Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

6809632015

FURTHER MATHEMATICS **9231/12**

Paper 1 Further Pure Mathematics 1 **May/June 2024**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

DC (LK) 328363/3

1 The cubic equation $2x^3 + x^2 - px - 5 = 0$, where p is a positive constant, has roots α, β, γ.

(a) State, in terms of p, the value of $\alpha\beta + \beta\gamma + \gamma\alpha$. [1]

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(b) Find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$. [2]

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(c) Deduce a cubic equation whose roots are $\alpha\beta, \beta\gamma, \alpha\gamma$. [1]

(d) Given that $\alpha^2+\beta^2+\gamma^2=\frac{1}{3}$, find the value of p. [2]

2 Prove by mathematical induction that $6^{4n} + 38^n - 2$ is divisible by 74 for all positive integers n. [6]

3 **(a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{N} r(r+1)(3r+4) = \tfrac{1}{12}N(N+1)(N+2)(9N+19).$$ [3]

(b) Express $\dfrac{3r+4}{r(r+1)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^{N} \frac{3r+4}{r(r+1)}\left(\frac{1}{4}\right)^{r+1}$$

in terms of N. [4]

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(c) Deduce the value of $\displaystyle\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)}\left(\frac{1}{4}\right)^{r+1}$. [1]

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4 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 14 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x-y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

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(b) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

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(c) Find the equations of the invariant lines, through the origin, of the transformation represented by **M**. [5]

(d) The triangle ABC in the x-y plane is transformed by **M** onto triangle DEF.

Given that the area of triangle DEF is $28\,\text{cm}^2$, find the area of triangle ABC. [2]

5 The points A, B, C have position vectors

$$2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}, \qquad 2\mathbf{i} + 4\mathbf{j} - \mathbf{k}, \qquad -3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [5]

The point D has position vector $2\mathbf{i}+\mathbf{j}+3\mathbf{k}$.

(b) Find the perpendicular distance from D to the plane ABC. [2]

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(c) Find the shortest distance between the lines AB and CD. [5]

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6 The curve C has equation $y = \dfrac{x^2 + ax + 1}{x + 2}$, where $a > \frac{5}{2}$.

(a) Find the equations of the asymptotes of C. [3]

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(b) Show that C has no stationary points. [4]

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(c) Sketch C, stating the coordinates of the point of intersection with the y-axis and labelling the asymptotes. [3]

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(d) **(i)** Sketch the curve with equation $y = \left|\dfrac{x^2 + ax + 1}{x + 2}\right|$. [2]

(ii) On your sketch in part **(i)**, draw the line $y = a$. [1]

(iii) It is given that $\left|\dfrac{x^2 + ax + 1}{x + 2}\right| < a$ for $-5 - \sqrt{14} < x < -3$ and $-5 + \sqrt{14} < x < 3$.

Find the value of a. [2]

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7 The curve C has polar equation $r^2 = (\pi - \theta)\tan^{-1}(\pi - \theta)$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C and state the polar coordinates of the point of C furthest from the pole. [3]

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(b) Using the substitution $u = \pi - \theta$, or otherwise, find the area of the region enclosed by C and the initial line. [7]

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(c) Show that, at the point of C furthest from the initial line,

$$2(\pi-\theta)\tan^{-1}(\pi-\theta)\cot\theta-\frac{\pi-\theta}{1+(\pi-\theta)^2}-\tan^{-1}(\pi-\theta)=0$$

and verify that this equation has a root for θ between 1.2 and 1.3. [5]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.