



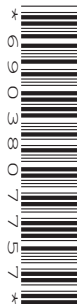
# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability &amp; Statistics

October/November 2024

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

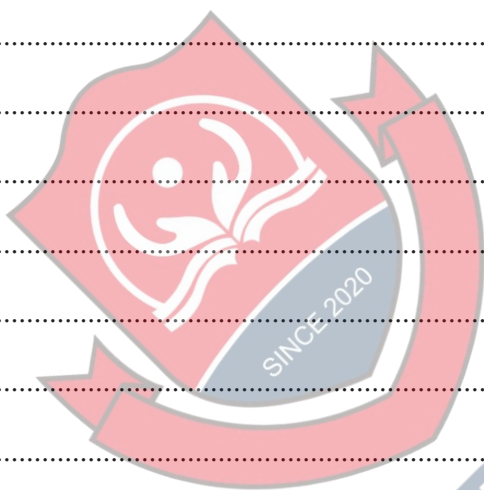


- 1 Ellie is investigating the heights of two types of beech tree,  $A$  and  $B$ , in a certain region. She has chosen a random sample of 60 beech trees of type  $A$  in the region, recorded their heights,  $x$  m, and calculated unbiased estimates for the population mean and population variance as 35.6 m and  $4.95 \text{ m}^2$  respectively.

Ellie also chooses a random sample of 50 beech trees of type  $B$  in the region and records their heights,  $y$  m. Her results are summarised as follows.

$$\Sigma y = 1654 \quad \Sigma y^2 = 54\,850$$

Find a 95% confidence interval for the difference between the population mean heights of type  $A$  and type  $B$  beech trees in the region. [6]



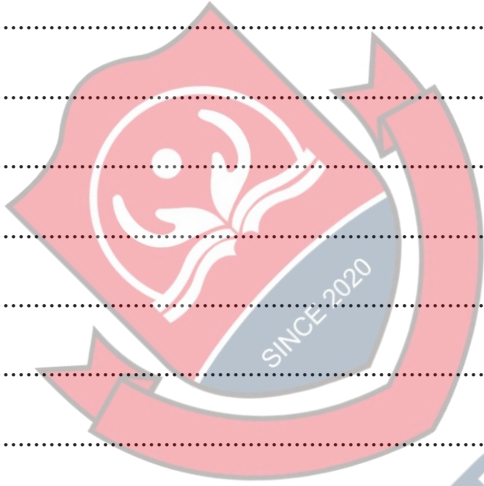


- 2 A school with a large number of students is updating its logo. Each student has designed a new logo and two teachers have each awarded a mark out of 50 for each logo. The marks awarded to a random sample of 12 students are shown in the following table.

| Student   | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ | $I$ | $J$ | $K$ | $L$ |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Teacher 1 | 36  | 38  | 40  | 36  | 22  | 34  | 45  | 44  | 48  | 35  | 28  | 30  |
| Teacher 2 | 38  | 42  | 32  | 41  | 32  | 41  | 42  | 50  | 36  | 44  | 42  | 41  |

One of the students claims that Teacher 2 is awarding higher marks than Teacher 1.

- (a) Carry out a Wilcoxon matched-pairs signed-rank test, at the 5% significance level, to test whether the data supports the claim. [7]

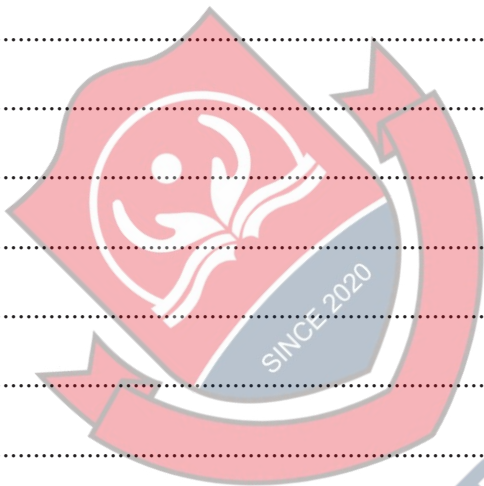


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- 4 The continuous random variable  $X$  has probability density function  $f$  given by

$$f(x) = \begin{cases} kx^3 & 0 \leq x < 1, \\ k(5-x) & 1 \leq x \leq 5, \\ 0 & \text{otherwise,} \end{cases}$$

where  $k$  is a constant.

- (a) Sketch the graph of  $f$ .

[1]



- (b) Show that  $k = \frac{4}{33}$ .

[2]

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[3]

[4]



- 5 Nikita has three coins. One coin is fair, one coin is biased so that the probability of obtaining a head is  $\frac{1}{3}$  and the third coin is biased so that the probability of obtaining a head is  $\frac{1}{5}$ . The random variable  $X$  is the number of heads that Nikita obtains when he throws all three coins at the same time.

(a) Find the probability generating function of  $X$ . [3]

Rajesh has two fair six-sided dice with faces labelled 1, 2, 3, 4, 5, 6. The random variable  $Y$  is the number of 4s that Rajesh obtains when he throws the two dice.

The random variable  $Z$  is the sum of the number of heads obtained by Nikita and the number of 4s obtained by Rajesh.

(b) Find the probability generating function of  $Z$ , expressing your answer as a polynomial. [4]

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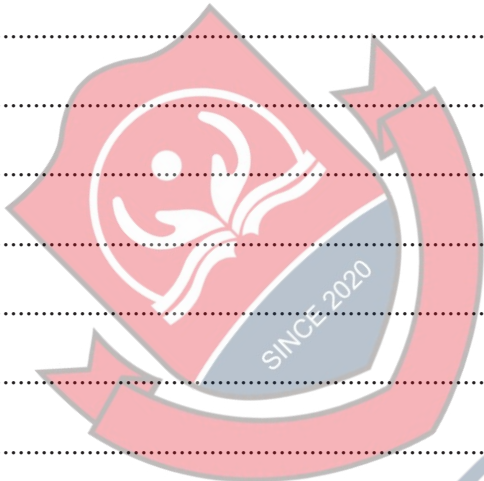


[2]

- 8.2      7.0      7.3      8.8      7.8      8.5      9.2      7.4

$$\Sigma y = 71.10 \quad \Sigma y^2 = 567.13$$

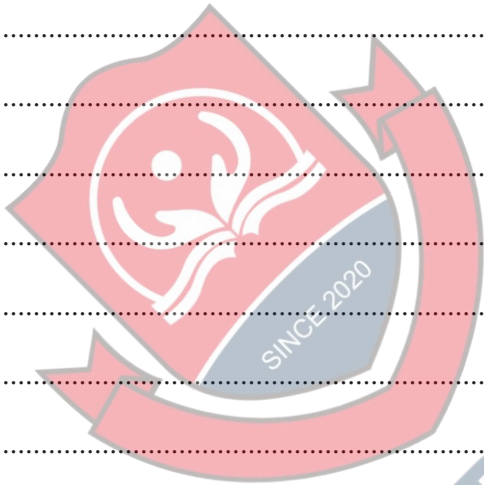
Test, at the 10% significance level, whether Ansal's suspicion is supported by the data. [8]



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If you use the following page to complete the answer to any question, the question number must be clearly shown.



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