Worked Solutions for ENGAA Past Papers

ENGAA S1 2019 - Question 1

1 Evaluate

$$\left(\sqrt{7}+\sqrt{3}\right)^2-\left(\sqrt{7}-\sqrt{3}\right)^2$$

A 0

B $2\sqrt{7}$

C $4\sqrt{7}$

D $2\sqrt{21}$

E 10

F $4\sqrt{21}$

G 20

ENGAA S1 2019 - Question 1 - Worked Solution

$$\left(\sqrt{7}+\sqrt{3}\right)^2-\left(\sqrt{7}-\sqrt{3}\right)^2$$

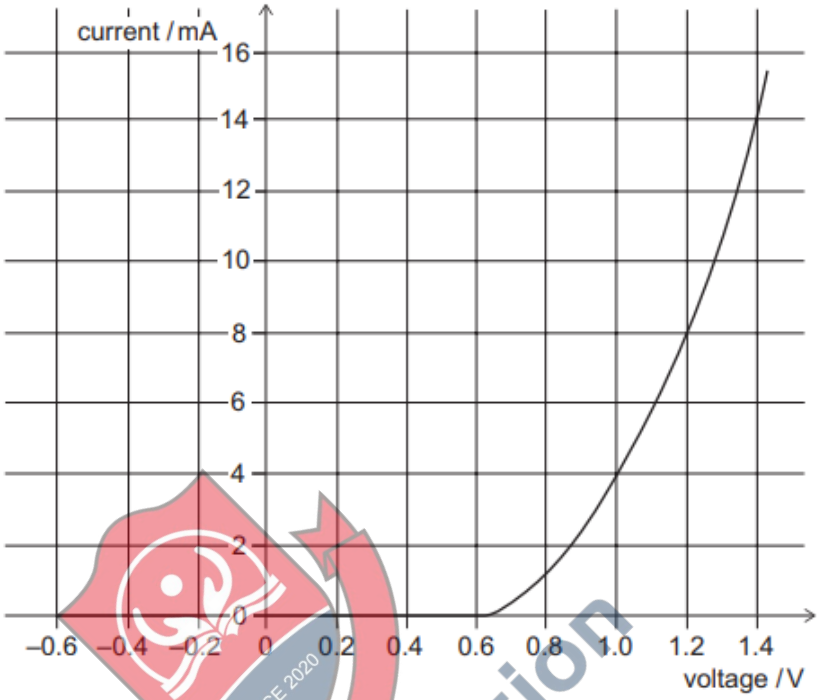
$$=7+2\sqrt{21}+3-\left(7-2\sqrt{21}+3\right)$$

$$=4\sqrt{21}$$

Answer is F

ENGAA S1 2019 - Question 2

2 The current–voltage graph for a diode is shown.



The diode is connected in series with a resistor and a 6.0 V battery. The current in the circuit is 8.0 mA.

What is the resistance of the resistor?

(Assume that the battery has negligible resistance.)

A 0.15 Ω

B 0.60 Ω

C 0.75 Ω

D 4.8 Ω

E 150 Ω

F 600 Ω

G 750 Ω

ENGAA S1 2019 - Question 2 - Worked Solution

$$V = IR \rightarrow R = \frac{V}{I}$$

$$I = 8mA$$

$$V = 1.2V$$

$$V_{res} = 6V - 1.2V = 4.8$$

$$R = \frac{4.8V}{8mA} = 600\Omega$$

Answer is F

ENGAA S1 2019 - Question 3

3 The equation gives y in terms of x:

$$y = 3 - 4\left(1 - \frac{x}{2}\right)^2$$

Which one of the following is a rearrangement for x in terms of y?

A $x = -2 \pm 2\sqrt{\frac{3-y}{4}}$

B $x = -2 \pm 2\sqrt{\frac{4-y}{3}}$

C $x = 1 \pm \sqrt{\frac{3-y}{4}}$

D $x = 1 \pm 2\sqrt{\frac{3-y}{4}}$

E $x = 2 \pm 2\sqrt{\frac{3-y}{4}}$

F $x = 2 \pm 2\sqrt{\frac{4-y}{3}}$

G $x = 2 \pm 2\sqrt{\frac{3+y}{4}}$

ENGAA S1 2019 - Question 3 - Worked Solution

$$y = 3 - 4\left(1 - \frac{x}{2}\right)^2$$

$$\Rightarrow \quad 1 - \frac{x}{2} = \pm\sqrt{3 - 4}$$

$$2 \mp 2\sqrt{3 - 4} = x$$

Answer is E

ENGAA S1 2019 - Question 4

4 Two electromagnetic waves P and Q travel in a vacuum and the ratio of their wavelengths is:

$$\frac{\text{wavelength of P}}{\text{wavelength of Q}} = 1.0 \times 10^{8}$$

Which row in the table shows the ratio of their speeds, the ratio of their frequencies, and identifies the possible natures of P and Q?

	$\frac{\textit{speed of P}}{\textit{speed of Q}}$	$\frac{\textit{frequency of P}}{\textit{frequency of Q}}$	*nature of* P	*nature of* Q
A	1.0	1.0×10^{-8}	microwave	X-ray
B	1.0	1.0×10^{-8}	microwave	radio wave
C	1.0	1.0×10^{8}	infrared	ultraviolet
D	1.0	1.0×10^{8}	visible light	infrared
E	1.0×10^{8}	1.0	gamma	X-ray
F	1.0×10^{8}	1.0	gamma	infrared
G	1.0×10^{8}	1.0×10^{16}	infrared	radio wave
H	1.0×10^{8}	1.0×10^{16}	visible light	ultraviolet

ENGAA S1 2019 - Question 4 - Worked Solution

Answer is A

ENGAA S1 2019 - Question 5

5 The resistance to the motion of a car is directly proportional to the square of the speed of the car.

The car increases its speed by 20%.

What is the percentage increase in the resistance to the motion of the car?

A 20%

B 24%

C 44%

D 120%

E 224%

F 240%

G 400%

ENGAA S1 2019 - Question 5 - Worked Solution

$$R \propto V^2$$

$$(12V)^2 = 1.44V^2$$

$$R \text{ increases by } 44\%$$

Answer is C

ENGAA S1 2019 - Question 6

6 A water-tight cylinder with a thin, freely moving piston contains $2.0 \times 10^{-3}\,m^3$ of trapped air at atmospheric pressure of $1.0 \times 10^5\,Pa$.

When the cylinder is submerged in water of constant density $1000\,kg\,m^{-3}$, the volume of air in the cylinder decreases to $4.0 \times 10^{-4}\,m^3$.

The piston is at a depth h below the surface of the water and the water surface is open to the atmosphere.

What is the depth h?

(gravitational field strength – $10\,N\,kg^{-1}$; assume that the temperature of the air remains constant and that air is an ideal gas)

A 40 m

B 50 m

C 60 m

D 400 m

E 500 m

F 600 m

ENGAA S1 2019 - Question 6 - Worked Solution

$$P_1V_1 = 1\times10^5\times2\times10^{-3} = P_2V_2 = P_2.4\times10^{-4}$$

$$P_2 = 5\times10^5$$

$$P_2 = P_1 + h\rho g = 10^5 + h(1000)(10)$$

$$h = 40m$$

Answer is A

ENGAA S1 2019 - Question 7

7 The equation of a curve is $y = px^2 + qx$ where p and q are constants.

The curve passes through the points $(2, 6)$ and $(4, -4)$.

What is the value of $q - p$?

A 1

B 2

C 5

D 6

E 9

F 16

ENGAA S1 2019 - Question 7 - Worked Solution

$$Substituting\ in\ equ$$

$$6 = 4p + 2q$$

$$-4 = 16p + 4q$$

$$p = -2$$

$$q = 7$$

$$\therefore q - p = 9$$

Answer is E

ENGAA S1 2019 - Question 8

8 The secondary coil of an ideal, 100% efficient transformer is connected to a resistor by cables of total resistance 1500 Ω. The current in the primary coil is 4.0 A. There are 240 turns in the primary coil and 4800 turns in the secondary coil.

What is the power produced as heat in the cables?

A 60 W

B 300 W

C 6000 W

D 24 000 W

E 120 000 W

F 9 600 000 W

ENGAA S1 2019 - Question 8 - Worked Solution

$$I_P = 4A \rightarrow I_S = \frac{240}{4500} . 4A = 0.2A$$

$$P = I^2 R = (0.2)^2 . 1500\Omega$$

$$P = 60W$$

Answer is A

ENGAA S1 2019 - Question 9

9 Which of the following is a simplification of

$$4-\frac{x(3x+1)}{x^2(3x^2-2x-1)}$$

A $\frac{12x^3-8x^2-7x-1}{x(3x-1)(x-1)}$

B $\frac{4x^2+4x-1}{x(x+1)}$

C $\frac{4x^2+4x+1}{x(x+1)}$

D $\frac{4x^2-4x-1}{x(x-1)}$

E $\frac{4x^2-4x+1}{x(x-1)}$

F $\frac{12x^3-8x^2-x+1}{x(3x-1)(x-1)}$

ENGAA S1 2019 - Question 9 - Worked Solution

Trick : the diameter probably has one of the top factors is 1.

$$3x^2-2x-1=(3x+1)(x-1)$$

$$4-\frac{1}{x(x-1)}=\frac{4x(x-1)-1}{x(x-1)}$$

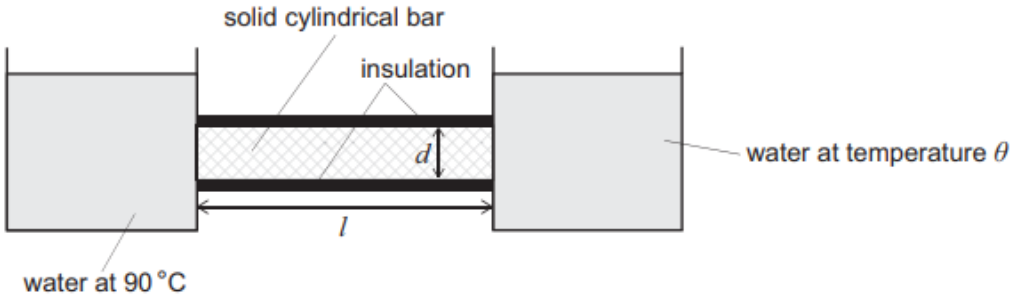
Answer is D

ENGAA S1 2019 - Question 10

10 Two tanks of water are connected by a solid cylindrical copper bar of length l and diameter d.

The bar is insulated.

One tank contains water at 90 °C and the other tank contains water at temperature θ.



For which of the following conditions is thermal energy conducted along the bar at the lowest rate?

	l / m	d / cm	θ / °C
A	0.40	4.0	20
B	0.40	4.0	40
C	0.40	8.0	20
D	0.40	8.0	40
E	0.80	4.0	20
F	0.80	4.0	40
G	0.80	8.0	20
H	0.80	8.0	40

ENGAA S1 2019 - Question 10 - Worked Solution

For lower thermal transfer

$how\ temperature\ difference\ -\ \theta\ =\ 40°C$

$thin\ pipe\ -\ d\ =\ 4.0$

$long\ pipe(so\ thermal\ energy\ has\ long\ way\ to\ travel)\ -\ l\ =\ 0.8$

Answer is F

ENGAA S1 2019 - Question 11

11 The ball for a garden game is a solid sphere of volume 192 cm^3.

For the children's version of the game the ball is a solid sphere made of the same material, but the radius is reduced by 25%.

What is the volume, in cm^3, of the children's ball?

A 48

B 81

C 96

D 108

E 144

ENGAA S1 2019 - Question 11 - Worked Solution

$$V_1 = \frac{4}{3}\pi r^3$$

$$V_2 = \frac{4}{3}\pi\left(\frac{4}{3}r\right)^3 = V_1 . \frac{3^3}{4^3}$$

$$192 . \frac{3^3}{4^3} = 81cm^3$$

Answer is B

ENGAA S1 2019 - Question 12

12 The radioactive isotope X becomes the stable isotope Y after a succession of decays involving only the emission of alpha and beta (β^-) particles.

During the decay of one nucleus from X to Y, a total of seven particles are emitted. It is known that more of these particles are alpha particles than beta particles.

The atomic number of X is Z and the mass number of X is A.

Which row in the table could give the atomic number and the mass number of Y?

	atomic number of Y	*mass number of* Y
A	$Z - 2$	$A - 12$
B	$Z - 5$	$A - 8$
C	$Z - 8$	$A - 20$
D	$Z - 10$	$A - 24$
E	$Z - 11$	$A - 16$

ENGAA S1 2019 - Question 12 - Worked Solution

$\alpha : Gives\ alpha\ particle\ off \quad A = -\frac{4}{2} = -2$

$\beta : Neutron \rightarrow proton + electron$

$A = 0 \quad Z = +1$

$ZAX \rightarrow n \times 24\alpha + (7 - n) - 10\beta + ??\, Y$

$n,\ the\ number\ of\ alpha\ particles\ gien\ off\ is\ greater\ than\ 7 - n$

$\therefore n = 4, 5\ or\ 6 \rightarrow eliminates\ first\ two\ options\ ,\ due\ to\ mass\ loss.$

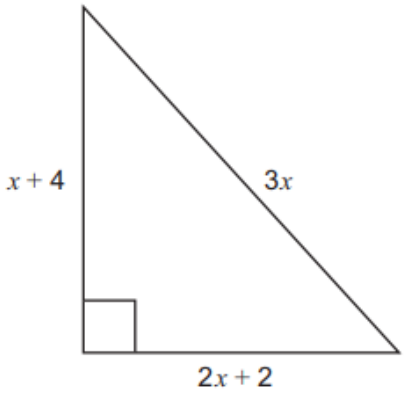
$Plugging\ C \nexists D \nexists E\ into\ above\ decay\ equation$

Only C holds

Answer is C

ENGAA S1 2019 - Question 13

13 The diagram shows a right-angled triangle, with sides of length $x + 4$, $2x + 2$ and $3x$, all in cm.



[diagram not to scale]

What is the area, in cm^2, of the triangle?

A 10

B 12

C 28

D 36

E 40

F 54

G 70

ENGAA S1 2019 - Question 13 - Worked Solution

Using Pythagoras' theorem

$$(x + 4)^2 + (2x + 2)^2 = (3x)^2$$

$$(x - 5)(x + 1) = 0$$

Lengths must be positive

$$x = 5,\ x \neq 1$$

$$A = \frac{1}{2}\ base \times height$$

$$A = \frac{1}{2}(5 + 4)(2 \times 5 + 2)$$

$$A = 54$$

Answer is F

ENGAA S1 2019 - Question 14

14 The kinetic energy of an object of mass 4.0 kg, travelling in a straight line, increases from 32 J to 200 J in 3.0 seconds due to a constant resultant force.

What is the value of this resultant force?

A 2.0 N

B 4.0 N

C 8.0 N

D 24 N

E 28 N

F 56 N

ENGAA S1 2019 - Question 14 - Worked Solution

$$T_0 = 32J = \frac{1}{2}mv_0^2 \rightarrow v_0 = 4$$

$$T_1 = 200J = \frac{1}{2}mv_1^2 \rightarrow v_1 = 10$$

By Newton's 2nd law:

$$F = ma = m - \frac{v-u}{\Delta t} = 4 . \frac{10-4}{3}$$

$$\therefore F = 8N$$

Answer is C

ENGAA S1 2019 - Question 15

15 *PR* and *QS* are the diagonals of a rhombus *PQRS*.

$PR = (3x + 2)$ cm

$QS = (8 - 2x)$ cm

The area of *PQRS* is 11 cm^2.

What is the difference, in cm, between the two possible lengths of *PR*?

A $2\frac{2}{3}$

B $4\frac{1}{2}$

C $5\frac{1}{3}$

D 8

E 14

ENGAA S1 2019 - Question 15 - Worked Solution

$$A = \frac{1}{2} \times PR \times QS = 11cm^2$$

$$\frac{1}{2} \times (3x + 2)(8 - 2x) = 11$$

$$(3x - 1)(x - 3) = 0$$

$$x = \frac{1}{3}, \; x = 3$$

$$PR = 3x + 2$$

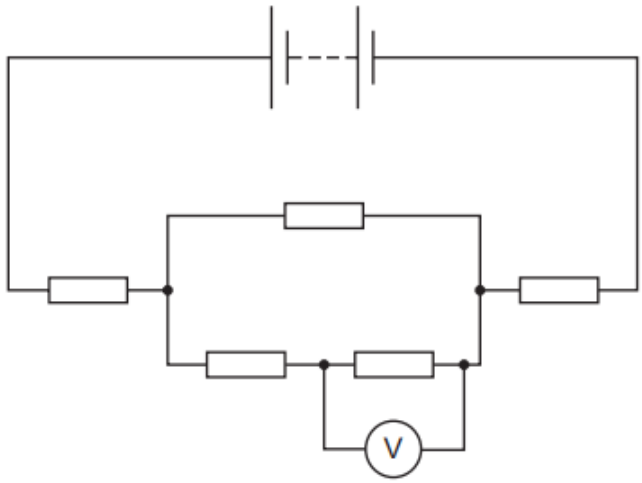
$$\therefore PR = 3, \; PR = 11$$

$$11 - 3 = 8$$

Answer is D

ENGAA S1 2019 - Question 16

16 In the following circuit, all five resistors have the same resistance.



The reading on the voltmeter is 1.0 V.

What is the voltage across the battery?

A 4.0 V

B 5.0 V

C 6.0 V

D 7.0 V

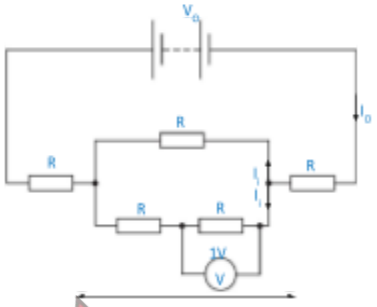
E 8.0 V

F 9.0 V

G 10 V

ENGAA S1 2019 - Question 16 - Worked Solution

$V_{parallel}=2V$



Parallel branch

$$R' = \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1}$$

$$R' = \frac{2}{3}R$$

$$V_{parallel} = 2V$$

$2V$ *measured across parallel branch*

By potential divider formula

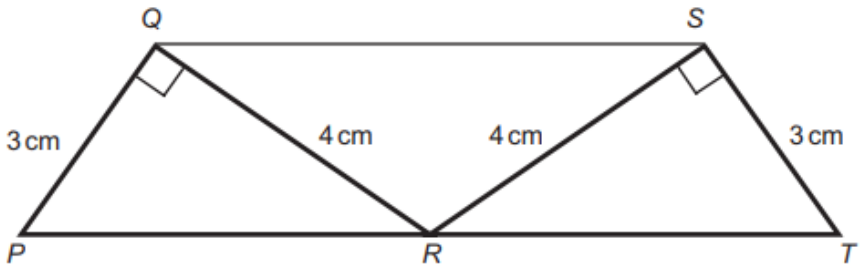
$$2V = \frac{\frac{2}{3}R}{\frac{2}{3}R+2R} - V_0$$

$$V_0 = 8V$$

Answer is E

ENGAA S1 2019 - Question 17

17



[diagram not to scale]

The diagram shows two congruent right-angled triangles *PQR* and *TSR* with right angles at *Q* and *S*, respectively.

$PQ = TS = 3\,\text{cm}$

$QR - SR - 4\,\text{cm}$

PRT is a straight line.

What is the length, in cm, of *QS*?

A 4

B $3\sqrt{2}$

C 5.2

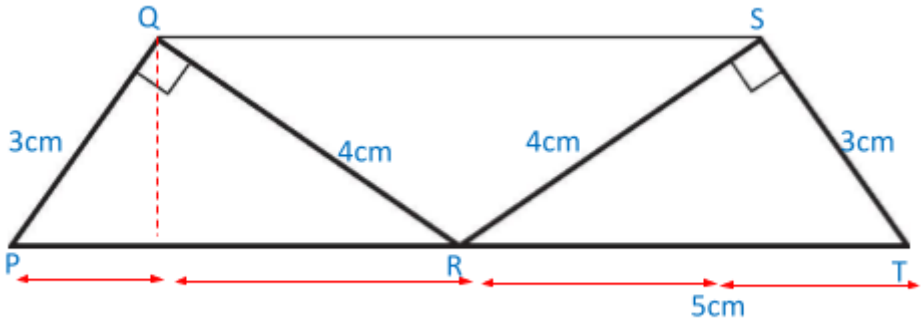
D $4\sqrt{2}$

E 6.4

F 8.2

G 10

ENGAA S1 2019 - Question 17 - Worked Solution



$QS = 2(5cm - \beta)$

By Pythagoras' theorem

$\beta^2 + \alpha^2 = 3^2 ---- ①$

$\alpha^2 + (5 - \beta)^2 = 4^2 ----- ②$

Subtracting ① from ②

$16 - 9 = 25 - 10\beta + \beta^2$

$7 = 25 - 10\beta$

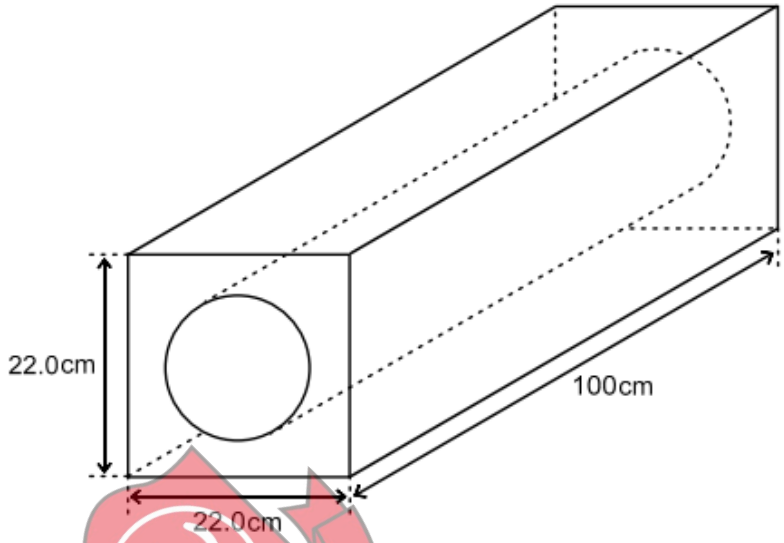
$\frac{7}{5} = 5 - 2\beta$

$QS = 10 - 2\beta = \frac{7}{5} + 5 = \frac{32}{5} = 6.4$

Answer is E

ENGAA S1 2019 - Question 18

18 A block is designed with a cylindrical channel to accommodate a hot-water pipe. The block is 100 cm long and it has a square cross-section of side 22.0 cm with a cylindrical hole in the middle, as shown in the diagram:



[diagram not to scale]

The diameter of the cylindrical hole is 14.0 cm and the density of the material from which the block is made is 0.100 g cm^{-3}.

What is the mass of the block?

(take π to be $\frac{22}{7}$)

A 1.32 kg

B 3.30 kg

C 13.2 kg

D 33.0 kg

E 132 kg

F 330 kg

G 1320 kg

H 3300 kg

ENGAA S1 2019 - Question 18 - Worked Solution

$$10^{-1} gcm^3 = 10^{-4}$$

$$V_{block} = V_{cuboid} - V_{cylinder}$$

$$V_{block} = (22 \times 22 \times 100) - \left(\frac{22}{7} \times 7 \times 7 \times 100\right)$$

$$V_{block} = 22 \times (22 - 7) \times 100$$

$$V_{block} = 22 \times 1500 cm$$

$$\rho = \frac{m}{V}$$

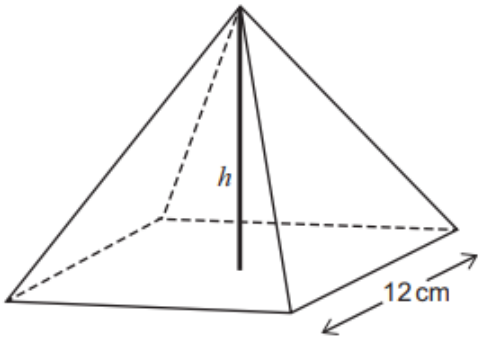
$$10^{-4} = \frac{m}{V}$$

$$m = 3.3kg$$

Answer is B

ENGAA S1 2019 - Question 19

19



[diagram not to scale]

A solid pyramid has a square base of side length 12 cm and a vertical height of h cm.

The volume of the pyramid, in cm^3, is equal to the total surface area of the pyramid, in cm^2.

What is the value of h?

(volume of pyramid = $\frac{1}{3}$ × area of base × vertical height)

A $\frac{72}{35}$

B $2\sqrt{3}$

C 6

D $\frac{144}{23}$

E 8

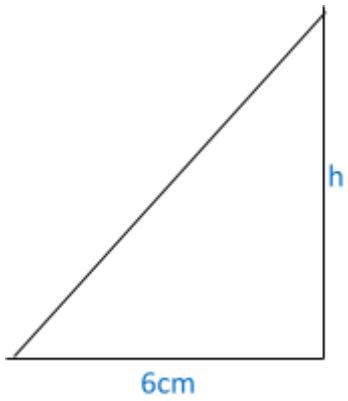
F $2\sqrt{21}$

ENGAA S1 2019 - Question 19 - Worked Solution

$A_b = 12^2 = 144$

$SA = A_b + 4\,.\,A_T = 144 + 4\times\left(\frac{1}{2}\times 12\times\sqrt{36 + h^2}\right)$

$Vol = \frac{1}{3}\times A_b \times h = \frac{144}{3}h$



Let Vol = SA

$144 + 24\sqrt{36 + h^2} = \frac{144}{3}h$

$3h(h - 8) = 0$

$h \neq 0 \qquad \therefore h = 8cm$

Answer is E

ENGAA S1 2019 - Question 20

20 A sample initially contains equal numbers of atoms of a radioactive isotope X and a stable isotope Y.

Isotope X has a half-life of 3 years and decays in a single stage to the stable isotope Y.

What is the ratio

$$\textit{number of atoms of X} : \textit{number of atoms of Y}$$

in the sample 6 years later?

A The sample contains only isotope Y.

B 1 : 7

C 1 : 4

D 1 : 3

E 7 : 4

ENGAA S1 2019 - Question 20 - Worked Solution

Initially

$$\frac{x}{y} = \frac{1}{1}$$

After two half lives

$$x \rightarrow \frac{1}{4}\ x$$

$$y \rightarrow \frac{1}{1}\ 4 + \frac{3}{4}\ 4$$

$$4 \rightarrow \frac{7}{4}\ 4$$

Thus ratio is

1 : 7

Answer is B

ENGAA S1 2019 - Question 21

21 Find the area of the shape bounded by the four lines:

$$2y + x = 4$$

$$x = -6$$

$$x = 0$$

$$y = 0$$

A 4

B 12

C 21

D 25

E 27

F 30

ENGAA S1 2019 - Question 21 - Worked Solution

$$y = -\frac{x}{2} + 2$$

$$A_2 = 6 \times 2 = 12$$

$$P_2 = (-6, 2)$$

$$P_3 = (0, 2)$$

$$P_1 = (-6, 5) \; via \; equqtion: \; y = -\frac{x}{2} + 2$$

$$A_1 = \frac{1}{2} \times 6 \times 3 = 9$$

$$A_1 + A_2 = 9 + 12 = 21$$

Answer is C

ENGAA S1 2019 - Question 22

22 A hydroelectric power station uses the water in a reservoir to power the generators. The water falls through a vertical height of 150 m to the turbines which power the generators.

The efficiency of the power station is 90% and the output power of the power station is 1800 MW.

The gravitational field strength is 10 N kg^{-1} and the density of water is 1000 kg m^{-3}.

What volume of water passes through the turbines in one minute?

A $6.48 \times 10^4\ m^3$

B $7.20 \times 10^4\ m^3$

C $8.00 \times 10^4\ m^3$

D $6.48 \times 10^7\ m^3$

E $7.20 \times 10^7\ m^3$

F $8.00 \times 10^7\ m^3$

ENGAA S1 2019 - Question 22 - Worked Solution

$$Energy\ in\ one: E = \frac{1800MW}{0.9} \times 60s$$

$$E = 120 \times 10^9 J$$

Power is generated by GPE

$$U = mgh = 1000kgm^{-3} \times V \times 10Nkg^{-1} \times 150m$$

$$let\ U = E_0$$

$$V = \frac{120 \times 10^9}{150 \times 10^4}$$

$$V = 0.8 \times 10^5$$

$$V = 8 \times 10^4$$

Answer is C

ENGAA S1 2019 - Question 23

23 The curve

$$y = x^3 + px^2 + qx + r$$

has a local maximum when $x = -1$ and a local minimum when $x = 3$

What is the value of p?

A –9

B –3

C –1

D 1

E 3

F 9

ENGAA S1 2019 - Question 23 - Worked Solution

$$y' = 3x^2 + 2px + q$$

$$y' = 0 \ when \ x = -1 \ and \ x = 3, \ sub \ in$$

$$3 - 2p + q = 0$$

$$27 + 6p + q = 0$$

$$27 - 3 + 6p + 2p + q - q = 0$$

$$p = -3$$

Answer is B

ENGAA S1 2019 - Question 24

24 A car P of mass 1000 kg is travelling north at 30 m s^{-1} along a straight, horizontal road when it hits another car Q which is directly ahead of P and travelling in the same direction. Car Q has a mass of 500 kg and is travelling at 20 m s^{-1}.

The collision lasts for 0.20 s and immediately after the collision car Q is moving north at 30 m s^{-1}.

What is the speed of P immediately after the collision and what is the size of the average resultant force that acts on Q during the collision?

(Assume that no external forces act on the cars during the collision.)

	speed of P / m s^{-1}	*average force on* Q / N
A	20	25 000
B	20	50 000
C	20	100 000
D	20	125 000
E	25	25 000
F	25	50 000
G	25	100 000
H	25	125 000

ENGAA S1 2019 - Question 24 - Worked Solution

Cons. of momentum

$$30\times1000 + 20\times500 = 1000\times v + 30\times500$$

$$v = 25ms^{-1}$$

Impulse on Q, via Newton's 2nd Law:

$$F = m\frac{v-u}{\Delta t} \rightarrow F = 500\frac{30-20}{0.2} = 25\times10^{3}N$$

Answer is E

ENGAA S1 2019 - Question 25

25 When simplified, $\frac{1}{(1-\sqrt{2})^3}$ is written in the form $a+b\sqrt{2}$ where a and b are integers.

What is the value of b?

A –7

B –5

C –1

D 1

E 5

F 7

ENGAA S1 2019 - Question 25 - Worked Solution

$$\frac{1}{(1-\sqrt{2})^3} \times \left(\frac{1+\sqrt{2}}{1+\sqrt{2}}\right)^3 = \left(\frac{1+\sqrt{2}}{1^2-\sqrt{2}^2}\right)^3$$

$$= \frac{(1+\sqrt{2})^3}{(-1)^3} = -\left(1 + 3\sqrt{2} + 3(2) + 2\sqrt{2}\right)$$

$$= -7 - 5\sqrt{2}$$

$$b = -5$$

Answer is B

ENGAA S1 2019 - Question 26

26 A metal wire of length 0.50 m has a uniform cross-sectional area of $4.0 \times 10^{-7}\ m^2$.

There is a current of 4.0 A in the wire.

What is the potential difference across the ends of the wire?

(resistivity of the metal = $1.6 \times 10^{-7}\ \Omega m$)

A 0.05 V

B 0.20 V

C 0.80 V

D 3.2 V

E 5.0 V

F 20 V

ENGAA S1 2019 - Question 26 - Worked Solution

$$\rho = \frac{RA}{l} \rightarrow R = \frac{\rho l}{A} = 0.8V$$

Answer is C

ENGAA S1 2019 - Question 27

27 It is given that

$$7\cos x + \tan x \sin x = 5$$

where $0° < x < 90°$

What are the possible values of $\tan x$?

A $\frac{1}{2}$ or $\frac{1}{3}$

B $\frac{1}{\sqrt{3}}$ or $\frac{1}{2\sqrt{2}}$

C $\frac{\sqrt{3}}{2}$ or $\frac{2\sqrt{2}}{3}$

D $\sqrt{3}$ or $2\sqrt{2}$

E 3 or 2

ENGAA S1 2019 - Question 27 - Worked Solution

$$7 + tan^2 x = 5secx$$

$$7 + sec^2 - 1 = 5secx$$

$$\lambda^2 - 5\lambda + 6 = 0 \qquad (\lambda = secx)$$

$$\lambda = \frac{5}{2} \pm \frac{1}{2}\sqrt{25 - 24}$$

$$secx = 2\,,\ 3$$

$$Using\ tan^2 x + 1 = sec^2 x$$

$$sec = 2$$

$$tanx = \sqrt{4 - 1}$$

$$tanx = \sqrt{3}$$

$$sec = 3$$

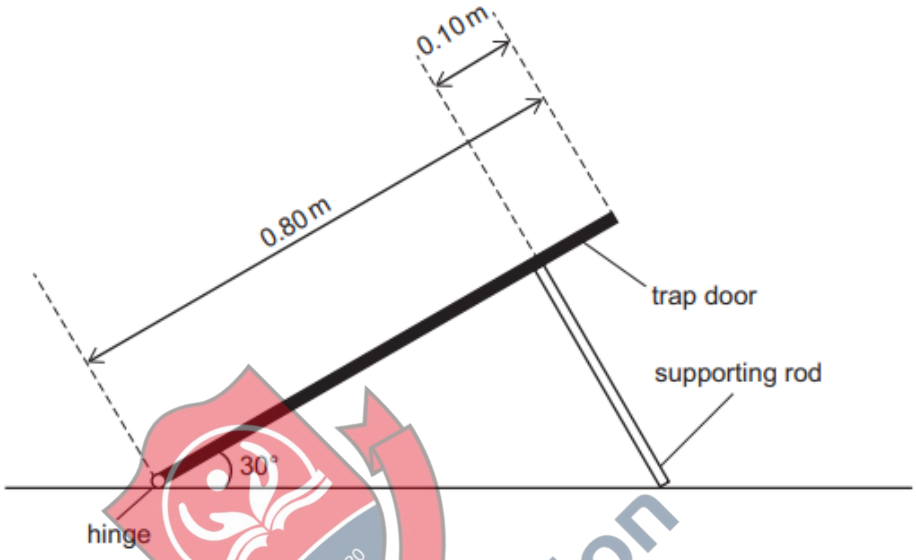
$$tanx = \sqrt{9 - 1}$$

$$tanx = 2\sqrt{2}$$

Answer is D

ENGAA S1 2019 - Question 28

28 A uniform square trap door of side 0.80 m and mass 14 kg has a smooth hinge at one edge and is held open at an angle of 30° to the horizontal. It is supported by a single rigid rod placed so that it meets the surface of the trap door at 90° at a distance 0.10 m from the top edge of the trap door, as shown.



What is the normal contact force exerted on the trap door by the rod?

(gravitational field strength = 10 N kg^{-1})

A 40 N

B $35\sqrt{3}$ N

C $40\sqrt{3}$ N

D 80 N

E $80\sqrt{3}$ N

F $280\frac{\sqrt{3}}{3}$ N

ENGAA S1 2019 - Question 28 - Worked Solution

Rotational moments / torques about hinge

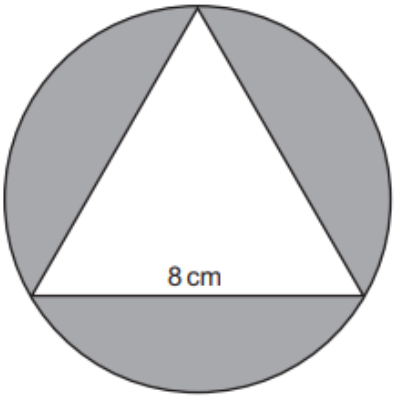
$(14kg \times 10Nkg \times \cos cos\ 30\ ) .\ 0.4m\ =\ N\ =\ 0.7m$

$$R = 40\sqrt{3}$$

Answer is C

ENGAA S1 2019 - Question 29

29 An equilateral triangle of side 8 cm is drawn so that its vertices lie on the circumference of a circle, as shown in the diagram.



What is the total of the three areas shaded in the diagram, in cm^2?

A $8(2\pi - 3)$

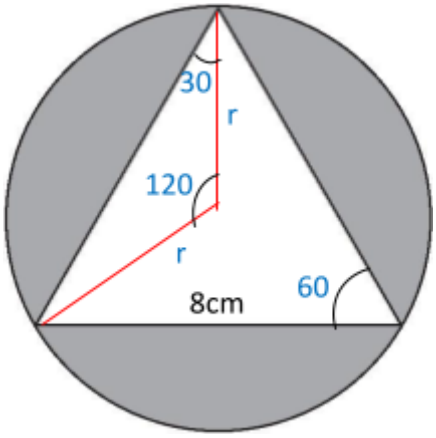
B $24(\pi - \sqrt{3})$

C $48(4\pi - \sqrt{3})$

D $\frac{16}{3}(4\pi - 6 - 3\sqrt{3})$

E $\frac{16}{3}(4\pi - 3\sqrt{3})$

ENGAA S1 2019 - Question 29 - Worked Solution



By Area law

$$A_{triangle} = \frac{1}{2} AB \sin sin\, C$$

$$= \frac{1}{2} \times 8 \times 8 \times \sin sin\, 60$$

$$A_{tri} = 16\sqrt{3}$$

$$By\ Sin\ law: \frac{r}{\sin sin\, 30} = \frac{8}{\sin sin\, 120}$$

$$r = \frac{8\sqrt{3}}{3}$$

$$A_{circ} = \pi r^2 = \pi \,.\, \frac{64}{9} \,.\, 3 = \pi \frac{64}{3}$$

$$A_{tri} = 16\sqrt{3}$$

$$A_{shade} = A_c - A_t = \frac{16}{3}\left(4\pi - 3\sqrt{3}\right)$$

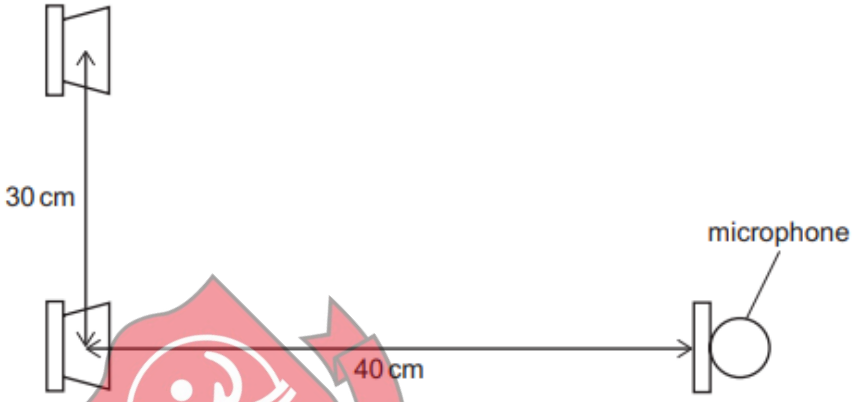
Answer is E

ENGAA S1 2019 - Question 30

30 Two small loudspeakers are placed side by side 30 cm apart.

They are connected to the same signal generator so that they emit sound of frequency 400 Hz in phase with one another.

The sounds both reach a microphone placed 40 cm directly in front of one of the two loudspeakers as shown.



What is the phase difference between waves from the loudspeakers as they arrive at the microphone?

(speed of sound – 320 $m\,s^{-1}$)

A 30°

B 36°

C 45°

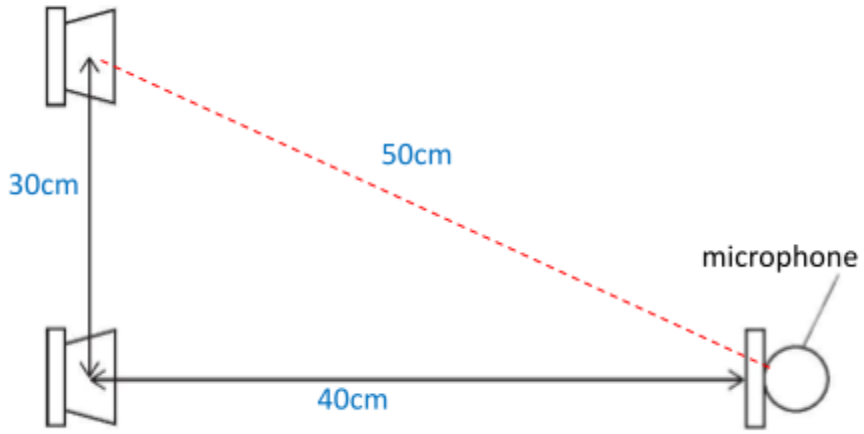
D 60°

E 72°

F 90°

G 120°

ENGAA S1 2019 - Question 30 - Worked Solution



10cm path difference

$\lambda = \frac{320}{400Hz} = 0.8m = 80cm$

$\varphi = \frac{d}{\lambda} . 2\pi = \frac{10cm}{80cm} . 2\pi = \frac{\pi}{4}$

$or\ in\ degrees: \varphi = \frac{d}{\lambda} . 360° = \frac{10}{80} . 360° = 45°$

Think of the phase difference in this case as the fraction of wavelength the top wave has travelled extra

Answer is C

ENGAA S1 2019 - Question 31

31 Which one of the following is the real solution of the equation

$$3\times5^{2x+1}-5^{x}-2=0$$

A $x=\log_5\left(\frac{1}{3}\right)$

B $x=\log_5\left(\frac{2}{5}\right)$

C $x=\log_5\left(\frac{3}{5}\right)$

D $x=\log_5\left(\frac{2}{3}\right)$

E $x=\log_5\left(\frac{5}{3}\right)$

F $x=\log_5\left(\frac{5}{2}\right)$

ENGAA S1 2019 - Question 31 - Worked Solution

$$3\times5\times\left(5^{x}\right)^{2}-5^{x}-2=0$$

$$let\ r=5^{x}$$

$$15r^{2}-r-2=0$$

$$r=\frac{2}{5}\ ,\quad r=-\frac{1}{3}$$

Logarithmic function only takes positive input.

$$r=5^{x}\rightarrow x=r$$

$$x=\frac{2}{5}$$

Answer is B

ENGAA S1 2019 - Question 32

32 An astronaut on the Moon throws a ball vertically upwards. The ball has a mass of 2.0 g and is thrown upwards at 80 m s^{-1}.

What is the maximum height gained by the ball?

(gravitational field strength close to the Moon's surface = 1.6 N kg^{-1})

A 25 m

B 50 m

C 320 m

D 2000 m

E 3200 m

F 4000 m

ENGAA S1 2019 - Question 32 - Worked Solution

$$v^2 - u^2 = 2aS$$

$$S = \frac{0^2 - 80^2}{-2.16}$$

0 velocity at the apex

Hence $v^2 = 0$

$$S = 2000m$$

Answer is D

ENGAA S1 2019 - Question 33

33 For a particular function $f(x)$, it is given that:

$$\int_{-2}^{2} 2f(x)\mathrm{d}x + \int_{2}^{4} f(x)\mathrm{d}x = 4$$

and also:

$$\int_{-2}^{2} 5f(x)\mathrm{d}x - \int_{-2}^{4} f(x)\mathrm{d}x = 7$$

Find the value of $\int_{2}^{4} f(x)\mathrm{d}x$

A $\frac{1}{3}$

B $\frac{11}{7}$

C $\frac{11}{6}$

D $\frac{13}{6}$

E $\frac{13}{3}$

ENGAA S1 2019 - Question 33 - Worked Solution

$$\int_{-2}^{2} 2f(x)dx + \int_{2}^{4} f(x)dx = 4 \quad ---- \ (1)$$

and also

$$\int_{-2}^{2} 5f(x)dx - \int_{-2}^{4} f(x)dx = 7$$

$$\int_{-2}^{2} 4f(x)dx + \int_{-2}^{2} fdx + \int_{4}^{-2} fdx = 7$$

$$\int_{-2}^{2} 4fdx - \int_{2}^{4} fdx = 7 \quad (taking\ 2(1)\ off\ this$$

$$\int_{-2}^{2} 4f\,dx - 2\int_{-2}^{2} 2f\,dx - \int_{2}^{4} f\,dx - 2\int_{2}^{4} f\,dx = 7 - 2\times 4$$

$$0 - 3\int_{2}^{4} f\,dx = -1$$

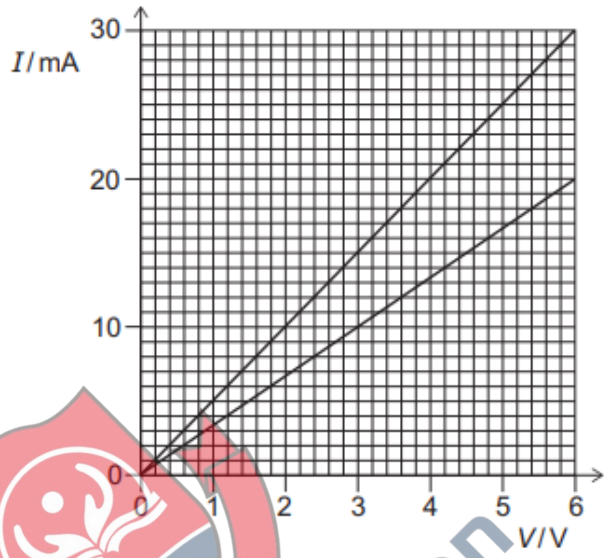
$$\therefore \int_{2}^{4} f\,dx = \frac{-1}{-3} = \frac{1}{3}$$

Answer is A

ENGAA S1 2019 - Question 34

34 A student has one 300 Ω resistor and another resistor of resistance *R*.

The student plots a graph of current *I* against potential difference *V* for the 300 Ω resistor and then for both resistors connected in parallel.



What is the resistance *R*?

A 3.3 Ω

B 5.0 Ω

C 10 Ω

D 100 Ω

E 200 Ω

F 600 Ω

G 1000 Ω

ENGAA S1 2019 - Question 34 - Worked Solution

$$V = IR$$

$$R = \frac{V}{I}$$

$$\frac{6V}{30\times10^{-3}mA} = 200\Omega$$

$$R^{'} = \left(\frac{1}{R} + \frac{1}{300}\right)$$

$$R^{'} = 200\Omega = \left(\frac{1}{R} + \frac{1}{300}\right)$$

$$\frac{1}{R} = \frac{1}{200} - \frac{1}{300} = \frac{100}{60000}$$

$$R = 600\Omega$$

Answer is F

ENGAA S1 2019 - Question 35

35 Given that

$$f(x) = \int_0^x (3+2t)^7 \, dt$$

what is the coefficient of x^4 in the expansion of $f(x)$ in powers of x?

A 70

B 162

C $\frac{2835}{4}$

D 3024

E 5670

F 15 120

G 22 680

ENGAA S1 2019 - Question 35 - Worked Solution

$$f(x) = \int_0^x (3 + 2t)^7 dt$$

$$f(x) = \int \left(C_0 + C_1 t + C_2 t^2 + C_3 t^3 + \ldots ++ C_7 t^7\right) dt$$

t^3 *term will go to* x^4

$$C_3 t^3 = \frac{7!}{3!4!} \cdot 3^4 (2t)^3 = 35 \times 81 \times 8 t^3$$

$$= 5670 \times 4 t^3$$

$$5670 \times \int_0^x 4t^3 = 5670 x^4$$

Answer is E

ENGAA S1 2019 - Question 36

36 A light, vertical, copper wire of length 2.4 m and uniform cross-sectional area $2.0 \times 10^{-6}\,m^2$ supports a load of mass 4.0 kg.

The Young modulus of copper is 1.2×10^{11} Pa.

What is the strain energy in the wire?

(gravitational field strength = $10\,N\,kg^{-1}$; assume that the wire obeys Hooke's law and that the cross-sectional area remains constant)

A 8.0×10^{-5} J

B 1.7×10^{-4} J

C 4.0×10^{-4} J

D 8.0×10^{-3} J

E 4.0×10^{-2} J

F 1.6×10^{-2} J

ENGAA S1 2019 - Question 36 - Worked Solution

$$r = \frac{F.l}{A.x}$$

$$x = \frac{4 \times 10 \times 2.4}{2 \times 10^{-6} \times 1.2 \times 10^{11}}$$

$$x = 4 \times 10^{-4}$$

$$Strain\ energy: \quad E = \frac{1}{2}Fx = \frac{1}{2} \times 4 \times 10 \times 4 \times 10^{-4}$$

$$E = 8 \times 10^{-3} J$$

Answer is D

ENGAA S1 2019 - Question 37

37 The three internal angles in a triangle are α, β and θ, and

$$3\tan\alpha - 2\sin\beta = 2$$

$$5\tan\alpha + 6\sin\beta = 8$$

What is the value of θ in degrees?

A 15

B 45

C 75

D 105

E 135

ENGAA S1 2019 - Question 37 - Worked Solution

$$a = \tan tan\,\alpha$$

$$b = \tan tan\,\beta$$

$$3a - 2b = 2$$

$$5a + 6b = 8$$

$$14a = 14$$

$$\tan tan\,\alpha = 1$$

$$\alpha = 45°$$

$$b = \frac{1}{2} = \sin sin\,\beta$$

$$\beta = 30°$$

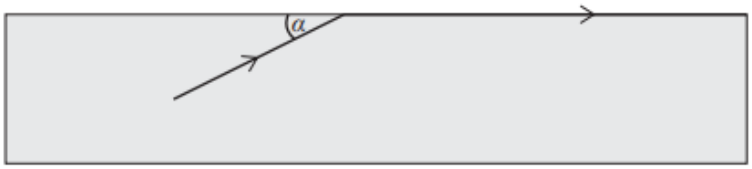
$$\theta = 180 - (45° + 35°) = 105°$$

Answer is D

ENGAA S1 2019 - Question 38

38 Light travelling in a transparent liquid strikes the surface from below. The angle between the surface of the liquid and the direction of travel of the light is α.

The light then travels along the surface between the liquid and the air as shown in the diagram.



Now, light travelling in air strikes the surface from above so that the angle between the surface and the direction of travel of this light is also α.

After the light strikes the surface from above, the angle between the surface and the direction of travel of the refracted light is β.

Which expression gives β?

(all angles are in degrees)

A $\cos\beta = \cos^2\alpha$

B $\cos\beta = \cos\alpha \sin^{-1}\left(\frac{1}{\alpha}\right)$

C $\sin\beta = \sin^2\alpha$

D $\beta = 90 - (90 - \alpha)^2$

E $\beta = 0$

F $\beta = \alpha$

ENGAA S1 2019 - Question 38 - Worked Solution

By Snell's law

$$\sin sin\,(90 - \alpha) = n \sin sin\,(90 - \beta)$$

$$\cos cos\,\alpha = n \cos cos\,\beta$$

Boundary condition

$$\sin sin\,(90 - \alpha) = n \sin sin\,(90 - 0) \rightarrow n = \frac{1}{\cos cos\,\alpha}$$

$$Thus \quad cos^2\alpha = \cos cos\,\beta$$

Answer is A

ENGAA S1 2019 - Question 39

39 Find the complete set of values of x for which

$$x^3 - 2x^2 - 7x - 4 > 0$$

A $x < -1$

B $x > -1$

C $-1 < x < 4$

D $x < -1$ or $x > 4$

E $x < 4$

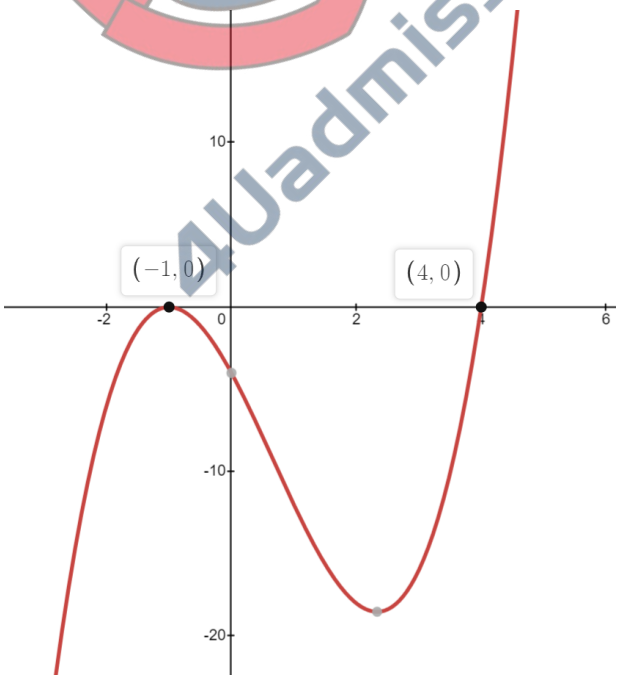
F $x > 4$

ENGAA S1 2019 - Question 39 - Worked Solution

$$f = x^3 - 2x^2 - 7x - 4 > 0$$

Use factor theorem to find that -1 is a root. This helps to factorise the expression.

$$(x + 1)^2(x - 4) > 0$$

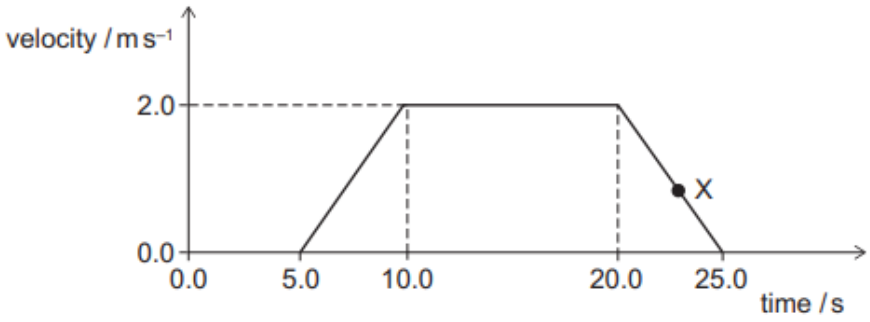


Drawing the graph helps see where it is above 0.

Answer is F

ENGAA S1 2019 - Question 40

40 The velocity–time graph is for an 80 kg person in a lift that is moving vertically upwards.



What is the magnitude of the contact force between the person and the lift floor at the time corresponding to X?

(gravitational field strength = 10 N kg^{-1})

A 640 N

B 768 N

C 800 N

D 832 N

E 960 N

ENGAA S1 2019 - Question 40 - Worked Solution

$$a = \frac{-2ms^{-1}}{5s} = 0.4ms^{-2}$$

$$F = mg + ma = 80\times10 - 0.4\times80$$

$$F = 80(10 - 0.4)$$

$$F = 768N$$

Answer is B