Worked Solutions for ENGAA Past Papers

ENGAA S2 2019 - Question 1

1 The ray diagram shows light passing from a vacuum into a medium.



Two angles, x and y, are shown on the diagram.

When x is 60°, y is 45°.

When x is 45°, what is the value of $\sin y$?



ENGAA S2 2019 - Question 1 - Worked Solution

 $\sin \sin 45 = \frac{\sqrt{2}}{2}$ $\sin \sin 60 = \frac{\sqrt{3}}{2}$

 $n\sin \sin x = \sin \sin y$

$$n \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{2}$$
$$n = \sqrt{\frac{2}{3}}$$

 $n. \sin \sin 45 = \sin \sin y$

$$\sin \sin y = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{3}}$$
$$\sin \sin y = \frac{1}{\sqrt{3}}$$

Answer is A



2 Identical resistors are used to produce three different arrangements X, Y and Z.



Each arrangement is connected, in turn, across the same battery which has a negligible internal resistance.

The total power developed in each of the arrangements is determined.

What is the order of the arrangements when placed in order of increasing power?



P = IRV = IR $\therefore P = \frac{V^2}{R}$

We want to rank them lowest power \rightarrow highest power

Thus: $highest R \rightarrow lowest R$

Χ,Υ,Ζ

Answer is A



3 A block of mass 2.0 kg is on a plane which is inclined to the horizontal at an angle of 30°.

The block is attached to a load of mass *M* by a light, inextensible string which passes over a smooth pulley.



ENGAA S2 2019 - Question 3 - Worked Solution



4 The battery in the circuit shown has an emf of 16 V and an internal resistance of 1.0Ω .



Which line in the table gives the voltmeter readings when switch S is in its open and closed states?

| | voltmeter reading /V de St | | |
|---|----------------------------|------------------|--|
| | when S is open | when S is closed | |
| A | $\frac{4.0}{3.0}$ | 2.0 | |
| в | 4.0 | 6.0 | |
| с | 4.0 | 2.4 | |
| D | 6.0 | 2.4 | |
| Е | 6.0 | 4.0 | |
| F | <u>48</u> 11 | 48 19 | |
| G | <u>48</u> 11 | 48 7.0 | |
| н | <u>128</u> 11 | <u>64</u> 7.0 | |

ENGAA S2 2019 - Question 4 - Worked Solution

$$V_{measured} = \frac{3}{R_0} \cdot 16V$$
$$V_{open} = \frac{13}{12} \cdot 16 = 4V$$
$$V_{closed} = \frac{3}{8} \cdot 16 = 6V$$

Answer is B



A stationary wave is set up in a medium in which the speed of the wave is 3.2 m s⁻¹. 5

The stationary wave is formed by the superposition of two longitudinal waves, each of amplitude 1.5 cm, travelling in opposite directions.

The distance between adjacent nodes in the stationary wave is 4.0 cm.

What is the total distance travelled by a particle at an antinode during a time interval of 1.0 minute?

- A 0m
- в 72 m
- С 144 m
- D 192 m
- 288 m Е
- 576 m F

ENGAA S2 2019 - Question 5 - Worked Solution

$$f = \frac{3.2ms}{0.08m} = \frac{v}{\lambda}$$

$$f = 40Hz$$

dmission in one minute : $40Hz \times 60s = 2400$ waves per min

Amplitude at antinode, where opposite waves superpose:

 $A = 1.5cm \times 2 = 3cm$

If the particle experiences one full wave it does 4 amplitue distance

$$d = 4A$$

Hence:

 $d_0 = 3cm \times 4 \times 2400 \min^{-1}$

- $= 28800 \, cm$
- = 288 m

Answer is E

6 A ray of light of single frequency *f* is travelling in a block of transparent material.

The ray strikes the boundary between the block and air at an angle θ to the boundary. When $\theta = 65^{\circ}$ the ray is just at the threshold of being totally internally reflected. Which of the following is an expression for the wavelength of the light in the material? (The speed of light in air is v_{air} .)



ENGAA S2 2019 - Question 6 - Worked Solution

$$\varphi = 90 - \epsilon$$

 $\frac{\sin \sin 90}{\sin \sin \varphi} = \frac{n_m}{n_{air}} = \frac{v_{air}}{v_{mat}} \quad by \ defination \ of \ optical \ index$

 $v_{mat} = v_{air} - \sin \sin \varphi$

 $\sin \sin \phi = \sin \sin (90 - \theta) = \cos \cos \theta$

$$v_{mat} = v_{air} \cdot \cos \cos 65$$

 $\lambda_{mat} = \frac{v_{air} \cos \cos 65}{f}$

Answer is A



Answer is E

8 The pressure exerted by a gas at constant temperature is directly proportional to its density.

A spherical bubble of gas forms at the bottom of a glass containing a fizzy drink.

The radius of the bubble at the point of formation, at the bottom of the drink, is R.

The depth of the liquid in the glass is h, and the density of the liquid of the drink is ρ .

Atmospheric pressure is P.

As the bubble rises, its radius changes.

Which expression gives the radius of the bubble when it is at a distance *x* below the surface of the drink?

(gravitational field strength = g; volume of sphere = $\frac{4}{3}\pi r^3$ where r is the radius; the mass and the temperature of the gas in the bubble remain constant)



ENGAA S2 2019 - Question 8 - Worked Solution P.V is constant

$$(h\rho g + P) \cdot 4\pi R^{3} = (x\rho g + P) \cdot 4\pi (R')^{3}$$

 $\therefore R' = (\frac{h\rho g + P}{x\rho g + P})^{\frac{1}{3}} R$

Answer is C

ENGAA S2 2019 - Question 9

9 A block of mass 2.0 kg slides directly down a smooth slope.

The slope is at an angle of 30° to the horizontal.

The block reaches a speed of $8.0 \,\mathrm{m\,s^{-1}}$, at which point the slope becomes rough and the block begins to decelerate.

After travelling a distance of 4.0 m down the rough slope the block comes to rest.

What is the magnitude of the average friction force between the block and the rough slope?

(gravitational field strength = 10 kg^{-1} ; assume that air resistance is negligible)



 $h = 4m \, . \, \sin \sin 30^{\circ}$ $\frac{1}{2}mu^{2} + mgh = \frac{1}{v}mv^{2} + mgh' + F \, . \, s$ Initial E = Final E + Work done by friction

 $\frac{1}{2}(2)8^{2} + 2 \times 10 \times 4 \sin \sin 30 = \frac{1}{2}(2)0^{7} + 2 \times 10 \times 0 + F.4$ $F = \frac{104}{4} = 26$

Answer is G



10 A non-uniform beam PQ of length 5.0 m and weight X rests on a pivot placed 3.0 m from end P. It is kept in equilibrium in a horizontal position by an upward force of magnitude 0.60X acting at end P.

The normal contact force at the pivot is 800 N.

What is the weight of the beam and how far is the centre of gravity of the beam from the pivot?

| | weight of beam / N | distance from pivot / m |
|---|--------------------|-------------------------|
| Α | 500 | 0.50 |
| в | 500 | 1.8 |
| С | 500 | 3.0 |
| D | 2000 | 0.50 |
| Е | 2000 | 1.8 |
| F | 2000 | 3.0 |

ENGAA S2 2019 - Question 7 - Worked Solution

ion 7 - Worked Solution Let d be the distance to the Center of Mass: Swimming torques about pivot. $d \cdot X = 3 \cdot (0.6X)$ d = 1.8m from pivot 0.6X + 800 = XX = 2000N

Answer is E

- A car is travelling along a straight road with constant acceleration. It passes a road sign.
 It travels 12.2 m in the 3rd second and 14.4 m in the 4th second after passing the road sign.
 What was the speed of the car as it passed the road sign?
 - A 2.20 m s⁻¹
 - **B** 4.50 m s⁻¹
 - C 6.70 ms⁻¹
 - D 7.80 m s⁻¹
 - E 13.3 ms⁻¹
 - F 37.2 m s⁻¹



Answer is C

12 A light spring has unstretched length 0.40m and spring constant 50Nm⁻¹.

The spring is stretched by a varying tension force that starts at a value of zero and increases at a constant rate of $0.20 \,\text{Ns}^{-1}$ up to a maximum value.

When the force reaches its maximum value, the strain energy of the spring is 0.25J.

What is the average power used to stretch the spring?

(Assume that the spring obeys Hooke's law.)

- **A** 0.010W
- B 0.020W
- C 0.040W
- **D** 0.080W
- E 1.0W F 2.0W G 4.0W H 8.0W ENGAA S2 2019 - Question 7 - Worked Solution

$$\frac{1}{2}Fx = 0.25$$
$$\frac{1}{2}kx^{2} = 0.25 \rightarrow PE$$
$$\frac{1}{2}kx^{2} = 25x^{2}$$
$$x = 10m$$
$$\frac{1}{2}F \cdot 10 = 0.25$$

 $\therefore F = 0.05N$

Rate of increasing force : $0.2Ns^{-1}$

$$\frac{0.05N}{0.2Ns^{-1}} = 25s$$
$$P = \frac{E}{t} = \frac{0.25J}{25s} = 0.01W$$

Answer is A

13 The circuit below contains three identical resistors, and two identical cells. When the switch is open, the power dissipated by resistor X is *P*.



What is the power dissipated by resistor X after the switch is closed?



ENGAA S2 2019 - Question 13 - Worked Solution

| Switch | $\sum v$ | $\sum R$ | Ι | Р |
|--------|----------|----------|------------------|--------------------|
| Open | 2V | 3R | $\frac{2}{3}V/R$ | $\frac{4}{9}V^2/R$ |
| Closed | V | 2R | $\frac{1}{2}V/R$ | $\frac{1}{4}V^2/R$ |

 $P' = \frac{9}{16}P$

Answer is B

ENGAA S2 2019 - Question 14

14 A car of mass *m* is pulling a caravan of mass *M*.

The caravan is connected to the car by a metal bar of length *l* and cross-sectional area A.

The Young modulus of the metal from which the bar is made is E.

The car and caravan have a constant forward acceleration a and there are total resistive forces D_1 acting on the car and D_2 acting on the caravan.

What is the extension of the bar?

(Assume that the bar obeys Hooke's law and that the cross-sectional area of the bar remains unchanged.)



ENGAA S2 2019 – Question 14 - Worked Solution

By Newton's 2^{nd} law on Caravan $T - D_2 = Ma$

By definition of Young's Modulus

$$E = \frac{T \times l}{A \times x} \to \frac{EAx}{l} - D_2 = Ma$$
$$x = \frac{(Ma + D_2)l}{EA}$$

Answer is C

15 Four resistors, P, Q, R and S, are connected to a battery with negligible internal resistance, as shown in the diagram.

P and S each have resistance x. Q and R each have resistance 2x.



A fifth resistor, T, which has resistance x, is to be added to the circuit in one of the following listed positions, as shown in the diagram:

- 1 in parallel with P
- 2 in series with Q
- 3 in parallel with R



Which of the positions for resistor T causes an increase in the magnitude of the voltmeter reading?

- A none of them
- B 1 only
- C 2 only
- D 3 only
- E 1 and 2 only
- F 1 and 3 only
- G 2 and 3 only
- H 1, 2 and 3



To maximize Voltmeter reading, increase difference between $V_1 \& V_2$

1 in parallel with $P \rightarrow \text{lowers } R_p$, lowers V_1 2 in series with $Q \rightarrow \text{increases } R_q$, lower V_1 3 in parallel with $R \rightarrow \text{makes } V_2 \text{ closer to } V_1$ Answer is E

16 A ball is thrown vertically upwards in air. The ball travels upwards to reach its highest point and then falls back down to its initial starting position. The velocity-time graph for the ball is shown.



ENGAA S2 2019 - Question 16 - Worked Solution

2 & 3 are based on reasoning of air resistance messing with the system's conservation. Answer is F



17 A stone is projected from level ground at an angle of 30° to the horizontal.

After 1.0s the stone lands on a ledge at height h above the level ground.

During this journey the vertical component of velocity of the stone is upwards for the first 0.60 s and downwards for the remaining 0.40 s.

What is the value of h?

(gravitational field strength = 10 N kg⁻¹; assume that air resistance is negligible)



Answer is A

18 A drawbridge system consists of a uniform ramp, of weight *W*, that is smoothly hinged at its lower end. The upper end is connected by a light, inextensible cable to a winch that is fixed to the wall in the position shown in the diagram.



The ramp is lowered slowly, at constant speed, from its closed (vertical) position ($\theta = 0^{\circ}$) to its open (horizontal) position ($\theta = 90^{\circ}$).

What is the maximum tension in the cable during this process?

(double-angle identities: $\sin 2\theta = 2\sin \theta \cos \theta$; $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$)

<u>w</u> 2 Α в $\sqrt{2}$ √3*W* С 2 D w 2W Ε √3 √2*W* F G 2*W*

ENGAA S2 2019 - Question 18 - Worked Solution



Answer is B

19 A particle of mass m has kinetic energy E when it collides with a stationary particle of mass M. The two particles coalesce.

Which of the following expressions gives the total kinetic energy transferred to other forms of energy in the collision?

A 0
B
$$\frac{ME}{(M+m)}$$

C $\frac{mE}{(M+m)E}$
E $\frac{(M+m)E}{M}$
F $\frac{mME}{(M+m)^2}$
G E
ENGAA S2 2019 - Question 19 - Worked Solution
KE of small particle:
 $E = \frac{1}{2}mv^2$
 $u = \sqrt{\frac{2E}{m}}$
Conservation of momentum
 $mu = (m + M)v$
 $v = \frac{m}{m+M}u$
Initial KE - Final KE
 $E_{lost} = \frac{1}{2}mv^2 - \frac{1}{2}(m + M)(\frac{m}{m+M})^2$
 $= E_0 - \frac{1}{2}\frac{m^2u^2}{m+M}$
 $= E_0(1 - \frac{m}{m+M})$
 $= E_0(1 - \frac{m}{m+M})$

Answer is B



20 The critical angle for light incident on a boundary from medium X to air is 45°.

The critical angle for light of the same frequency incident on a boundary from medium Y to air is 60°.

There is a boundary between medium X and medium Y. Light of the same frequency travelling in one of these mediums is incident on this boundary.

In which direction of incidence is there a critical angle at this boundary, and within what range is this critical angle?

| | direction of incidence | critical angle | |
|---|------------------------|---------------------|--|
| Α | X to Y | between 0° and 30° | |
| в | X to Y | between 30° and 45° | |
| С | X to Y | between 45° and 60° | |
| D | X to Y | between 60° and 90° | |
| Е | Y to X | between 0° and 30° | |
| F | Y to X | between 30° and 45° | |
| G | Y to X | between 45° and 60° | |
| н | Y to X | between 60° and 90° | |



critical angel is from slower optical medium to higher so,

 $x \rightarrow y$ $n_x \sin \sin \theta_c = n_y \sin \sin 90^\circ$ $\sin \sin \theta_c = \frac{n_y}{n_x} = \frac{\sin \sin 45^\circ}{\sin \sin 60^\circ} = \frac{\sqrt{2}/2}{\sqrt{3}/2} = \frac{\sqrt{2}}{\sqrt{3}} \cong 0.8$ $\sin \sin 45 = \frac{\sqrt{2}}{2} \cong 0.7$

$$\sin \sin 60 = \frac{\sqrt{3}}{2} \cong 0.87$$

 θ_c in between 45° & 60°

Answer is C

