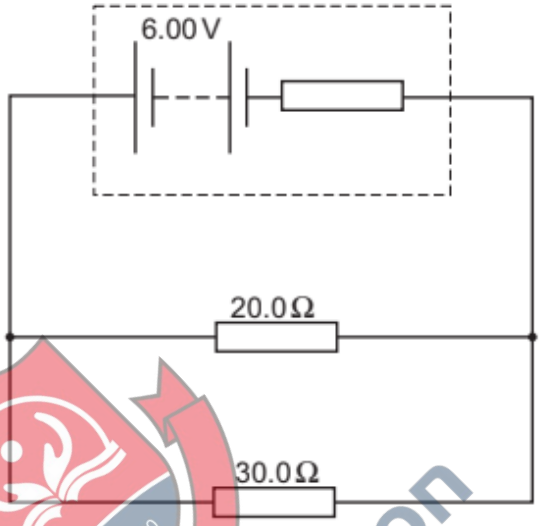
$$P_{R}=\frac{W_{R}}{t}=\frac{19.8}{1.5}=13.2W$$

Answer is B.

ENGAA S2 2018 - Question 12

12 A circuit contains a battery with internal resistance and two resistors, connected as shown in the diagram.

The emf of the battery is 6.00 V. The pd across the 20.0 Ω resistor is 4.80 V.



What is the internal resistance of the battery?

A 0.0800 Ω

B 0.333 Ω

C 0.480 Ω

D 3.00 Ω

E 12.0 Ω

F 12.5 Ω

G 15.0 Ω

ENGAA S2 2018 - Question 12 - Worked Solution

Parallel circuit so:

$$4.8 = 6 - V_r$$

$$V_r = 1.2 \quad \text{(p.d lost in internal resistance)}$$

$$I_1 = \frac{V}{R} = \frac{4.8}{20} = 0.24A$$

$$I_2 = \frac{V}{R} = \frac{48}{30} = 0.16A$$

$$I = 0.24 + 0.16$$

$$I = 0.4A$$

$$V_r = I_r$$

$$r = \frac{V_r}{I} = \frac{1.2}{0.4} = 3\Omega$$

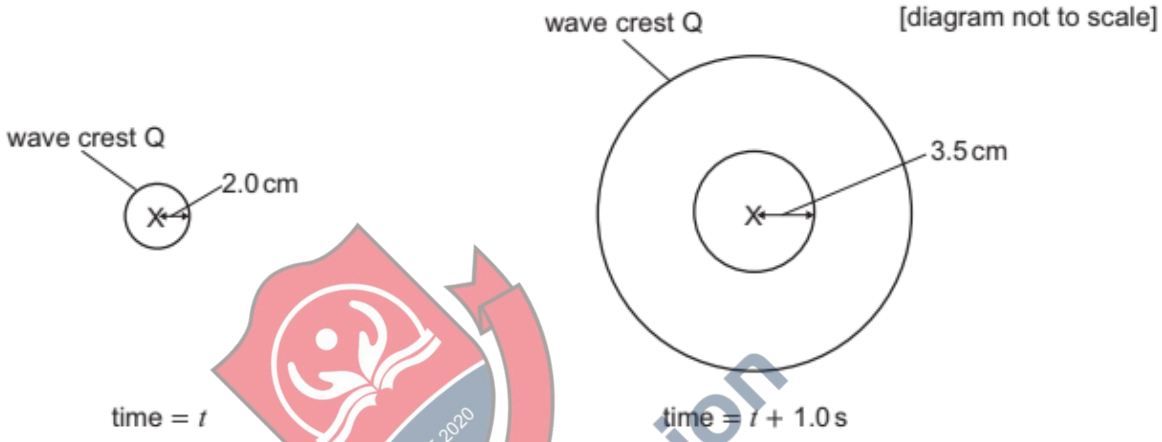
Answer is D.

ENGAA S2 2018 - Question 13

13 A stick at position X dips into water every 0.80 s, creating a circular wave which travels at a constant speed.

The diagrams show the wave crests at a time t and 1.0 s later. One of the wave crests, labelled Q, appears in both diagrams.

In each diagram, the distance from X to a wave crest is labelled.



What is the wavelength of the wave?

A 1.2 cm

B 1.5 cm

C 2.0 cm

D 3.5 cm

E 4.4 cm

F 6.0 cm

G 7.5 cm

H 14 cm

ENGAA S2 2018 - Question 13 - Worked Solution

Period of wave:

T = 0.8s

$$f = \frac{1}{T} = 1.25\ Hz$$

Speed of wave:

$$V = \lambda t$$

$$= 1.25\lambda$$

Also:

$$V = \frac{2}{t} = 1.25\lambda$$

$$t = \frac{8}{5\lambda} \quad ①$$

But also:

$$V = \frac{3.5+\lambda}{t+1} = \frac{2}{t} \quad ②$$

①→②

$$3.5 + \lambda = \frac{5\lambda}{8}\left(1 + \frac{8}{5\lambda}\right) \times 2$$

$$3.5 + \lambda = \frac{5\lambda}{4} + 2$$

$$1.5 = \frac{\lambda}{4}$$

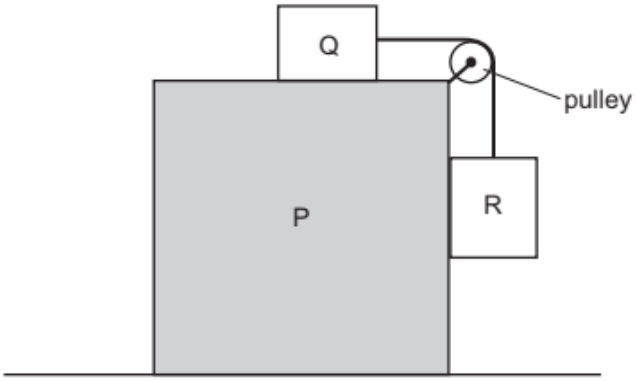
$$\lambda = 6\ cm$$

Answer is F.

ENGAA S2 2018 - Question 14

14 A block P has a smaller block Q resting on its top surface.

Q is connected to a hanging block, R, by a light, inextensible string. The string passes over a smooth pulley which is connected to block P, as shown in the diagram.



The masses of blocks P, Q and R are m_P, m_Q and m_R respectively.

The surfaces of the three blocks are smooth.

P is accelerated horizontally to the right by an external force. While this is happening, Q and R do not move relative to P.

What is the acceleration of P?

(gravitational field strength = g)

A $\frac{m_Q g}{m_R}$

B $\frac{m_R g}{m_Q}$

C $\frac{m_R g}{m_R + m_Q}$

D $\frac{m_Q g}{(m_P + m_Q + m_R)}$

E $\frac{m_R g}{(m_P + m_Q + m_R)}$

F $\frac{(m_Q + m_R) g}{(m_P + m_Q + m_R)}$

ENGAA S2 2018 - Question 14 - Worked Solution

R doesn't move w . r . t P, but P only moves horizontally, so R doesn't move vertically in lab frame:

$$T = M_R g$$

$$M_Q a_Q = T$$

$$a_Q = \frac{M_R g}{M_Q}$$

$a_Q = a_P$ as a doesn't move relative to P.

$$a_P = \frac{M_R g}{M_Q}$$

Answer is B.

ENGAA S2 2018 - Question 15

15 A solid cube with a total surface area of 96 cm^2 is suspended from a spring of spring constant $2.0 \times 10^4\ N\,m^{-1}$ and causes the spring to extend by 1.6×10^{-4} m.

The cube is removed from the spring and placed on a horizontal surface where it rests with one face on the surface.

What is the pressure exerted by the cube on the surface and what is the density of the material from which the cube is made?

(gravitational field strength = $10\ N\,kg^{-1}$)

	pressure / $N\,m^{-2}$	*density* / $kg\,m^{-3}$
A	2.0×10^1	5.0×10^{-2}
B	2.0×10^1	5.0×10^4
C	3.3×10^2	5.0×10^{-2}
D	3.3×10^2	5.0×10^3
E	3.3×10^2	5.0×10^4
F	2.0×10^3	5.0×10^{-2}
G	2.0×10^3	5.0×10^3
H	2.0×10^3	5.0×10^4

ENGAA S2 2018 - Question 15 - Worked Solution

$$SA = 6L^2 = 96$$

$$L^2 = 16$$

$$L = 4\ cm$$

Mass of cube:

$$kx = mg$$

$$m = \frac{2.0\times10^4\times1.6\times10^{-4}}{10}$$

$$= 0.32\ kg$$

$$P = \frac{F}{A}$$

$$= \frac{mg}{A}$$

$$= \frac{3.2}{L^2}$$

$$= 2000\ Nm^{-2}$$

Density;

$$\rho = \frac{m}{v}$$

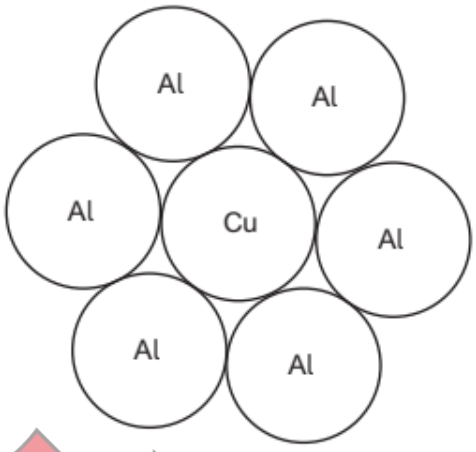
$$= \frac{0.32\ kg}{(0.04m)^3}$$

$$= 5000\ kg$$

Answer is G.

ENGAA S2 2018 - Question 16

16 A power cable consists of a cylindrical copper (Cu) wire surrounded by six cylindrical aluminium (Al) wires. All the wires are of the same cross-sectional area as shown:



The table gives the densities and resistivities of aluminium and copper.

material	*density*	*resistivity*
aluminium	d	3ρ
copper	$3d$	2ρ

The cable has mass M and length L.

Which expression gives the resistance between the two ends of the cable?

A $\frac{18\rho dL^2}{5M}$

B $\frac{21\rho dL^2}{M}$

C $\frac{81\rho dL^2}{5M}$

D $\frac{180\rho dL^2}{M}$

E $\frac{12\rho dL^2}{5M}$

F $\frac{28\rho dL^2}{3M}$

G $\frac{36\rho dL^2}{5M}$

H $\frac{80\rho dL^2}{M}$

ENGAA S2 2018 - Question 16 - Worked Solution

$$P = \frac{RA}{L} \quad (P = respectively)$$

$$d = \frac{m}{v} (d = density)$$

$$V = A \times L$$

$$d = \frac{m}{AL} \Rightarrow m = ALd$$

Now for whole cable:

$$M = 6 \times M_A + M_C$$

$$= 6 \times ALd + AL(3d)$$

$$= 9ALd$$

$$A = \frac{M}{9Ld}$$

$$R_C = \frac{(2\rho)\times L}{A}$$

$$= \frac{2\rho L}{M}\times 9Ld$$

$$= \frac{18\rho d v^2}{M}$$

$$R_a = \frac{(3\rho)L}{A}$$

$$= \frac{3\rho L}{M}\times 9Ld$$

$$= \frac{27\rho d L^2}{M}$$

$$R_T = \left(\frac{6}{R_a} + \frac{1}{R_c}\right)^{-1}$$

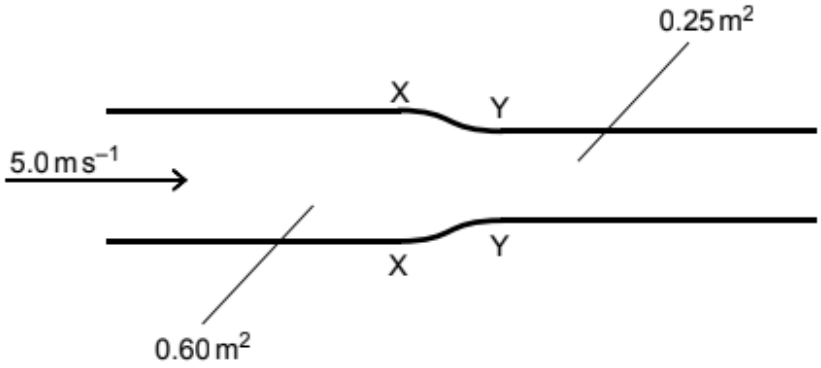
$$= \left(\frac{6M}{27\rho d L^2} + \frac{M}{18\rho d L^2}\right)^{-1}$$

$$= \frac{18\rho d L^2}{5M}$$

Answer is A.

ENGAA S2 2018 - Question 17

17 Oil of density 800 kg m^{-3} is pumped through a pipe of circular cross-sectional area 0.60 m^2 at a speed of 5.0 m s^{-1}. Between X and Y, the cross-sectional area of the pipe decreases to 0.25 m^2.



What is the resultant force exerted on the oil as it passes from X to Y?

A 0 N

B 7000 N

C 12 000 N

D 16 800 N

E 19 000 N

F 24 000 N

G 40 800 N

H 143 000 N

ENGAA S2 2018 - Question 17 - Worked Solution

A soil is incompressible, flow rate at X = flow rate at Y

(Otherwise oil would build up between X and Y)

$$A_X V_X = A_Y V_Y$$

$$V_Y = \frac{A_X V_X}{A_Y}$$

$$= \frac{0.6 \times 5}{0.25}$$

$$= 12\, ms^{-1}$$

$$\Delta V = V_X - V_Y$$

$$= 7\, ms^{-1}$$

In a timest volume flows through X

$$V = \frac{M}{\rho}$$

$$\Delta t = \frac{V}{A_X V_X} = \frac{M}{\rho} \times \frac{1}{0.6 \times 5} = \frac{M}{3\rho}$$

Then:

$$F = \frac{\Delta P}{\Delta t}$$

$$= M\Delta V \times \frac{3\rho}{M}$$

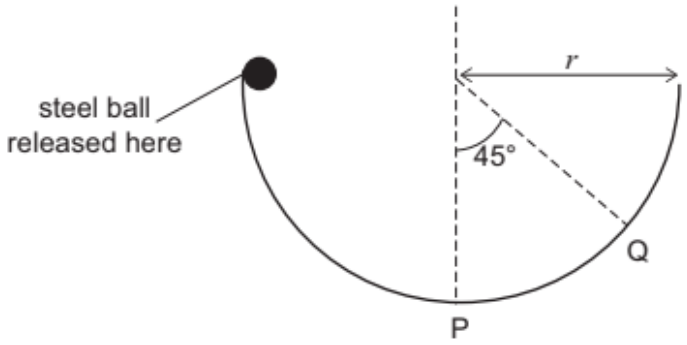
$$= 3\rho \times 7$$

$$= 21\rho = 21 \times 800 = 16800\, N$$

Answer is D.

ENGAA S2 2018 - Question 18

18 A small steel ball of mass m is released from the top of a semi-circular ramp of radius r as shown in the diagram:



[diagram not to scale]

After being released, the ball moves around the semi-circle to the lowest point at position P and then rises to a maximum height on the other side at position Q before falling down again.

Assume that the friction force acting on the ball has a constant magnitude whilst the ball is moving.

What is the kinetic energy of the ball as it first passes position P?

(gravitational field strength = g)

A $mgr(\sqrt{2}-1)$

B $mgr\left(1-\frac{\sqrt{2}}{3}\right)$

C $mgr\left(1-\frac{\sqrt{2}}{4}\right)$

D $\frac{2mgr}{3}$

E $\frac{3mgr}{4}$

F $mgr\left(\frac{1+\sqrt{2}}{3}\right)$

G mgr

H $mgr\left(2-\frac{\sqrt{2}}{2}\right)$

ENGAA S2 2018 - Question 18 - Worked Solution

F = friction

Distance moved:

$$d_Q = r\theta$$

$$= \frac{3\pi}{4}r$$

Initially:

E = 0

At Q:

$$E_Q = mgr\cos cos\ 45°$$

By conservation of energy:

$$E - E_Q = FdQ$$

$$mgr\cos cos\ (45) = \frac{3\pi}{4}rF$$

$$F = \frac{2\sqrt{2}}{3\pi}mg$$

At P:

$$E_P = E_k - mgr$$

$$E - E_P = F \times dp$$

$$mgr - E_k = \frac{\pi}{2}r \times F$$

$$E_k = mgr - \frac{\pi}{2}r \times \frac{2\sqrt{2}}{3\pi}mg$$

$$= mgr\left[1 - \frac{\sqrt{2}}{3}\right]$$

Answer is B.