Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER



**FURTHER MATHEMATICS** **9231/32**

Paper 3 Further Mechanics **October/November 2024**

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ($g$) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].



This document has **16** pages. Any blank pages are indicated.




[Turn over

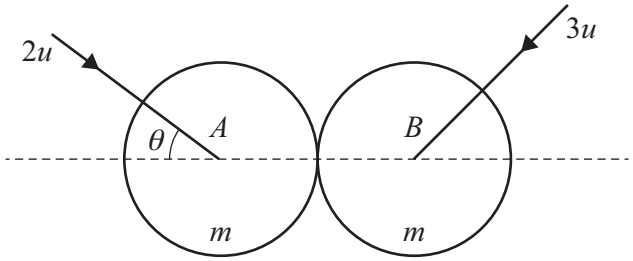
**1** A particle of mass 2 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 100 N. The other end of the string is attached to a fixed point $O$ on a smooth horizontal surface. The particle is moving in a horizontal circle about $O$ with the string taut and with constant angular speed 5 radians per second.

Find the extension of the string. [3]

**2** A particle $P$ of mass $m$ is attached to one end of a light elastic spring of natural length $a$ and modulus of elasticity $5mg$. The other end of the spring is attached to a fixed point $O$. The spring hangs vertically with $P$ below $O$. The particle $P$ is pulled down vertically and released from rest when the length of the spring is $\frac{3}{2}a$.

Find the distance of $P$ below $O$ when $P$ first comes to instantaneous rest. [4]

3




The diagram shows two identical smooth uniform spheres $A$ and $B$ of equal radii and each of mass $m$. The two spheres are moving on a smooth horizontal surface when they collide with speeds $2u$ and $3u$ respectively. Immediately before the collision, $A$'s direction of motion makes an angle $\theta$ with the line of centres and $B$'s direction of motion is perpendicular to that of $A$. After the collision, $B$ moves perpendicular to the line of centres. The coefficient of restitution between the spheres is $\frac{1}{3}$.

**(a)** Find the value of $\tan\theta$. [3]

**(b)** Find the total loss of kinetic energy as a result of the collision. [2]

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**(c)** Find, in degrees, the angle through which the direction of motion of $A$ is deflected as a result of the collision. [2]

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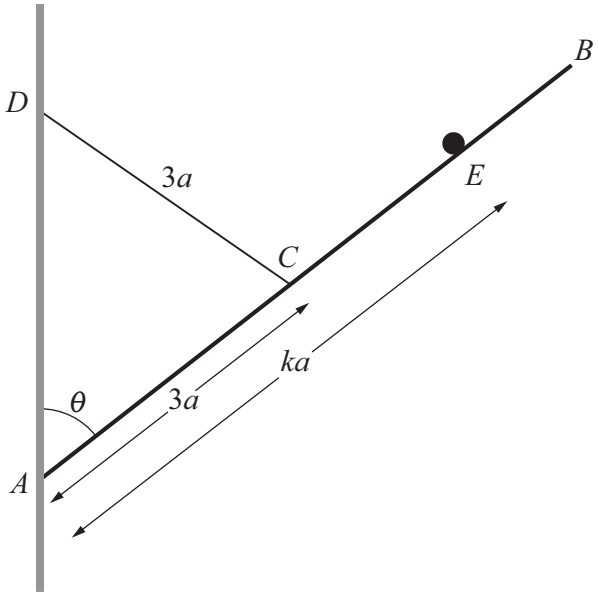


The end $A$ of a uniform rod $AB$ of length $6a$ and weight $W$ is in contact with a rough vertical wall. One end of a light inextensible string of length $3a$ is attached to the midpoint $C$ of the rod. The other end of the string is attached to a point $D$ on the wall, vertically above $A$. The rod is in equilibrium when the angle between the rod and the wall is $\theta$, where $\tan\theta = \frac{3}{2}$. A particle of weight $W$ is attached to the point $E$ on the rod, where the distance $AE$ is equal to $ka$ $(3 < k < 6)$ (see diagram). The rod and the string are in a vertical plane perpendicular to the wall. The coefficient of friction between the rod and the wall is $\frac{1}{3}$. The rod is about to slip down the wall.

**(a)** Find the value of $k$. [5]

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**(b)** Find, in terms of $W$, the magnitude of the frictional force between the rod and the wall. [2]

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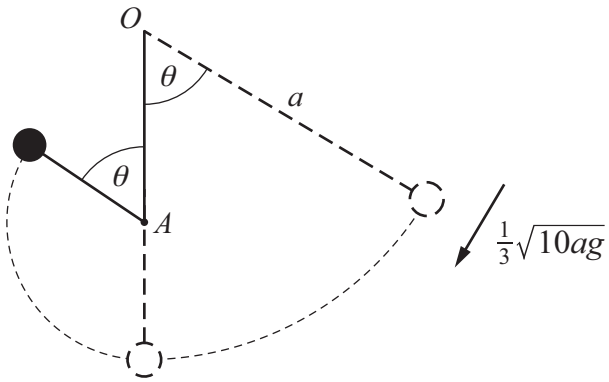
**5** A particle $P$ is projected from a point $O$ on horizontal ground with speed $u$ at an angle $\theta$ above the horizontal, where $\tan\theta = \frac{1}{3}$. The particle $P$ moves freely under gravity and passes through the point with coordinates $(3a, \frac{4}{5}a)$ relative to horizontal and vertical axes through $O$ in the plane of the motion.

**(a)** Use the equation of the trajectory to show that $u^2 = 25ag$. [2]

At the instant when $P$ is moving horizontally, a particle $Q$ is projected from $O$ with speed $V$ at an angle $\alpha$ above the horizontal. The particles $P$ and $Q$ reach the ground at the same point and at the same time.

**(b)** Express $V^2$ in the form $kag$, where $k$ is a rational number. [6]

6




A particle $P$ of mass $m$ is attached to one end of a light inextensible string of length $a$. The other end of the string is attached to a fixed point $O$. The particle $P$ is held with the string taut and the string makes an angle $\theta$ with the downward vertical through $O$. The particle $P$ is projected at right angles to the string with speed $\frac{1}{3}\sqrt{10ag}$ and begins to move downwards along a circular path. When the string is vertical, it strikes a small smooth peg at the point $A$ which is vertically below $O$. The circular path and the point $A$ are in the same vertical plane. After the string strikes the peg, the particle $P$ begins to move in a vertical circle with centre $A$. When the string makes an angle $\theta$ with the upward vertical through $A$ the string becomes slack (see diagram). The distance of $A$ below $O$ is $\frac{5}{9}a$.

**(a)** Find the value of $\cos\theta$. [6]

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**(b)** Find the ratio of the tensions in the string immediately before and immediately after it strikes the peg. [4]

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7 A particle $P$ of mass $m$ kg is held at rest at a point $O$ and released so that it moves vertically under gravity against a resistive force of magnitude $0.1mv^2$ N, where $v$ m s$^{-1}$ is the velocity of $P$ at time $t$ s.

**(a)** Find an expression for $v$ in terms of $t$. [6]

The displacement of $P$ from $O$ at time $t$ s is $x$ m.

**(b)** Find an expression for $v^2$ in terms of $x$. [5]

## Additional page

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