Worked Solutions for ENGAA Past Papers

ENGAA S2 2020 - Question 1

1 Spring P has spring constant $1.0\,\mathrm{N\,cm^{-1}}$ and spring Q has spring constant $3.0\,\mathrm{N\,cm^{-1}}$.

The two springs are connected in series.

The springs are stretched by 6.0 cm in total.

What is the extension of spring P?

(The springs have negligible mass and obey Hooke's law.)

A 1.5 cm

B 2.0 cm

C 3.0 cm

D 4.0 cm

E 4.5 cm

ENGAA S2 2020 - Question 1 - Worked Solution

$$k_P : k = 1 : 3$$

$$x_P : x_Q = 3 : 1$$

$$x_P = \frac{6cm}{4} \cdot 3 = 4.5cm$$

Answer is E

ENGAA S2 2020 - Question 2

2 A single strand of wire has a radius of 2.0×10^{-4} m and length 15 m. The resistivity of the material from which the wire is made is $4.8 \times 10^{-7}\,\Omega\,$m.

Twelve strands of this wire are connected in parallel to make a cable.

What is the resistance of the cable?

A $\frac{\pi}{2160}\,\Omega$

B $\frac{\pi}{180}\,\Omega$

C $\frac{\pi}{15}\,\Omega$

D $\frac{15}{\pi}\,\Omega$

E $\frac{180}{\pi}\,\Omega$

F $\frac{2160}{\pi}\,\Omega$

ENGAA S2 2020 - Question 2 - Worked Solution

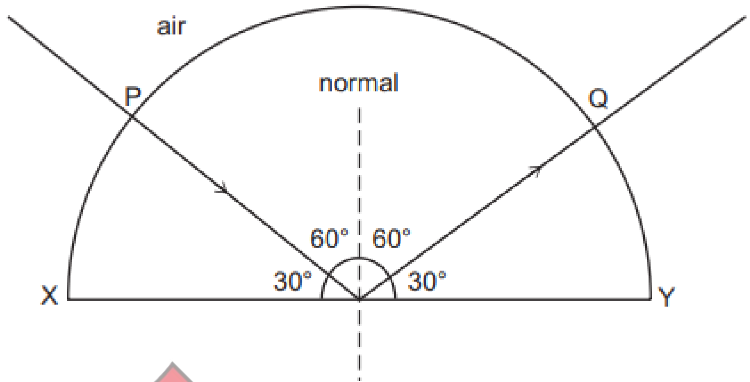
$$A = \pi r^2 = \pi\,.\,4\times10^{-8}$$

$$R = \frac{\rho\,.\,l}{12A} = \frac{15}{\pi}$$

Answer is D

ENGAA S2 2020 - Question 3

3 A ray of light is directed into a semicircular transparent block, entering at P. The direction of the ray is adjusted until it strikes the centre of the flat face XY of the block at the critical angle and reflects to Q as shown.



The length of XY is L.

The speed of light in air is c.

What is the time taken by the light to travel from P to Q in the block?

A $\frac{L\sqrt{3}}{2c}$

B $\frac{L}{c}$

C $\frac{2L}{c\sqrt{3}}$

D $\frac{L\sqrt{3}}{c}$

E $\frac{2L}{c}$

F $\frac{4L}{c\sqrt{3}}$

ENGAA S2 2020 - Question 3 - Worked Solution

$$V = C \,.\ \sin sin\, 60 = \frac{\sqrt{3}}{2}\, C$$

$$d = R + R = L$$

$$t = \frac{d}{s} = \frac{2L}{\sqrt{3}C}$$

Answer is C

ENGAA S2 2020 - Question 4

4 A solid cube with sides of length 20 cm is made from material with density 2000 kg m^{-3}. The cube is suspended, in equilibrium, from an initially unstretched spring, and this results in the spring gaining strain energy of 3.2 J.

What is the spring constant of the spring?

(gravitational field strength = 10 N kg^{-1}; the spring obeys Hooke's law)

A 40 N m^{-1}

B 80 N m^{-1}

C 400 N m^{-1}

D 800 N m^{-1}

E 4000 N m^{-1}

F 8000 N m^{-1}

ENGAA S2 2020 - Question 4 - Worked Solution

$$m = \rho V = 16 kg$$

$$E_S = \frac{1}{2} Fx$$

$$= \frac{F^2}{2k}$$

$$k = \frac{(ma)^2}{2E_s} = 4000 Nm^{-1}$$

Answer is E

ENGAA S2 2020 - Question 5

5 A projectile is fired upwards from the ground at an angle of 60° to the vertical at a speed of $20\,\text{m}\,\text{s}^{-1}$.

It travels a horizontal distance d and lands with a downwards vertical component of velocity of $4.0\,\text{m}\,\text{s}^{-1}$ on ground that is height h above the starting point of the projectile.

What are d and h?

(gravitational field strength = $10\,\text{N}\,\text{kg}^{-1}$; assume that air resistance is negligible)

	d / m	h / m
A	$6.0\sqrt{3}$	4.2
B	$6.0\sqrt{3}$	5.8
C	$10\sqrt{3} - 4.0$	4.2
D	$10\sqrt{3} - 4.0$	14.2
E	$10\sqrt{3} + 4.0$	5.8
F	$10\sqrt{3} + 4.0$	14.2
G	$14\sqrt{3}$	4.2
H	$14\sqrt{3}$	5.8

ENGAA S2 2020 - Question 5 - Worked Solution

Consider vertical component

$$v^2 . u^2 = 2as$$

$$(-4)^2 - (20 \sin sin\, 30)^2 = 2(-10)h$$

$$h = 4.2m$$

To find d we need vertical flight time

$$v = u + at$$

$$-4 = 20 \sin sin\, 30 - 10t$$

$$t = 1.4\text{s}$$

Horizontally:

$$d = v . t = 20 \cos cos\, 30 \times 1.4$$

$$d = 14\sqrt{3}$$

Answer is G

ENGAA S2 2020 - Question 6

6 Diagram 1 shows the positions of nine equally spaced particles in a medium.

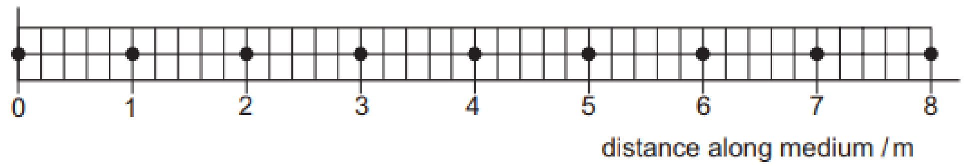


Diagram 1

Diagram 2 shows the positions of the same nine particles, at a particular time, while a longitudinal wave is travelling through the medium.

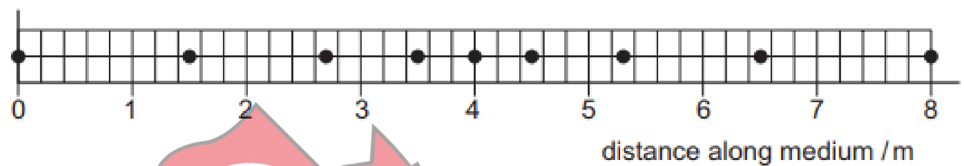


Diagram 2

What is the amplitude of the wave?

A 0.4 m

B 0.5 m

C 0.6 m

D 0.7 m

E 2.0 m

F 4.0 m

G 6.0 m

H 8.0 m

ENGAA S2 2020 - Question 6 - Worked Solution

Maximum displacement I 0.7m

Answer is D

ENGAA S2 2020 - Question 7

7 A spaceship with mass 8.0×10^4 kg travels at constant velocity and has 1.0×10^{12} J of kinetic energy.

An external impulse of 8.0×10^7 kg m s^{-1}, lasting for 2.0 s, is applied to the spaceship acting in the opposite direction to the motion of the spaceship.

What is the average rate of loss of kinetic energy of the spaceship during the application of the impulse?

A 9.5×10^{10} W

B 1.8×10^{11} W

C 2.2×10^{11} W

D 3.2×10^{11} W

E 3.6×10^{11} W

F 7.2×10^{11} W

ENGAA S2 2020 - Question 7 - Worked Solution

$$T = \frac{1}{2}mv^2$$

$$\therefore v = \sqrt{\frac{2T}{m}} = 5000ms^{-1}$$

$$F = m\frac{v-u}{\Delta t}$$

$$F\Delta t = mv - mu$$

$$8\times10^7\times2 = 8\times10^9 \times \left(v_2 - 5000\right)$$

$$v_2 = 4000ms^{-1}$$

$$T' - T = \frac{1}{2}m\left(v_2^{\ 2} - v_1^{\ 2}\right)$$

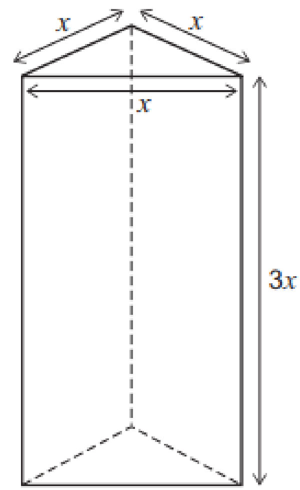
$$= -\ 3.6\times10^{11}J$$

$$P = \frac{\Delta T}{t} = \frac{-3.6\times10^{11}}{2s} = -\ 1.8\times10^{11}W$$

Answer is B

ENGAA S2 2020 - Question 8

8 The diagram shows a solid triangular prism.



The sides of the triangular cross section of the prism are of length x.

The height of the prism is $3x$.

The uniform density of the prism is ρ.

The gravitational field strength is g.

What is the minimum pressure the prism can exert when it rests on level ground?

A $3\rho g$

B $3\rho g x$

C $\dfrac{\rho g}{4}$

D $\dfrac{\rho g x}{4}$

E $\dfrac{\sqrt{3}\rho g}{4}$

F $\dfrac{\sqrt{3}\rho g x}{4}$

ENGAA S2 2020 - Question 8 - Worked Solution

$P = \frac{F}{A}$ $\quad$ $As\ F = \rho V g\ is\ constant,\ we\ ant\ to\ maximise\ A\ in\ order\ to\ minimise\ P$

$$P = \rho g \frac{V}{A}$$

$$= \rho g h$$

$$= \rho g \frac{\sqrt{3}}{4}$$

Answer is F

ENGAA S2 2020 - Question 9

9 An apple of mass m_a is placed on a uniform metre rule with the centre of gravity of the apple at the 10 cm mark. The rule is balanced on a pivot placed at the 35 cm mark.

The apple is replaced with an orange of mass m_o. The rule now balances with the pivot at the 40 cm mark.

What is the ratio $\frac{m_a}{m_o}$?

A $\frac{5}{9}$

B $\frac{4}{5}$

C $\frac{5}{6}$

D $\frac{6}{5}$

E $\frac{5}{4}$

F $\frac{9}{5}$

ENGAA S2 2020 - Question 9 - Worked Solution

Torques:

$$m_a \,.\, 25 = m_r \,.\, 15$$

$$m_o \,.\, 30 = m_r \,.\, 10$$

$$\frac{m_a}{m_o} \cdot \frac{25}{30} = \frac{15}{10}$$

$$\frac{m_a}{m_o} = \frac{9}{5}$$

Answer is F

ENGAA S2 2020 - Question 10

10 A cyclist travels at a constant speed of $12\,\text{m s}^{-1}$ on level ground. During this time the power needed to maintain a constant speed is 900 W. The total weight of the cyclist and bicycle is 850 N.

The cyclist now cycles up a slope at the same constant speed. The slope is at an angle of 30° to the horizontal.

What is the driving force on the bicycle as it travels up the slope?

(Assume that the magnitude of the resistive forces is constant.)

A 75 N

B 350 N

C 500 N

D $\left(425\sqrt{3} - 75\right)$N

E 775 N

F $\left(425\sqrt{3} + 75\right)$N

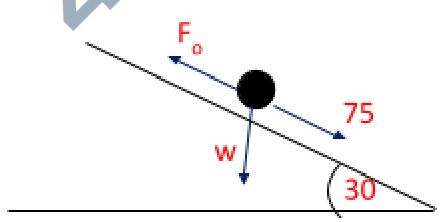
G 925 N

ENGAA S2 2020 - Question 10 - Worked Solution

$$d = s\,.\,t = 12cm$$

$$W = F\,.\,d$$

$$F = 75N$$



$$Weight\ down\ slope:\ \ W \sin sin\ 30$$

$$thus\ for\ balanced\ forces\ on\ slope:$$

$$F_o - 75 - 850 \sin sin\ 30 \ = 0$$

$$F_o = 500N$$

Answer is C

ENGAA S2 2020 - Question 11

11 Three identical resistors can be combined in four different arrangements.

One of the arrangements has a resistance of $18\,\Omega$.

A different arrangement has a resistance of $8.0\,\Omega$.

What are the resistances of the other two arrangements?

(All three resistors contribute to the total resistance in all arrangements.)

A $2.0\,\Omega$ and $4.0\,\Omega$

B $2.0\,\Omega$ and $9.0\,\Omega$

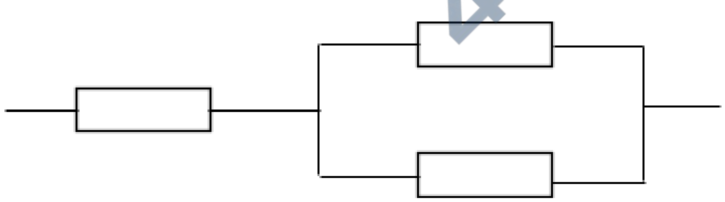
C $4.0\,\Omega$ and $12\,\Omega$

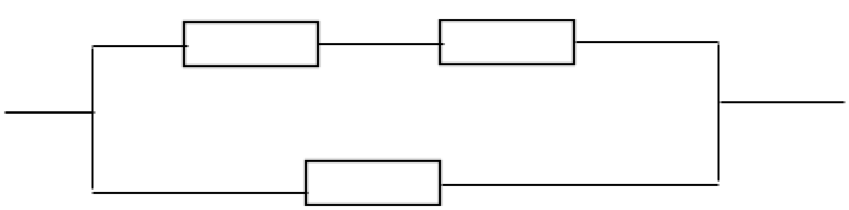
D $4.0\,\Omega$ and $36\,\Omega$

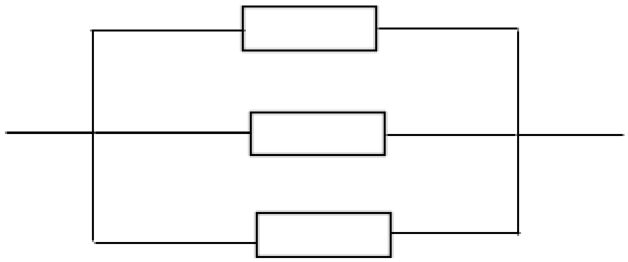
E $36\,\Omega$ and $162\,\Omega$

F $81\,\Omega$ and $162\,\Omega$

ENGAA S2 2020 - Question 11 - Worked Solution







Let each resistor have resistance R

$R_0 = 3R$

$R_o = \frac{3}{2}R$

$R_o = \frac{3}{2}R$

$R_o = \frac{1}{3}R$

$If\ R = 6\ ,\ R_o \in \{2\Omega\ ,\ 4\Omega\ ,\ 9\Omega\ ,\ 18\Omega\}$

$R = 12\ ,\ R_o\{4\Omega\ ,\ 8\Omega\ ,18\Omega\ ,\ 36\Omega\}$

$thus\ R = 12\Omega$

Answer is D

ENGAA S2 2020 - Question 12

12 A 4.0 kΩ fixed resistor is connected in series with a light dependent resistor (LDR) across a 100 V dc power supply.

The current in the LDR is 5.0 mA.

The intensity of light falling on the LDR now decreases and the voltage across the fixed resistor changes by 50%.

What is the change in the resistance of the LDR as a result of the change in intensity?

A 8.0 kΩ

B 12 kΩ

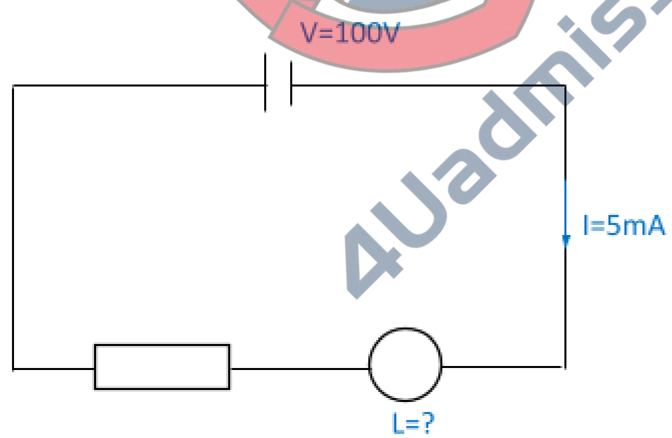
C 16 kΩ

D 20 kΩ

E 32 kΩ

F 36 kΩ

ENGAA S2 2020 - Question 12 - Worked Solution



$$V_{R_o} = IR = 20V$$

$$VR' = 10V$$

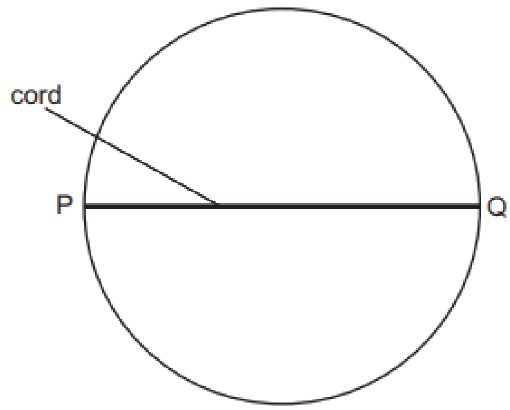
$$VR' = 100V \cdot \frac{4k\Omega}{L+4k\Omega} = 10v$$

$$L = 20000\Omega$$

Answer is D

ENGAA S2 2020 - Question 13

13 An elastic cord with spring constant k is fixed to two points P and Q on the diameter of a ring so that the cord is taut but unstretched. The radius of the ring is r.



The midpoint of the cord is then pulled and fixed to a point on the ring halfway between P and Q.

What is the energy stored in the elastic cord?

A $\frac{1}{2}kr^2$

B $2kr^2$

C $\frac{1}{2}(\sqrt{2}-1)kr^2$

D $2(\sqrt{2}-1)kr^2$

E $\frac{1}{2}(3-2\sqrt{2})kr^2$

F $2(3-2\sqrt{2})kr^2$

ENGAA S2 2020 - Question 13 - Worked Solution

$$r' = \sqrt{r^2 + r^2}$$

$$= \sqrt{2}r$$

$$E = \frac{1}{2}k\Delta x^2 = \frac{1}{2}k\left(2 . \sqrt{2}r - 2r\right)^2$$

.

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$$E = 2\left(3 - 2\sqrt{2}\right)kx^2$$

Answer is F

ENGAA S2 2020 - Question 14

14 An object of mass M experiences a resultant force of magnitude F. The force acts in a single horizontal direction with a magnitude that varies with time t according to

$$F = X + Y\sqrt{t}$$

where X and Y are constants.

The object is at rest at $t = 0$.

What is the magnitude of the momentum of the object at time $t = T$?

A $T\left(X + \frac{2}{3}Y\sqrt{T}\right)$

B $T\left(X + Y\sqrt{T}\right)$

C $\frac{T}{M}\left(X + \frac{2}{3}Y\sqrt{T}\right)$

D $\frac{T}{M}\left(X + Y\sqrt{T}\right)$

E $\frac{Y}{2\sqrt{T}}$

F $\frac{Y}{2M\sqrt{T}}$

ENGAA S2 2020 - Question 14 - Worked Solution

$$F = \frac{dP}{dt}$$

$$thus \quad P = \int F dt$$

$$P_T = \int_{t=0}^{t=T} \left(x + y\sqrt{t}\right)dt$$

$$t = 0$$

$$= \left| xt + \frac{2}{3}yt^{3/2} \right|_0^T$$

$$P_T = T\left(x + \frac{2}{3}y\sqrt{T}\right)$$

Answer is A

ENGAA S2 2020 - Question 15

15 A trolley of mass 3.0 kg is moving horizontally along a smooth track. Its displacement x from a point at time t is given by the equation:

$$x = 8 + 4t + 2t^2$$

where x is in metres and t is in seconds.

How much work is done on the trolley between times $t = 0$ and $t = 5.0$ s?

A 12 J

B 24 J

C 78 J

D 270 J

E 840 J

F 864 J

G 936 J

ENGAA S2 2020 - Question 15 - Worked Solution

$$V = \frac{dx}{dt} = 4 + 4t$$

$$V(t = 0) = 4$$

$$V(t = 5) = 24$$

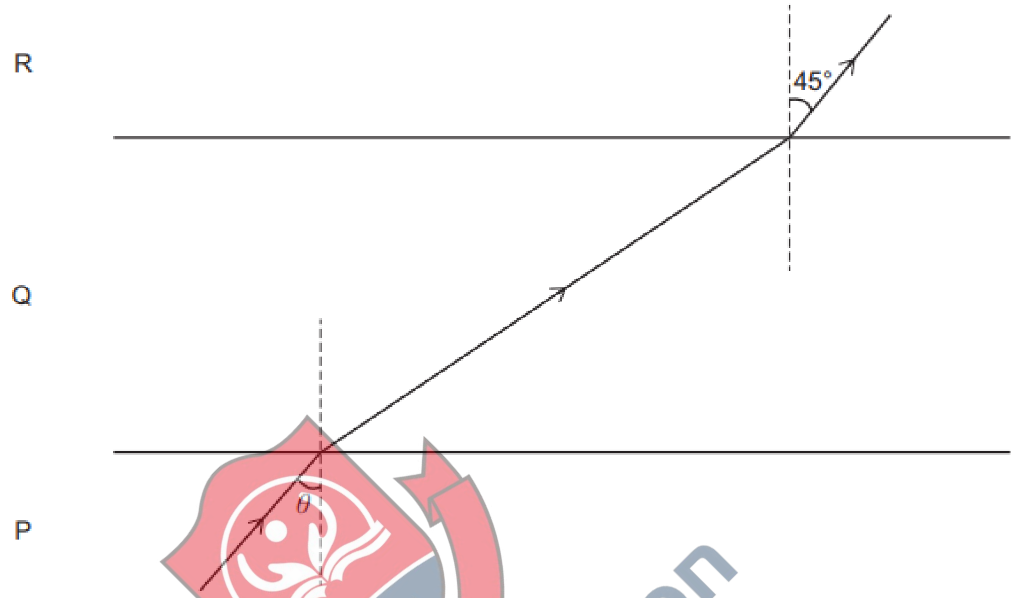
$$W = \Delta T = \frac{1}{2}m\left(24^2 - 4^2\right)$$

$$W = 840J$$

Answer is E

ENGAA S2 2020 - Question 16

16 The diagram shows a ray of light passing through three mediums, P, Q and R. The boundaries between the three mediums are parallel.



[diagram not to scale]

The ratio of the speed of light in medium P to the speed of light in medium Q is $2:\sqrt{5}$

The ratio of the speed of light in medium Q to the speed of light in medium R is $3:\sqrt{6}$

What is the value of $\sin\theta$?

A $\frac{\sqrt{2}}{2}$

B $\frac{\sqrt{3}}{2}$

C $\frac{\sqrt{3}}{6}$

D $\frac{\sqrt{5}}{5}$

E $\frac{\sqrt{15}}{5}$

F $\frac{\sqrt{15}}{6}$

ENGAA S2 2020 - Question 16 - Worked Solution

$$\sqrt{6}\sin sin\,\theta = 3\sin sin\,45$$

$$\sqrt{5}\sin sin\,\theta = 2\sin sin\,\theta$$

$$\sin sin\,\theta = \frac{2}{\sqrt{5}}\,.\,\frac{3}{\sqrt{6}}\,.\sin sin\,45$$

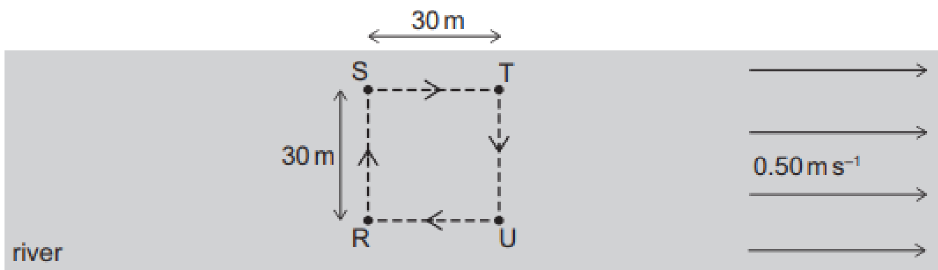
$$\sin sin\,\theta = \frac{\sqrt{15}}{5}$$

Answer is E

ENGAA S2 2020 - Question 17

17 Water in a wide river flows at a constant speed of 0.50 m s^{-1}. A swimmer swims around a square path of side 30 m marked out by 4 posts R, S, T and U which are fixed to the river bed, as shown.

The swimmer has a constant speed of 1.0 m s^{-1} relative to the water.



How long does it take for the swimmer to swim around the square path once?

A $(60 + 24\sqrt{5})$ s

B $(60 + 40\sqrt{3})$ s

C $(80 + 24\sqrt{5})$ s

D $(80 + 40\sqrt{3})$ s

E 120 s

F 140 s

ENGAA S2 2020 - Question 17 - Worked Solution

$$ST : 1ms^{-1}$$

$$TU : 0.5 - 1$$

$$UR : \ - 0.5$$

$$RS : 0.5 + 1$$

$$For\ vertical\ \ v = \sqrt{1^2 - 0.5^2}$$

$$= \frac{\sqrt{3}}{2} ms^{-1}$$

$$vert : 2\times\frac{30m}{\sqrt{3}/2} = 40\sqrt{3}\ s$$

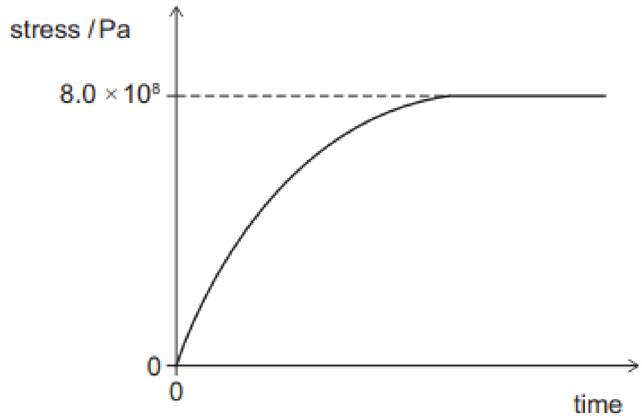
$$hoz : t = \frac{30}{1.5} + \frac{30}{0.5} = 80\ s$$

$$\therefore d = 80 + 40\sqrt{3}$$

Answer is D

ENGAA S2 2020 - Question 18

18 The stress in a steel cable increases with time and is then maintained at a constant value, as shown. The wire does not reach its limit of proportionality.



The table shows properties of the steel used in the cable and the dimensions of the cable.

length / m	*cross-sectional area* / m^2	*Young modulus* / Pa
4.0	2.0×10^{-4}	2.0×10^{11}

How much work was done to stretch the cable?

A 320 J

B 1.28 kJ

C 2.56 kJ

D 320 kJ

E 640 kJ

F 1.60 MJ

G 6.40 MJ

ENGAA S2 2020 - Question 18 - Worked Solution

$$F = \frac{y\,.\,Ax}{l}$$

$$\therefore x = stress\,.\,\frac{l}{y}$$

$$W = \int F dx = \int \frac{y\,Ax}{l} dx$$

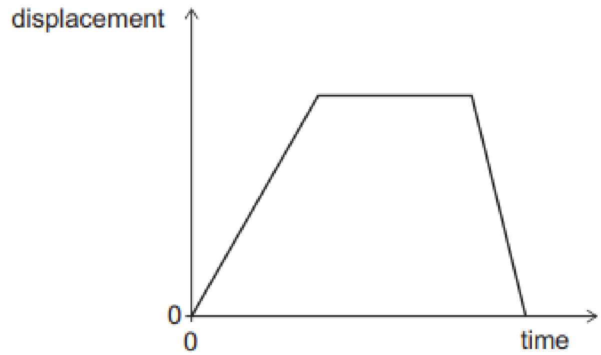
$$W = \frac{1}{2} \frac{yA}{l} x^2$$

$$W = 1280J$$

Answer is B

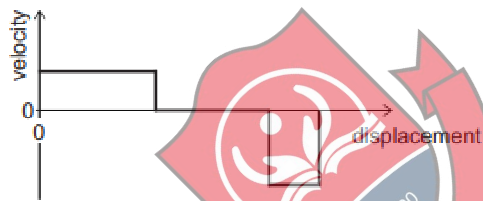
ENGAA S2 2020 - Question 19

19 The following graph shows how the displacement of an object travelling along a straight, horizontal track varies with time.

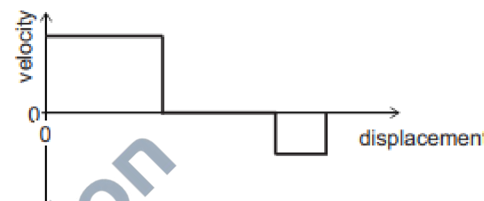


Which graph shows the velocity of this object against displacement?

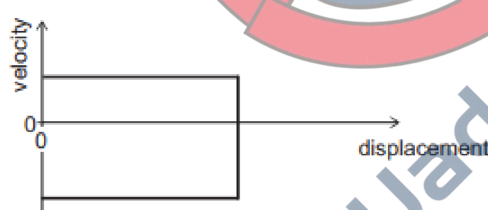
A



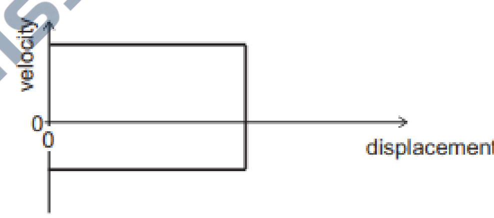
B



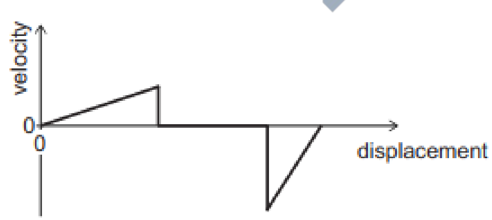
C



D



E



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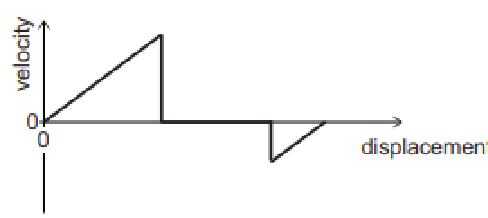

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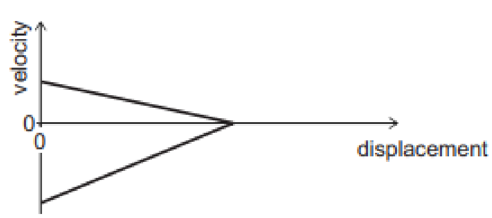

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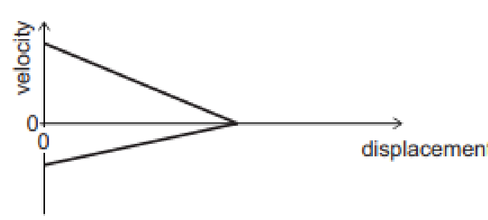

ENGAA S2 2020 - Question 19 - Worked Solution

Answer is C

ENGAA S2 2020 - Question 20

20 A cell has emf E and internal resistance r that varies with current I according to:

$$r = kI^2$$

where k is a constant.

A variable resistor is connected to the terminals of the cell. The resistance of the variable resistor is adjusted.

Which expression gives the resistance of the variable resistor, in terms of k and E, that causes maximum power dissipation in it?

A $3\left(\frac{kE^2}{2}\right)^{\frac{1}{3}}$

B $3\left(\frac{kE^2}{4}\right)^{\frac{1}{3}}$

C $3\left(\frac{kE^2}{9}\right)^{\frac{1}{3}}$

D $3\left(\frac{kE^2}{16}\right)^{\frac{1}{3}}$

E $\left(2kE^2\right)^{\frac{1}{3}}$

F $\left(4kE^2\right)^{\frac{1}{3}}$

G $\left(9kE^2\right)^{\frac{1}{3}}$

H $\left(16kE^2\right)^{\frac{1}{3}}$

ENGAA S2 2020 - Question 20 - Worked Solution

By ohm's law

$$E = I(R + r)$$

$$r = kI^2$$

$$\therefore IR = E - kI^3$$

$$P = I^2R = EI - kI^4$$

$$\frac{dP}{dI} = E - 4kI^3$$

$$\frac{dP}{dI} = 0 \quad when\, E = 4kI^3$$

OR

$$I = \left(\frac{E}{4k}\right)^{\frac{1}{3}}$$

$$IR = E + kI^3$$

$$= IR + \frac{E}{4}$$