



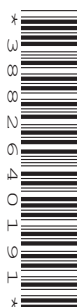
Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

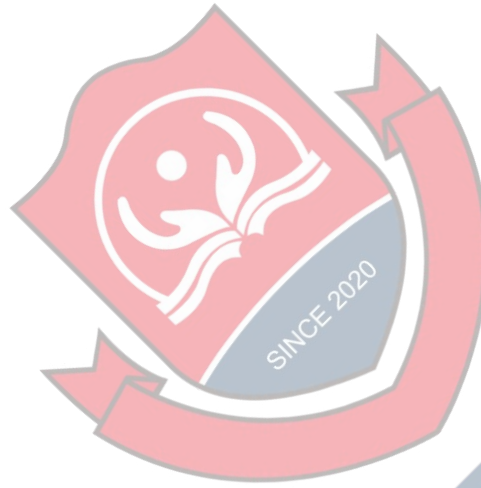
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



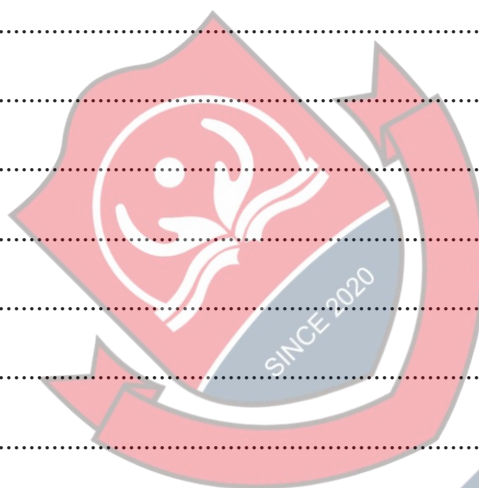
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- 1 Find the exact value of $\int_2^{\frac{7}{2}} \frac{1}{\sqrt{4x-x^2-1}} dx$.

[5]



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$$x = \cosh t, \quad y = \sinh t, \quad \text{for } 0 < t \leq \frac{3}{5}.$$

(a) Show that $s = \int_0^{\frac{3}{5}} \sqrt{\cosh 2t} \, dt$.

[4]

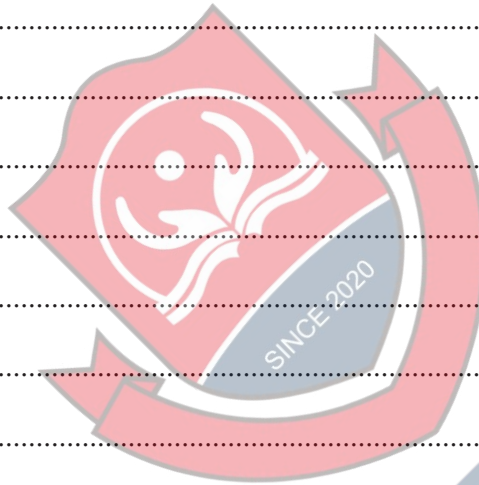
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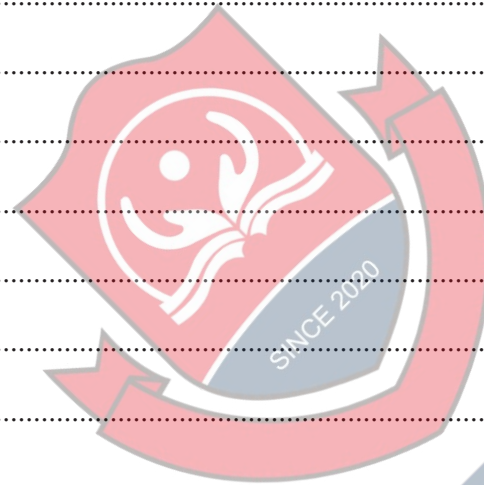
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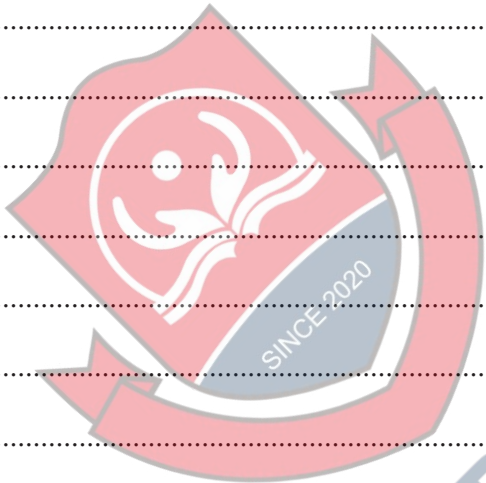
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$x^3 + 2xy + 8y^3 = -12.$

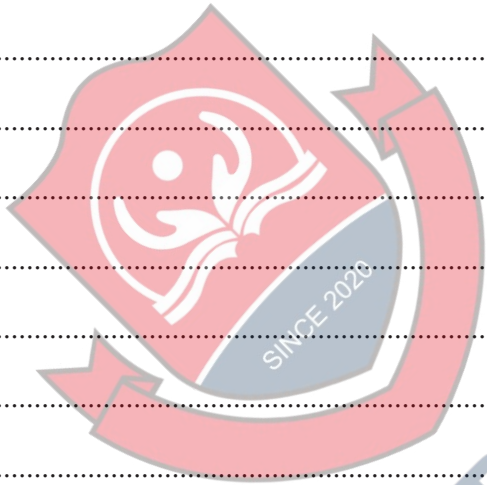
(a) Show that, at the point $(-2, -1)$ on C , $\frac{dy}{dx} = -\frac{1}{2}.$ [3]

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[5]



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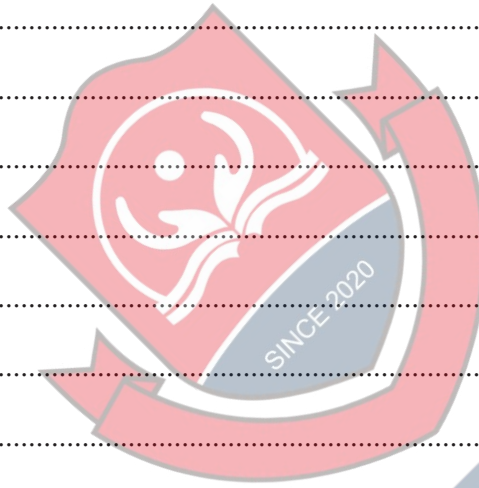
- (b) Use a similar method to find, in terms of N , an upper bound for $\sum_{r=1}^N \frac{1}{r^2}$. [3]

- (c) Deduce lower and upper bounds for $\sum_{r=1}^{\infty} \frac{1}{r^2}$. [2]

[illegible]

$$\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 25x = 338 \sin t.$$

[7]

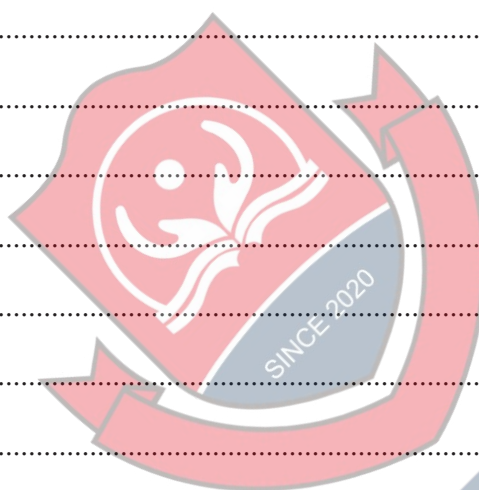
The image features a red and white circular logo in the top left corner, which includes a stylized flower and the text 'SINCE 2020'. A large, light blue diagonal watermark with the text '4U admission' is overlaid across the entire page. The background is white with horizontal dotted lines.

(b) Show that, for large positive values of t and for any initial conditions,

$$x \approx R \sin(t - \phi),$$

where the constants R and ϕ are to be determined.

[3]



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7 (a) Show that

$$\frac{d}{dx} \left(\frac{x}{2} \sqrt{x^2 - 9} - \frac{9}{2} \cosh^{-1} \frac{x}{3} \right) = \sqrt{x^2 - 9}. \quad [3]$$

(b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \sqrt{x^2 - 9},$$

given that $y = 1$ when $x = 3$. Give your answer in the form $y = f(x)$. [9]







8 The planes Π_1 and Π_2 do not intersect and are both perpendicular to $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. The line l intersects Π_1 at the point $(1, 6, 0)$ and intersects Π_2 at the point $(3, -6, 0)$.

(a) Find Cartesian equations of Π_1 and Π_2 . [3]

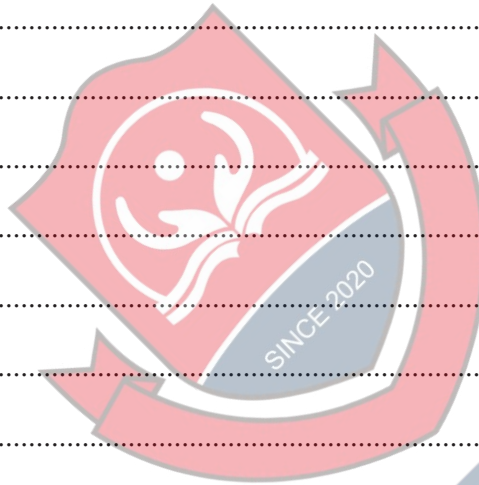
(b) Express the vector equation of l in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{a} + \lambda \mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are vectors to be determined, and hence show that for points on l , $\frac{1}{2}x + \frac{1}{12}y = 1$ and $z = 0$. [2]

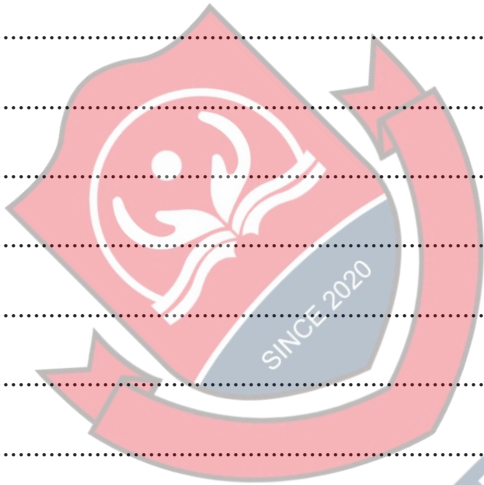




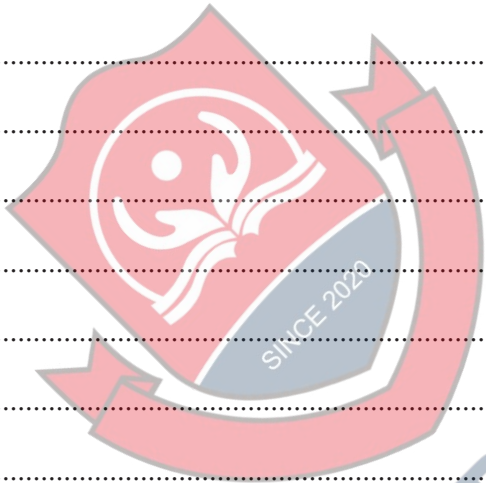
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(c) Show that the characteristic equation of $\mathbf{A}$ is $-\lambda^3 + 3\lambda^2 + \frac{7}{4}\lambda = 0$ and hence find the eigenvalues of $\mathbf{A}$. [3]

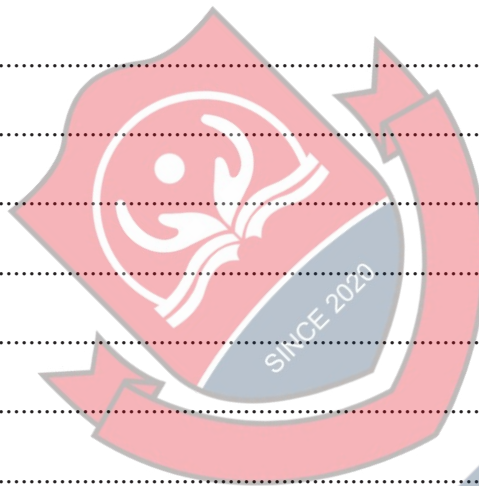
The image features a large, light blue watermark reading "4U admission" diagonally across the page. In the top left corner, there is a partial view of the University of Utah logo, which includes a red shield with a white stylized flower and the text "SINCE 2020". The background is white with horizontal dotted lines.



The logo of the Ministry of National Education (T.C. Millî Eğitim Bakanlığı) is prominently displayed in the center. It features a stylized sun with rays and a book, symbolizing knowledge and enlightenment. The text 'SINCE 2020' is written below the sun, and '4Uadmission' is written in a large, bold, sans-serif font across the bottom of the logo.



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