

- 1 The cubic equation $2x^3 + x^2 - px - 5 = 0$, where p is a positive constant, has roots α, β, γ .

(a) State, in terms of p , the value of $\alpha\beta + \beta\gamma + \gamma\alpha$.

[1]

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(b) Find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$.

[2]



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- (c) Deduce a cubic equation whose roots are $\alpha\beta, \beta\gamma, \alpha\gamma$. [1]

[illegible]

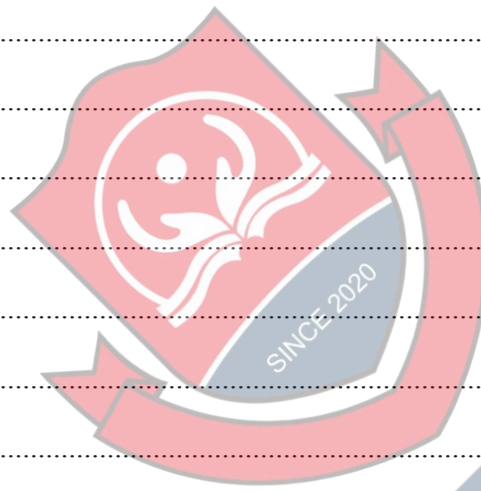
- (d) Given that $\alpha^2 + \beta^2 + \gamma^2 = \frac{1}{3}$, find the value of p . [2]

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- 2 Prove by mathematical induction that $6^{4n} + 38^n - 2$ is divisible by 74 for all positive integers n . [6]



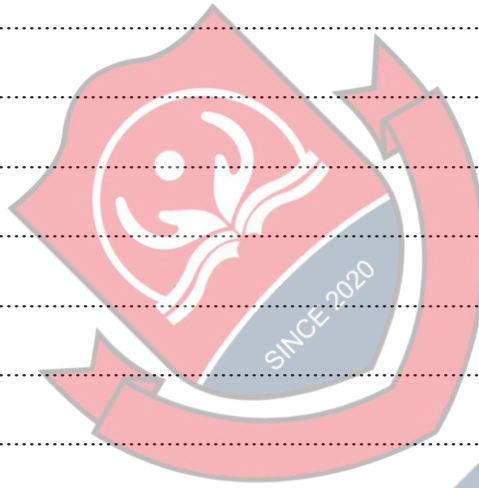
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- 3 (a) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^N r(r+1)(3r+4) = \frac{1}{12}N(N+1)(N+2)(9N+19). \quad [3]$$



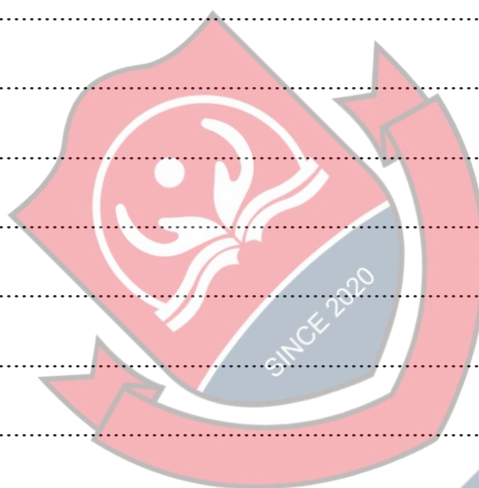
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- (b) Express $\frac{3r+4}{r(r+1)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^N \frac{3r+4}{r(r+1)} \left(\frac{1}{4}\right)^{r+1}$$

in terms of N .

[4]



- (c) Deduce the value of $\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)} \left(\frac{1}{4}\right)^{r+1}$.

[1]

4 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 14 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x - y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

(b) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

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- (c) Find the equations of the invariant lines, through the origin, of the transformation represented by $\mathbf{M}$. [5]

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- (d) The triangle ABC in the x - y plane is transformed by $\mathbf{M}$ onto triangle DEF .

Given that the area of triangle DEF is 28 cm^2 , find the area of triangle ABC . [2]

[illegible]

5 The points A, B, C have position vectors

$$2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k},$$

$$2\mathbf{i} + 4\mathbf{j} - \mathbf{k},$$

$$-3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k},$$

respectively, relative to the origin O .

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]



The point D has position vector $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

- (b) Find the perpendicular distance from D to the plane ABC . [2]

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- (c) Find the shortest distance between the lines AB and CD . [5]



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- 6** The curve C has equation $y = \frac{x^2 + ax + 1}{x + 2}$, where $a > \frac{5}{2}$.

(a) Find the equations of the asymptotes of C .

[3]

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(b) Show that C has no stationary points.

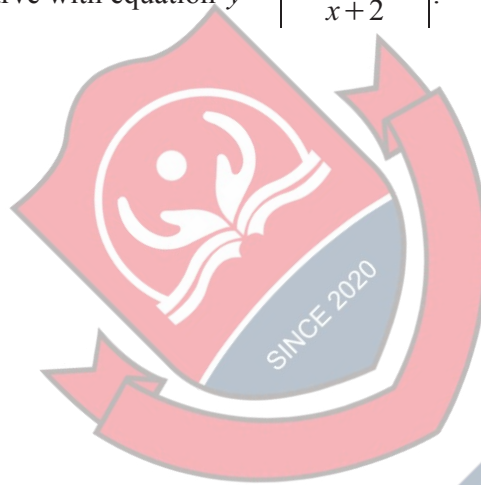
[4]



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- (c) Sketch C , stating the coordinates of the point of intersection with the y -axis and labelling the asymptotes. [3]

- (d) (i) Sketch the curve with equation $y = \left| \frac{x^2 + ax + 1}{x + 2} \right|$. [2]



- (ii) On your sketch in part (i), draw the line $y = a$. [1]

- (iii) It is given that $\left| \frac{x^2 + ax + 1}{x + 2} \right| < a$ for $-5 - \sqrt{14} < x < -3$ and $-5 + \sqrt{14} < x < 3$.

Find the value of a . [2]

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7 The curve C has polar equation $r^2 = (\pi - \theta) \tan^{-1}(\pi - \theta)$, for $0 \leq \theta \leq \pi$.

- (a) Sketch C and state the polar coordinates of the point of C furthest from the pole. [3]

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- (b) Using the substitution $u = \pi - \theta$, or otherwise, find the area of the region enclosed by C and the initial line. [7]

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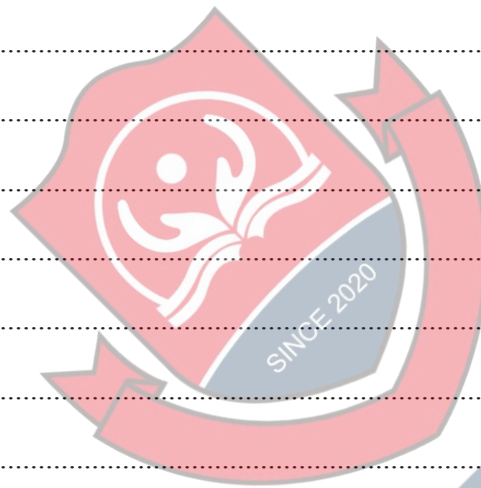
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- (c) Show that, at the point of C furthest from the initial line,

$$2(\pi - \theta) \tan^{-1}(\pi - \theta) \cot \theta - \frac{\pi - \theta}{1 + (\pi - \theta)^2} - \tan^{-1}(\pi - \theta) = 0$$

and verify that this equation has a root for θ between 1.2 and 1.3.

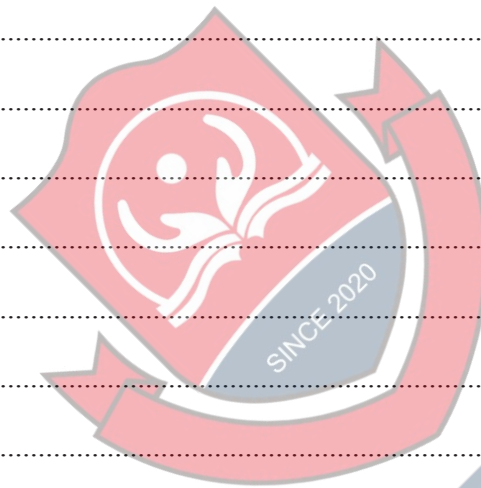
[5]



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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.



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