MATHEMATICS

Paper 9709/11
Pure Mathematics 1 (11)

Key messages

The question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' This means that clear working must be shown to justify solutions, particularly in syllabus items such as quadratic equations and trigonometric equations. In the case of quadratic equations, for example, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient for certain marks to be awarded. It is also insufficient to quote only the formula: candidates need to show values substituted into it. When factorising, candidates should ensure that the factors always expand to give the coefficients of the quadratic equation.

Candidates should take care to read each question carefully and to note the number of marks attached to each question.

General comments

Some very good responses were seen but the paper proved very challenging for many candidates. In AS and A Level Mathematics papers the knowledge and use of basic algebraic and trigonometric methods from IGCSE or O Level is expected, as stated in the syllabus.

Comments on specific questions

Question 1

(a) A well answered question with many candidates getting full marks. Successful candidates factorised the whole expression or factorised the first two terms before completing the square. Candidates should check that their final expression is equivalent to the original expression.

(b) A generally well answered question with many achieving full marks. Most candidates did not use their answer from part **(a)**. The most common method where a majority of candidates gained full marks was one of substitution for x^2 before factorising. Of those who did not get full marks the main reasons were not showing a factor of 3 before factorising or not showing their substitution.

Question 2

(a) Successful students scored all 4 marks by correctly stating the transformations, beginning with a stretch followed by a translation. Candidates who started with a translation invariably used the vector $\begin{pmatrix}0\\-2\end{pmatrix}$ rather than $\begin{pmatrix}0\\-\frac{2}{3}\end{pmatrix}$. Where marks were lost it was due to a failure to fully describe the transformation e.g. translation down 2, rather than translation down 2 vertically.

(b) Most candidates gave the answer in the form f $(x) = a \sin x + b$ and many of these gave the correct answer.

Question 3

(a) Candidates that gained credit found $20 \times 27 \times a^3 = 160$ and hence found the correct value of a. Candidates should take care when simplifying expressions involving powers e.g., simplifying $(ax)^3$ to a^3x^3 rather than ax^3

(b) Successful candidates found the coefficient of x^2 using $6C2 \times 3^4 \times a^2$. As in **part (a)**, some candidates made errors in the use of indices by using $(ax)^2 = ax^2$. Candidates who were not awarded the B1 were often able to gain the M1 mark for $160 \times 1 - 2 \times$ *their* $6C2 \times 3^4 \times a^2$

Question 4

(a) Successful candidates used the given formula to find f (2.0001) and gave k correct to 5 decimal places.

(b) Successful candidates used $\frac{y_2 - y_1}{x_2 - x_1}$ and the given coordinates of A and E to find the gradient of AE.

(c) Successful candidates used the values of the gradients of the chords to find f' (2). Some candidates found f'(x) using differentiation but often went astray due to sign errors or poor algebra.

Question 5

(a) Many candidates gained full marks on this question using a substitution, factorising, and simplifying. Some candidates worked on both sides of the equation to prove the identity.

(b) Some candidates gained full marks on this question. Other candidates rearranged the equation to find and solve a quadratic which had an additional, invalid, solution. Candidates who wrote $\cos x = \frac{-1}{4}$ were unable to gain any marks.

Question 6

(a) Many candidates gained one mark on this question by drawing a curve in the 4th quadrant but only a few candidates gained full marks. The mirror line of $y = x$ was often omitted and the curve was often not in the correct position within the 4th quadrant.

(b) A number of candidates were familiar with finding the inverse of a function and many gained 2 of the 3 marks available. Some candidates considered the range of $f^{-1}(x)$ and found

$f^{-1}(x) = -\sqrt{\frac{2}{x-4}}$

(c) Successful candidates solved the equation f(x) = 4.5 and used the diagram to deduce that the solution is negative. Other candidates were familiar with solving equations of this form but did not consider the two square roots of 4 and gave an answer of 2 rather than –2

(d) There were several answers that gained the mark with the most common one being a statement that the two curves do not intersect. Incorrect answers included 'negative numbers do not have a square root' and 'a function cannot equal its inverse'.

Question 7

(a) The most common approach was to find angle $\theta = \frac{\pi}{2} - \cos^{-1}\frac{10}{15}$ *or* $\sin^{-1}\frac{10}{15} = 0.7297$

(b) For many this was a successful question with good application of the correct radian formulae and identification of the necessary parts needed for a complete solution. Some candidates stopped after finding only the perimeter or area. Candidates are to be reminded to note the number of marks available for each question.

Question 8

(a) Successful candidates gained full marks on this question whilst many gained some marks. Some candidates set up two simultaneous equations in *d* and *p,* solved them to find *d* and *p* and then found the 10th term. Other successful candidates set up one equation in *p*, solved it to find *p* and then found the 10th term. Some candidates didn't gain credit for their answers because of sign errors, poor algebra, confusing *p* and *d* or using sum of the series formula instead of the *n*th term formula.

(b) Overall, candidates were more successful in solving **part (b)** than **part (a)** with some candidates gaining full marks. Candidates were familiar with setting up a quadratic equation from a geometric sequence and many then solved the equation to find *q* or *r* and hence the sum to infinity. Some candidates did not show a complete method for solving the quadratic and were unable to gain full marks.

Question 9

Many candidates were able to integrate, and more successful candidates stated that the volume of revolution $= \pi \int \frac{1}{(5x-4)^{\frac{2}{3}}}\,dx$ and integrated appropriately. Other candidates found the area under the curve rather than the volume of revolution and they are to be reminded to read each question carefully to gain maximum credit. Few candidates found the volume of the cylinder but those that did used integration or the formula for the volume of a cylinder.

Question 10

Successful candidates used substitution to find $(x-3)^2 + (mx-9)^2 = 18$, simplified this to $(m^2+1)x^2 - (6 + 18m)x + 72 = 0$ and then used $b^2 - 4ac = 0$ to set up a quadratic in m; $m^2 + 6m - 7 = 0$. Candidates who reached this point were generally then able to find the two points correctly. Some candidates used the substitution $y = mx + c$ and were unable to make any significant progress. A few candidates mistakenly used $m = \frac{1}{9}$.

Question 11

(a) Many candidates differentiated successfully and solved $-\frac{12}{x^4} + \frac{3}{x^2} = 0$ to find $x = \pm 2$. A few candidates stated an incorrect solution of $x = 0$. A fully correct solution was found by a small number of candidates using a variety of notations.

(b) Candidates that were successful found the equation of the tangent and normal and equated them to find the point of intersection. The area of the triangle was found by a variety of methods including integration and Pythagoras with the area of a triangle. However, most candidates were unable to make much progress.

MATHEMATICS

Paper 9709/12
Pure Mathematics 1 (12)

Key messages

Candidates would benefit from spending time to ensure they understand the terminology required in responses and the structure of examination papers at Advanced Level. For example, the correct terminology for transformation questions is 'translation', followed by a column vector, or 'stretch', with the word 'factor' and both its size and direction clearly indicated.

If information is given in the introduction to the question, then it is true and relevant for the whole question. However, if information is given in **part (a)**, then it is only true and relevant for that part of the question. The phrase 'it is given instead' is often used to emphasise this in a subsequent part of a question. Many candidates failed to appreciate this distinction in **Question 8**.

Where exact answers are requested, such as in **Question 5(a)**, then a rounded decimal is not acceptable. Exact answers should not contain fractions divided by fractions but should be simplified. Premature rounding often means that the final answer is inaccurate. If a final answer is requested to three significant figures, then intermediate working should be carried out to at least four significant figures.

Care needs to be taken to ensure that the whole of a given function is used and not just part of it. This was a particular issue in both **Questions 9** and **10**.

General comments

The paper was generally found to be accessible for candidates and many excellent scripts were seen. Candidates seemed to have sufficient time to finish the paper. Presentation of work was mostly good, although some answers still seem to be written in pencil and then overwritten with ink. This practice produces a very unclear image when the script is scanned and makes it difficult to mark. Consequently, appropriate marks may not be awarded. Centres should strongly advise candidates not to do this.

Comments on specific questions

Question 1

This question was a good start to the paper for most candidates. Many were able to obtain the required coefficients, form a correct equation and solve it. Common errors included obtaining $80a$ instead of $80a^2$ and multiplying the wrong coefficient by 12.

Question 2

Many fully correct descriptions were given, and full marks were obtained for answers as succinct as:

'A stretch of factor 4 in the y-direction followed by a translation of $\begin{pmatrix} 3 \\ -8 \end{pmatrix}$.'

It is very important that the correct terminology is used and that the required order is clear.

Question 3

Many fully complete answers were seen for **part (a)** although it is important that '= 0' is retained from the question and not simply inserted at the end of the answer. In **part (b)**, many candidates were unable to gain a mark as they used a calculator to solve the quadratic equation rather than factorisation or another valid method. Candidates need to be aware that simply quoting the quadratic formula is insufficient: values need to be substituted into it. A significant number of candidates seemed unable to obtain the two required answers for $\sin^{-1}\left(-\frac{3}{4}\right)$ in the given range. The value of -48.6 from a calculator was commonly given as part of the solution, as was 131.4.

Question 4

Part (a) proved to be straightforward for most candidates, but the other two parts were much more challenging for many of them. In **part (b)**, it was common for only one part of the required inequality for the range to be given, as was confusion between $<$ and $\leqslant$. It was often unclear what candidates meant when they referred to 'it' being many-to-one, as this could have applied to the given function or the inverse. Candidates are encouraged to simply state that "g^{-1} does not exist because g is not one to one". In **part (c)**, most candidates were able to find $f\left(\frac{25}{16}\right)$, $hf(x)$ and equate them, but many were unable to solve the resulting equation. Those who did manage this often failed to discount $3-2\sqrt{2}$ as a possible solution.

Question 5

In **part (a)** many candidates were able to find a correct answer but not in an exact form. In **part (b)(i)**, most candidates could correctly find the sum to infinity, but many were unsure how to present it: $\frac{\tan\theta}{1-\cos\theta}$ was acceptable as there are no embedded fractions (fractions divided by fractions), whereas $\frac{\frac{\sin\theta}{\cos\theta}}{1-\cos\theta}$ and $\frac{\tan\theta}{1-\frac{\sin\theta}{\tan\theta}}$ were not acceptable. **Part (b)(ii)** was generally well done, but some weaker candidates found the tenth term instead of the required sum.

Question 6

Part (a) was very well done by most candidates with the vast majority using differentiation to find the minimum point. Those who attempted to complete the square sometimes became confused and thought $x = 2$ rather than $\sqrt{x} = 2$. A significant number of candidates failed to see the connection between **part (a)** and **part (b)** and wasted time finding the co-ordinates of *A* and *B* again. The integration to find the areas was generally well done, but candidates need to remember to clearly show the limits substituted into the functions. Questions like **6(b)** generally involve subtracting one area from another; most candidates are more likely to score full marks if they work out the areas separately and then subtract. This avoids having to deal with applying the minus sign to all the terms of the second integral.

Question 7

Several different methods were used to find the two possible values in **part (a)**, but the most common and most successful method was to replace y in the circle equation with $2a - x$ and then use the discriminant. Errors often occurred when candidates were expanding the brackets, and they are encouraged to take care when performing this process and collecting like terms. The 18 on the right-hand side of the equation was sometimes forgotten or ignored. **Part (b)** proved to be challenging and was omitted by about 30% of candidates. Many who did attempt it unnecessarily spent time finding the point of intersection of the tangent and the circle rather than using the centre of the circle. Using a diagram may well have avoided this waste and should always be encouraged for this type of question. Others seem confused about what was being asked for and were unable to make any progress. Again, a diagram may well have made clear what was required.

Question 8

About 40% of candidates scored no marks on **part (a)(i)**, with a number attempting to use the arc length to show the given result. Many candidates showed this successfully using right angled triangle trigonometry. **Part (a)(ii)** produced a mixed response. Many candidates could find the areas of the sectors, but the height of the trapezium was more challenging, with some simply using 0.2 or 0.4 instead of calculating it. When working out several areas that are to be added together, clear labelling of those areas can enable candidates to be awarded method marks even if their areas are wrong. This should always be encouraged. **Part (b)** again proved challenging and was omitted by almost 20% of candidates. Of those who did attempt it, many failed to appreciate that because the radius had changed *EF* was no longer 2.4.

Question 9

A small but significant number of candidates missed out the $-6x$ and only considered the first part of the function. This significantly changed the nature of the question and therefore no marks could be awarded in **part (a)**. Care needs to be taken by candidates to avoid this type of mistake. Those who considered the whole of the derivative usually realised what was required, although weaker candidates sometimes differentiated or integrated unnecessarily. It is important to note that if the candidate gave their final answer as two separate inequalities, then the word 'and' rather than 'or' needed to be used. In **part (b)**, those candidates who realised that integration was required were almost always able to do so correctly, but there was confusion over the implication of $f(1) = -1$. Some candidates failed to include a constant of integration and others set $f(x) = 0$.

Question 10

This question proved challenging for many candidates, especially **part (b)** which was omitted by 20% of them. In **part (a)**, most candidates knew how to start their response and the differentiation was generally done well. Many seemed unsure how to find the required x value and were, therefore, unable to make any further progress. In **part (b)**, many failed to see the link with the first part of the question and spent time finding the x-coordinate which had been previously found. Others assumed that the gradient of the perpendicular bisector would be $\frac{1}{3}$ because the gradient of the curve was -3, or failed to find the midpoint of *AB*. About half of the candidates were able to find the correct equation but about half of these lost the final mark as they did not give their answer in the required form.

MATHEMATICS

Paper 9709/13
Pure Mathematics 1 (13)

Key messages

it is important that centres remind their students to note the ninth bullet point of the examination instructions on the front page of the examination paper: 'You must show your working clearly; no marks will be given for unsupported answers from a calculator.' Examiners are unable to give credit when, for example, quadratic equation solutions, definite integration results, gradients, simultaneous equation solutions, intersection points and sums of series arrive from little or no working. Even when these results are correct they are not acceptable. Examiners have made allowances when these results are used in later stages of a solution but this is not automatic. Centres should not allow calculators with equation solvers, programmable calculators or graphical calculators to be used in this examination.

General comments

Most candidates appeared well prepared for this exam and the vast majority attempted most questions. With a few exceptions questions were interpreted correctly and an appropriate method of solution selected. The questions involving series and calculus were particularly well answered whilst those involving transformation geometry caused problems for many candidates.

Comments on specific questions

Question 1

This question was generally well answered by most candidates. Many candidates demonstrated that the application of the binomial expansion was well understood and the multiplication necessary to obtain the required term was carried out successfully.

Question 2

(a) This part was well answered by a minority of candidates. Those who used an algebraic method were occasionally successful as were those who used calculus to find the coordinates of B. Where candidates identified the graph was the result of a simple translation of $y = k \cos x$ they usually reached the correct answers by considering where the graph is zero and where it is a minimum. This question part was often omitted by some candidates.

(b) In contrast to **part (a)** this part was usually completed correctly. Some candidates chose to work in degrees and some mixed the units but managed to find the correct result. Finding the inverse trigonometric function was generally done accurately and most answers were given in the required exact form.

Question 3

(a) The formulae for arc length and sector area were used effectively by most candidates to produce two equations in r and θ. Although elimination of θ was the efficient route to a quadratic equation there were numerous correct equations in θ found through elimination of r. Sight of clear method of solution for these quadratics was the only way full marks could be awarded. At this stage it was sufficient to find both pairs of solutions or just the appropriate pair. Those candidates who showed no working after obtaining the two initial equations in r and θ, yet still managed to reach correct solutions, could gain no more than half marks.

(b) To gain the method mark here answers had to use the reflex angle found in **part (a)**. The simplest route to the area using the sine of angle ACB often enabled candidates to reach the correct answer very quickly. Many other correct answers were seen using various lengths within the triangle ACB. It was very rare to see an answer confusing degrees and radians gain any credit.

Question 4

(a) This part was completed successfully by most candidates. Clearing the brackets then using $\tan\theta = \sin\theta / \cos\theta$ and the trigonometric identity was the most frequently seen method. Very few candidates presented their answers in a form other than that required by the question.

(b) In contrast to **part (a)** many incorrect or incomplete answers were seen to this part. Again answers (here sin $2x$) were produced without any working thus losing method marks and many answers showed that the '$2x$' was not considered at all. When it was considered and the first angle found correctly the second angle was often found as $180 - x$ rather than $90 - x$. Rounding of the values of $2x$ was often responsible for an accuracy error in the value of x.

Question 5

(a) The procedure for finding a stationary point was demonstrated by most candidates. Problems arose in quite a few cases in the differentiation of $\frac{1}{2x}$ when it was written incorrectly as $2x^{-1}$.

Setting the differentiated expression to zero and a clear, complete solution to the resulting simple cubic equation invariably gained the available method mark but, as in previous questions, the necessary working was not always seen.

(b) The great majority of candidates used the sign of the second derivative to correctly identify the stationary point as a minimum. Candidates were mostly aware that they needed to show the value of the second derivative to be able to state that it was positive. The discontinuity at $x = 0$ appeared to deter the use of the change in gradient around the stationary point to identify its' nature.

(c) With a correct gradient from **part (a)** many candidates went on to show or state that this was always positive and conclude the function must be increasing in the given range. A significant number of candidates did not realise that the substitution of individual values into the gradient and the subsequent positive results was not sufficient evidence to conclude that the gradient was always positive.

Question 6

(a) This question was answered well by many candidates. A good understanding of the need to integrate in order to find the equation of the curve was in evidence as was the understanding of the process for finding the constant of integration. Few errors were seen but one is worthy of mention: re-writing the differential as $-20(5x - 3)^2$. This resulted in no marks for the question.

Those few candidates who thought that an equation of the form $y = mx + c$ was required for the equation of a straight line could only gain a maximum of half marks for a correct differentiation.

(b) Many candidates found this part much more difficult than **part (a)**. Some candidates knew that x was replaced by $2x$ to show the stretch and that the resulting equation had x replaced by $x - 2$ and y replaced by $y - 10$ to show the translation. Of these candidates, there were a number who didn't realise that the order of the transformations could not be interchanged. Common errors were to replace x with $\frac{x}{2}$ or to multiply $(5x - 3)$ by 2. When dealing with the translation part of the question those who replaced x by $x - 2$ were in the minority, however nearly all candidates replaced y with $y - 10$. After dealing with the stretch the most common mistake was to replace $10x - 3$ with $10x - 3 - 2$.

Question 7

(a) This part was answered correctly by most candidates using one of the two standard sum to *n* terms formulae. Occasionally misinterpretation of the question resulted in the assumption the *n*th term was 127.5.

(b) A minority of candidates used the *n*th term formula successfully to find the appropriate first and last terms between the given values and went on to find the correct value of the required sum. The use of inequalities produced mixed results with many answers using $S_{40} – S_{11}$ rather than the correct $S_{40} – S_{10}$. Where clear working was shown candidates gained some credit for attempts to find *n* for either of the given values. When *n* appeared as a decimal it was not always apparent to candidates that an appropriate integer value should be deduced from this.

Question 8

This unstructured question was answered well and many correct answers were seen. A common method was to find the coordinates of A and B and the centre of the circle, use these to find the equations of AC and BC and then the equations of the perpendiculars at A and B. These were then solved simultaneously to give the required point. Other successful methods involved finding the gradient of the circle at A and B to obtain the tangent equations and the use of geometry with similar triangles or trigonometry. The use of a clear sketch was a feature of some of the most straightforward and shortest solutions; the sketch enabling candidates to see that the *y*-coordinate of the required point must be –1. When finding the *x* coordinates of A and B it was essential that the solution to the quadratic equation obtained from setting $x = 0$ in the circle equation was shown.

Question 9

(a) The process of finding the equation of the tangent to a curve appeared to be understood by the majority of candidates. The required differentiation, however, proved very challenging for some. Occasionally after a correct differentiation and a correct calculation of the gradient at $x = 3$,some candidates went on to find the equation of the normal instead of the tangent.

(b) Like **part (a)** the process of finding the volume of rotation of the arc of a curve was understood well by many candidates and many completely correct answers were seen. Those few who did not square *y* were then faced with an integral they could not find. Those candidates who presented the answer without integration could only gain one mark if they presented the correct integral.

Question 10

(a) The expression for the n^{th} term of a geometric progression was used effectively in most answers to obtain a quadratic in r^2. The candidates who showed their solution to this equation usually went on to gain full marks in this part.

(b) Successful application of the formula for the sum of the first *n* terms of a geometric progression was seen in most answers.

(c) Producing a list of terms for the new GP enabled candidates to find the first term and common ratio which were then usually used correctly to find the sum to infinity. Some candidates used a value of the common ratio greater than 1 and didn't realise that the source of this value should be carefully checked.

Question 11

(a) The negative coefficient of x^2 caused problems for some candidates who resolved this by changing the sign of every term. Candidates are to be reminded to then apply the reverse procedure to their result. Nevertheless many completely correct completed square forms were seen. The correct range was not always quoted from a correct completed square form. Sketches which might have helped were rarely seen. Answers which ignored the request to complete the square could only gain the mark for the range.

(b) In the most favoured solution method candidates found the inverse of g, then $g^{-1}(f(x))$ with very few confusing this with $f(g^{-1}(x))$. Setting the discriminant of the quadratic equation formed from $g^{-1}(f(x)) = g(x)$ to zero was the next stage although a few successful methods involving the properties of equal rooted quadratics were seen. Again the required root of the quadratic was often stated with no evidence of solution of the quadratic losing method marks. Some very good algebraic manipulation and completely correct solutions were seen. A small number of candidates avoided using the inverse of g by using gg(x) = f(x) to form their quadratic equation.

MATHEMATICS

Paper 9709/21
Pure Mathematics 2 (21)

Key messages

When dealing with questions involving trigonometry, it is important to ensure that the correct angle units are used. Candidates are advised to read each question carefully and ensure that their final answer is in the correct form required.

General comments

There did not appear to be any issues with the timing and spacing for candidates to write their answers. Many candidates would appear to have prepared well for the examination.

Comments on specific questions

Question 1

Most candidates were able to obtain the first mark for differentiation. Attempts to equate the derivative to zero and simplify to obtain an expression in terms of cos x were varied. Very few completely correct solutions were seen. Some candidates weren't able to manipulate the trigonometric terms to obtain the equation $\cos x = \sqrt[3]{0.4}$. A number of candidates gave an answer in degrees, highlighting the need to read the question carefully and take note of what units angles need to be given in.

Question 2

There were many mixed responses to this question. Some candidates did not manage to make use of implicit differentiation in this question in order to progress. Of those that did, too many did not differentiate, with respect to x, the term $x^2 \ln y$ as a product. Some errors were made when candidates chose to rearrange their equation to obtain in the form $\frac{dy}{dx} = \ldots$ This was not necessary as a substitution into the initial gradient function would have probably yielded less errors.

Question 3

(a) Some candidates did not manage to sketch a graph of the modulus of a function. Of those that realised that a V shaped graph was needed, many placed the graph incorrectly, with the vertex on the y axis, or the x-axis or not touching an axis at all. For correctly sketched modulus graphs, most candidates were able to position the straight line graph correctly.

(b) Many candidates, having failed to score marks in **part (a)** were able to obtain at least 3 of the 4 available marks, by obtaining the critical values from either two linear equations or inequalities, or by using a squaring method to obtain a quadratic equation or inequality. Problems arose when attempting to obtain the final inequality, although a correct solution to **part (a)** would have helped. Many candidates did not really connect the two parts, not realising that **part (a)** was intended to help with the inequality in **part (b)**.

(c) It was pleasing to see that many candidates did relate their answer to **part (b)** with **part (c)**, with $e^{0.1N}$ being equated to the larger critical value obtained in **part (b)**. Only a few candidates wrote down an integer answer as required in the question. Answers such as 11.79 or 11.8 were seen rather than the required answer of 11.

Question 4

(a) Many candidates were able to state that $3\tan 2\theta$ could be written as $\frac{6\tan\theta}{1-\tan^2\theta}$. Some candidates left the expression for $\tan\left(\theta+45^\circ\right)$ in the form $\frac{\tan\theta+\tan 45^\circ}{1-\tan\theta\tan 45^\circ}$ rather than substitute in $\tan 45^\circ=1$ immediately. Substitution in later often meant that simplification errors were made. Most realised that the two terms needed to be combined but many chose to multiply the terms so that a common denominator of $\left(1-\tan^2\theta\right)\left(1-\tan\theta\right)$ was obtained. These candidates were rarely able to simplify to the given answer correctly. Most success was had by the candidates who realised that the common denominator of the two fractions was $\left(1-\tan^2\theta\right)$.

(b) Most candidates were able to use the given answer and obtain the correct quadratic equation in terms of $\tan\theta$. This quadratic equation did not factorise, which caused problems for some candidates. It was rare to see both correct solutions, with the acute angle solution being the most common answer given.

Question 5

(a) Many candidates were able to differentiate the given equation as a quotient and obtain a correct gradient function. There were occasional errors with the differentiation of the exponential term. Less success was had trying to show the given result. Equating the gradient function to zero and collecting the constant terms initially, followed by a division of each term by $6e^{2x}$ or equivalent, usually yielded the most success.

(b) The most successful candidates used the change of sign method by considering the values of $x-\frac{1}{2}e^{-2x}-\frac{1}{6}$ or an equivalent form, when x takes the values of 0.35 and 0.45. It is important that these values are stated to show a change in sign, together with a conclusion. Other methods were equally acceptable, but candidates are to be reminded to provide sufficient detail in the form of an argument and inequality.

(c) Some candidates were not able to use iterative methods and thus didn't make any progress in this question part. It was important that those candidates that did attempt this question part gave their working to the correct level of accuracy and then their final answer, after having done sufficient iterations, also to the correct level of accuracy. In this question a starting value of 0.4 converged to the required answer quite quickly.

Question 6

(a) A small minority of candidates produced correct solutions. It was clear that many candidates were not able to correctly apply with the trapezium rule.

(b) Very few correct solutions were seen. Whilst many candidates obtained a correct integrand, very few attempted to use an appropriate double angle formula and where unable to make much progress as a result. This is a common issue each session, with candidates usually being able to deal with the double angle formula for $\sin 2A$ but not the variations for $\cos 2A$.

Question 7

(a) Many candidates were able to obtain full marks in this question part. The most successful solutions were from candidates who used algebraic long division. Candidates using synthetic division often did not take the factor of 3 into account and ended up with an incorrect quotient of $9x^2 + 12$. If candidates choose to use synthetic division, then they should be aware of the correct process when dealing with linear factors where the coefficient of x is not 1.

(b) Not many candidates made use of the result from **part (a)**, not making a connection between the integrand in **part (b)** and the result from **part (a)**. Of those candidates that did, integration and the substitution of limits was very often correct, but often the final manipulation of logarithms led to an incorrect final result. Candidates are advised to always check to see if the work done in one question part could be of use in subsequent question parts. The word 'Hence' is not always used.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2 (22)

Key messages

In ‘show that’ questions, it is essential to show each step of working so that sufficient detail is shown to reach a given answer. This is also covered in the instructions in the rubric on the front of the paper, with which candidates should be familiar. Candidates should also ensure that they are fully meeting the demands of the questions by ensuring answers are given in the required form and the correct level of accuracy has been used.

General comments

Many candidates were usually able to make a reasonable attempt at the paper. There appeared to be no timing issues and candidates appeared to have sufficient room in which to answer their questions.

Comments on specific questions

Question 1

Most candidates were successful in gaining the first 3 marks of this question, either by forming 2 linear equations or a quadratic equation in order to find the critical values. Unfortunately, a number of candidates identified an incorrect domain for their final answer. A simple check using a couple of different numbers would have helped such candidates determine whether or not their answer was correct.

Question 2

Very few candidates were able to gain full marks in this question. Having been instructed to use logarithms, most candidates were able to write 6^{2x-1} as $(2x-1)\ln 6$. Very few candidates were able to take logarithms of the right-hand side of the given equation correctly, with $(3x+2)\ln 5$ being the most common error. The few candidates who were able to write the right-hand side of the given equation correctly as $\ln 5 + 3x + 2$, usually reached the final correct answer. The application of the laws of logarithms was problematic probably because of the inclusion of the exponential term.

Question 3

(a) Most candidates were able to differentiate the given equation correctly, although there were the occasional slips with signs. Many correct answers were seen.

(b) Some candidates had difficulties showing that the *x*-coordinate of P was $\ln 2$. It was expected that a substitution of $y = 0$ be used and the subsequent equation of $e^{3x} = 8$ obtained. As the answer to this question part was given, it was essential that candidates show sufficient detail to justify the given answer. In this case, a statement of $x = \frac{1}{3}\ln 8$ followed by $x = \ln 8^{\frac{1}{3}}$ or equivalent and then $x = \ln 2$ would gain the candidate the allocated 2 marks. There were mixed responses to the remaining part of this question. Common errors included sign and coefficient errors in the integrals. Again, it is important that the substitution of the limits into a definite integral is shown clearly.

Question 4

This unstructured question tested the problem-solving skills of the candidates. Many candidates attempted to find the gradient function in terms of *t*, but didn't succeed with differentiating $\cos^2 t$ using the chain rule. Other errors such as sign errors and coefficient errors often meant that candidates obtained an incorrect gradient at the point in question. These candidates were often able to obtain subsequent method marks provided an attempt at differentiation had taken place and the equation of a normal was being considered. Despite correct solutions, some candidates were unable to obtain full marks as they did not present the normal equation in the required form.

Question 5

(a) Many candidates were able to obtain full marks in this question part. The most successful solutions were from candidates who used algebraic long division. Candidates using synthetic division often did not take the factor of 3 into account and ended up with an incorrect quotient of $9x^2+12x-3$. If candidates choose to use synthetic division, then they should be aware of the correct process when dealing with linear factors where the coefficient of *x* is not 1.

(b) Not many candidates made use of the result from **part (a)**, not making a connection between the integrand in **part (b)** and the result from **part (a)**. Of those candidates that did, integration and the substitution of limits was very often correct, but often the final manipulation of logarithms led to an incorrect final result. Candidates are advised to always check to see if the work done in one question part could be of use in subsequent question parts. The word 'Hence' is not always used.

Question 6

(a) Most candidates were able to obtain both the available marks, apart from the occasional error when differentiating the logarithmic term. Candidates should be guided by the mark allocation, in a question such as this, there is no simplification that can be done.

(b) Candidates often have difficulty with questions of this type. Most realised that the gradient at the point M was zero and attempted to rearrange their answer to **part (a)** accordingly. This type of question is often done using a measure of observation – attempting to see which terms 'remain together'.

(c) The most successful candidates used the change of sign method by considering the values of $\frac{x+3}{\ln(2x+1)}-0.5-x$ or an equivalent form when x takes the values of 2.5 and 3.0. It is important that these values are stated to show a change in sign, together with a conclusion. Other methods were equally acceptable, but candidates often did not provide enough detail in the form of an argument and inequality.

(d) Some candidates were not able to make any progress with iterative methods as evidenced by many blank solution spaces. It was important that those candidates that did attempt this question part gave their working to the correct level of accuracy and then their final answer, after having done sufficient iterations, also to the correct level of accuracy. In this question a starting value of 2.75 converged to the required answer very quickly.

Question 7

(a) A straightforward proof, provided that candidates gave sufficient detail. It was essential that the use of the double angle formula be seen clearly, followed by the result $\frac{1}{\cos\theta}=\sec\theta$.

(b) Again, it was necessary for candidates to identify that they needed to use the result from **part (a)** to re-write the given equation in a much simpler form. Many candidates were able to obtain a correct quadratic equation in terms either $\sec\theta$ or $\cos\theta$. Some candidates were then unable to get any correct solutions. A number of candidates did not succeed with dealing with negative angles or radians, so practice at these types of equations is useful.

(c) Very few correct responses were seen. This question part was often not attempted and those that did were not able to use the result from **part (a)** to form $\int 2\sec^2\left(\frac{x}{2}\right)dx$.

MATHEMATICS

Paper 9709/23
Pure Mathematics 2 (23)

Key messages

In ‘show that’ questions, it is essential to show each step of working so that sufficient detail is shown to reach a given answer. This is also covered in the instructions in the rubric on the front of the paper, with which candidates should be familiar. Candidates should also ensure that they are fully meeting the demands of the questions by ensuring answers are given in the required form and the correct level of accuracy has been used.

General comments

Many candidates were usually able to make a reasonable attempt at the paper. There appeared to be no timing issues and candidates appeared to have sufficient room in which to answer their questions.

Comments on specific questions

Question 1

Most candidates were successful in gaining the first 3 marks of this question, either by forming 2 linear equations or a quadratic equation in order to find the critical values. Unfortunately, a number of candidates identified an incorrect domain for their final answer. A simple check using a couple of different numbers would have helped such candidates determine whether or not their answer was correct.

Question 2

Very few candidates were able to gain full marks in this question. Having been instructed to use logarithms, most candidates were able to write 6^{2x-1} as $(2x-1)\ln 6$. Very few candidates were able to take logarithms of the right-hand side of the given equation correctly, with $(3x+2)\ln 5$ being the most common error. The few candidates who were able to write the right-hand side of the given equation correctly as $\ln 5 + 3x + 2$, usually reached the final correct answer. The application of the laws of logarithms was problematic probably because of the inclusion of the exponential term.

Question 3

(a) Most candidates were able to differentiate the given equation correctly, although there were the occasional slips with signs. Many correct answers were seen.

(b) Some candidates had difficulties showing that the x-coordinate of P was $\ln 2$. It was expected that a substitution of $y = 0$ be used and the subsequent equation of $e^{3x} = 8$ obtained. As the answer to this question part was given, it was essential that candidates show sufficient detail to justify the given answer. In this case, a statement of $x = \frac{1}{3}\ln 8$ followed by $x = \ln 8^{\frac{1}{3}}$ or equivalent and then $x = \ln 2$ would gain the candidate the allocated 2 marks. There were mixed responses to the remaining part of this question. Common errors included sign and coefficient errors in the integrals. Again, it is important that the substitution of the limits into a definite integral is shown clearly.

Question 4

This unstructured question tested the problem-solving skills of the candidates. Many candidates attempted to find the gradient function in terms of *t*, but didn't succeed with differentiating $\cos^2 t$ using the chain rule. Other errors such as sign errors and coefficient errors often meant that candidates obtained an incorrect gradient at the point in question. These candidates were often able to obtain subsequent method marks provided an attempt at differentiation had taken place and the equation of a normal was being considered. Despite correct solutions, some candidates were unable to obtain full marks as they did not present the normal equation in the required form.

Question 5

(a) Many candidates were able to obtain full marks in this question part. The most successful solutions were from candidates who used algebraic long division. Candidates using synthetic division often did not take the factor of 3 into account and ended up with an incorrect quotient of $9x^2 + 12x - 3$. If candidates choose to use synthetic division, then they should be aware of the correct process when dealing with linear factors where the coefficient of *x* is not 1.

(b) Not many candidates made use of the result from **part (a)**, not making a connection between the integrand in **part (b)** and the result from **part (a)**. Of those candidates that did, integration and the substitution of limits was very often correct, but often the final manipulation of logarithms led to an incorrect final result. Candidates are advised to always check to see if the work done in one question part could be of use in subsequent question parts. The word 'Hence' is not always used.

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(b) Candidates often have difficulty with questions of this type. Most realised that the gradient at the point M was zero and attempted to rearrange their answer to **part (a)** accordingly. This type of question is often done using a measure of observation – attempting to see which terms 'remain together'.

(c) The most successful candidates used the change of sign method by considering the values of $\frac{x+3}{\ln(2x+1)} - 0.5 - x$ or an equivalent form when x takes the values of 2.5 and 3.0. It is important that these values are stated to show a change in sign, together with a conclusion. Other methods were equally acceptable, but candidates often did not provide enough detail in the form of an argument and inequality.

(d) Some candidates were not able to make any progress with iterative methods as evidenced by many blank solution spaces. It was important that those candidates that did attempt this question part gave their working to the correct level of accuracy and then their final answer, after having done sufficient iterations, also to the correct level of accuracy. In this question a starting value of 2.75 converged to the required answer very quickly.

Question 7

(a) A straightforward proof, provided that candidates gave sufficient detail. It was essential that the use of the double angle formula be seen clearly, followed by the result $\frac{1}{\cos\theta} = \sec\theta$.

(b) Again, it was necessary for candidates to identify that they needed to use the result from **part (a)** to re-write the given equation in a much simpler form. Many candidates were able to obtain a correct quadratic equation in terms either $\sec\theta$ or $\cos\theta$. Some candidates were then unable to get any correct solutions. A number of candidates did not succeed with dealing with negative angles or radians, so practice at these types of equations is useful.

(c) Very few correct responses were seen. This question part was often not attempted and those that did were not able to use the result from **part (a)** to form $\int 2\sec^2\left(\frac{x}{2}\right)\mathrm{d}x$.

MATHEMATICS

Paper 9709/31
Pure Mathematics 3 (31)

Key messages

- Read the questions very carefully and make sure that your response matches the demand.
- Use mathematical notation correctly. The omission of brackets in algebraic arguments can turn a response with potential into not being worthy of credit.
- Do not overwrite work as it may be difficult to read when scanned.
- Obtain a copy of the syllabus at the start of the course and ensure that you have studied all of the topics required.

General comments

There were some candidates who demonstrated a high level of mathematical skill and reasoning. These candidates were usually able to respond to all parts of all questions.

Some candidates did not make an attempt at a question; the frequency of the blank responses was greater towards the end of the paper. This included complex numbers and vectors, both topics that candidates often omit, but there were also gaps in questions requiring basic calculus, which has been a strength in the past.

Comments on specific questions

Question 1

The binomial expansion allowed many candidates to gain credit. There were several slips with signs and in simplifying expressions. The major sources of errors were due to the use of x rather than $2x$ in the expansion of $(1-2x)^{\frac{1}{2}}$ and incorrect (and unnecessary) attempts to expand $(3+x)$. A few candidates gave only the term in x^2 and not the terms up to and including the term in x^2.

Question 2

The most successful start was to combine the two logarithm terms to obtain $\ln(x(x-5))$. This usually resulted in a correct quadratic equation. Most candidates solved the equation correctly, but some did not reject the negative solution.

The most common error was to state $\ln(x-5) = \ln x - \ln 5$.

Question 3

Very few candidates started by stating that $y \ln a = \ln b + \ln x$. The majority started by writing down equations that used the given values of $\ln x$ as values of x, and consequently they made no useful progress. A minority of candidates obtained the relevant values of x and used the original form of the equation to form their simultaneous equations.

Question 4

(a) Candidates who were familiar with the polar form for a complex number usually obtained the correct modulus for *u*. The argument proved to be more challenging, with many responses giving values in the wrong quadrant.

(b) A small minority of candidates were aware of the results for dividing complex numbers in polar form, but the majority did not have a correct strategy for this part. A small number did obtain the correct modulus.

Question 5

The majority of candidates recognised the function as a quotient and selected a correct method for differentiation. The candidates were less successful in applying the chain rule: it was unusual to see correct derivatives for both $e^{\sin x}$ and $\cos^2 x$. Depending on the errors made, it was still possible to gain further method marks for forming and solving a quadratic equation in $\sin x$.

Question 6

(a) Although other alternatives would be acceptable, candidates were expected to start with a sketch showing $y = e^x - 3$ and $y = \operatorname{cosec}\frac{1}{2}x$ for the given interval. There were some recognisable attempts to sketch the exponential graph, with some showing the correct intercept on the vertical axis. The trigonometrical graph was less successful, with many candidates not recognising that $\operatorname{cosec}\frac{1}{2}x$ is the reciprocal of $\sin\frac{1}{2}x$.

(b) Several correct answers were seen. Most errors were caused by not working in radians, or by using an incorrect trigonometric ratio to evaluate $\operatorname{cosec}\frac{1}{2}x$.

(c) The candidates who attempted this item were usually successful. Most errors occurred when candidates attempted to work back from $e^x - 3 = \operatorname{cosec}\frac{1}{2}x$ but did not get as far as the iterative formula (with subscripts).

(d) There were several fully correct solutions. Errors were usually due to not working in radians, not being able to evaluate $\operatorname{cosec}\frac{1}{2}x$, or not working to the required number of decimal places.

(e) The majority of candidates who had a correct solution to **part (d)** were able to say how many iterations were needed to determine the root correct to 2 decimal places.

Question 7

(a) Several candidates recognised the first locus as a circle and drew a circle of the correct size in the correct place. There were some circles of the correct size, but with the centre in the wrong place, often at $-3 + 2i$. The second locus was often not recognised, with only a minority of candidates showing the perpendicular bisector of the correct line.

(b) A small minority of candidates understood that they needed to find the shortest distance between the circle and the line, but most did not have a correct strategy for this. This part was left blank by a number of candidates.

Question 8

A minority of candidates showed some awareness of the method of integration by substitution, and these candidates usually made good progress. Candidates were required to state that $\frac{d}{dx}(1 - \sin x) = -\cos x$. Without this, no useful progress could be made.

Question 9

(a) The minority of candidates that found the scalar product of the direction vectors usually succeeded in demonstrating the given result.

(b) Those candidates who understood the process for finding the point of intersection of the two lines often obtained the correct position vector.

(c) There were few responses to this part that succeeded in gaining marks.

(d) There were very few correct responses. The preceding parts of the question had been establishing all the required information but the majority of candidates did not reach this stage.

Question 10

(a) The candidates were expected to use implicit differentiation to differentiate $2x = \tan y$ and to use the outcome to establish the given result. A few candidates followed this process correctly. The most common approach was to rewrite the given equation as $y = \tan^{-1} 2x$ and to quote the result from the formula booklet – an approach that scored no marks.

(b) Some candidates did recognise this as a question requiring integration by parts, but only a minority recognised the relevance of the result from **part (a)**. Most solutions that started correctly did not progress beyond $\int \frac{x^2}{1+4x^2}\,\mathrm{d}x$.

Question 11

(a) This task required candidates to use the given information to establish the differential equation. The majority of candidates who offered a solution assumed the given result and showed that it was consistent with the data provided. There were no marks available for a solution by verification.

(b) The question indicates that partial fractions are required, so several candidates earned some marks for this despite not achieving a correct separation of the variables. There were several attempts to complete the integration required, with the most common error being a sign error in $\int \frac{1}{300-x}\,\mathrm{d}x$. The question asks for the answer as a single logarithm and some candidates with a correct solution did not make that final step.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3 (32)

Key messages

- Read the questions very carefully and make sure that your response matches the demand. For example, finding the point of intersection of two lines is not the same as showing that the two lines intersect.
- Use mathematical notation correctly. The omission of brackets in algebraic arguments can turn a response with potential into not being worthy of credit. For example $(2y-1)\ln a = (x-y)\ln b$ and $2y-1\ln a = \text{x} - y \ln b$ are not identical.
- If a question asks you to prove a particular result then the response needs to be very clear about what each fact demonstrates. For example, if a question asks you to demonstrate that a triangle is isosceles, it is not sufficient to find the lengths of the sides – you need to say that because two of the sides have equal lengths then the triangle is isosceles. You must make the deductions, do not leave that to the examiner.
- Candidates who give a first draft of a response in pencil and then write over it in ink should be aware that the response doesn't scan well.
- When a question asks for an exact answer, that is not the same as stating all the decimal places that your calculator will give you. The answer might be a value involving surds, or multiples of a constant such as π or e.

General comments

The work of the candidates covered the full range of the marks available. The better work was clearly set out, followed sequentially down the page and was easy to follow.

There were several instances in this paper where candidates omitted key brackets and never recovered them in their subsequent working. This lack of precision in the use of notation was also seen in the presentation of integrals and derivatives.

Comments on specific questions

Question 1

(a) Many candidates identified that the sketch should be a symmetrical V shape. For a sketch of this nature the points of intersection with the axes should be shown. In this example the points of intersection should be at 2*a* on both axes. The most common errors were a sketch of the wrong shape, a sketch of the correct shape but in the wrong place or missing / incorrect markings on the axes. Several candidates attributed a value to *a*.

(b) Some candidates worked from a correct sketch and added the line $y = 2x - 3a$. These candidates usually deduced that there was only one critical value and they often obtained the correct answer. Many candidates chose to square the inequality to remove the modulus signs. There were many slips in the algebra, particularly because of the involvement of *a* in the working. The critical values $x = a$ and $x = \frac{5}{3}a$ were often seen. It was unusual for $x = a$ to be rejected.

Question 2

There were two aspects of this question that caused difficulties: the numerator of the fraction is of the same degree as the denominator, and the denominator is not given in factorised form. Neither of these points delayed the candidates, but they did result in many errors. A large minority of candidates did what they were expected to do and obtained the three constants with no difficulty. Some started by dividing the numerator by the denominator and some worked from the form $A + \frac{B}{2x+3} + \frac{C}{x-4}$.

Most errors stemmed from attempting to use $\frac{6x^2 - 9x - 16}{2x^2 - 5x - 12} = \frac{A}{2x+3} + \frac{B}{x-4}$ or from the incorrect factorisation $2x^2 - 5x - 12 = \left(x + \frac{3}{2}\right)(x+4)$.

Question 3

(a) The majority of candidates made a correct start, usually by taking logarithms of both sides of the equation to obtain $(2y-1)\ln a = (x-y)\ln b$, or equivalent. It was common for the brackets to be omitted. Another common feature of the work this year was incorrect ‘cancelling’ to obtain $(2y-1)a = (x-y)b$. Many candidates who obtained a correct form for the equation did not go on to compare it with a standard form for the equation of a straight line in order to reach the required conclusion.

(b) This part of the question was comparatively well done. Some candidates worked from their equation obtained in **part (a)** and some reverted to the original index form. There were several fully correct solutions. A common error was to get to the form $y = \frac{\ln b}{\ln b^7}x + \frac{\ln b^3}{\ln b^7}$ and not to simplify further.

Question 4

Many candidates coped well with the implicit differentiation. The notation *y*' was used by some candidates; this was often accurate but in some cases candidates confused their *y*' with *y*. A few candidates retained the 6 after differentiation.

Finding the value of *x* when $y = 1$ was much more problematic. Some candidates did not realise that this was necessary and many candidates manipulated the exponential equation incorrectly. It was common for candidates to take logarithms of the terms separately, often obtaining the incorrect value $x = \frac{\ln 6}{3}$. Those candidates who showed clear formation of the correct quadratic in e^x often obtained $e^x = 2$.Some candidates did not reject the negative root of the quadratic equation and continued with two solutions. A few candidates created difficulties for themselves by substituting *x* for e^x in their quadratic equation.

Sign errors in rearranging and substituting to obtain the gradient were relatively frequent, and fully correct solutions were unusual.

Candidates who substituted the values prior to rearranging saved themselves a lot of writing and were often more accurate.

Question 5

(a) It was pleasing to note many fully correct responses. Some candidates obtained correct relevant values but did not go on to draw a clear conclusion. Most errors were due to working in degrees rather than in radians.

(b) The majority of candidates demonstrated the given result correctly. Those candidates who chose to work back from $e^{2x} = 5 + \cos 3x$ often stopped before they reached the iterative formula (with subscripts).

(c) There were many correct responses. Most errors were due to not working in radians. Some candidates did not work to the required degree of accuracy.

Question 6

(a) Many candidates scored all four marks for this item. The main errors in the differentiation were in dealing with $\frac{d}{dx}e^{-ax}$; the *a* was often incorrectly placed, and the sign incorrect. Candidates often obtained the correct *x*-coordinate. Some did not go on to find the *y*-coordinate, and some did not substitute for *x* when trying to obtain *y*.

(b) The majority of candidates recognised the need to use integration by parts. There were many sign errors and errors in dealing with the coefficients, with many candidates scoring only the two method marks. The substitution of the limit $\frac{2}{a}$ gave rise to many algebraic errors. It was common to see $\frac{2}{a} \div a = 2$. Candidates were expected to simplify their answer to a 2-term form. Many final answers were left with $-\frac{2}{a^2e^2} - \frac{1}{a^2e^2}$ not combined as a single term. A small number of candidates attributed a value to *a* and scored no marks.

Question 7

(a) Numerous different approaches were seen. The most common was also the most efficient method -factorising using the difference of two squares. This yielded the correct result almost immediately. Also common was the use of Pythagoras. Many successful solutions were seen starting from $\cos^4\theta - \sin^4\theta = \left(\frac{1+\cos 2\theta}{2}\right)^2 - \left(\frac{1-\cos 2\theta}{2}\right)^2$. A common issue was candidates' use of notation; often leaving out θ or not carefully distinguishing between $\cos^2\theta$ and $\cos 2\theta$. A common error was to start with $\cos^4\theta - \sin^4\theta \equiv (\cos^2\theta - \sin^2\theta)^2$.

(b) Those candidates spotted the link with **part (a)** usually scored the mark for obtaining $\frac{1}{2}\sin 2\theta$.

Completing the rest of the integral proved to be much more challenging. The candidates who recognised the final term as $\text{Sin}^2 2\theta$ were often able to make the correct double angle substitution to obtain a form that could be integrated. It was also possible to use the double angle formulae to replace $\sin^2\theta$ and $\cos^2\theta$ but this was a longer route which generated sign errors and often stopped short of a correct expression with $\cos 4\theta$. Incorrect integration such as $\sin^2 2\theta$ becoming $\frac{1}{2}\sin^3 2\theta$ or $\frac{1}{3}\cos^3 2\theta$ was common. Many candidates did not demonstrate an understanding of the forms of trigonometric functions that can be directly integrated and those for which the use of an identity is necessary first.
There were errors in substituting limits, often caused by a lack of brackets and not showing the initial substitution fully.

Question 8

(a) Most candidates understood the correct form for the equation of the required line and gave a correct response. The most common error was in the form of the equation. This should be **r** = ….., but it was very common to see $l_1 = \ldots$There were some slips in the arithmetic, and a small minority of candidates who tried to involve the position vector of *A*.

(b) Many candidates did not answer the question set; they demonstrated that the two lines intersect but they did not find the coordinates of the point of intersection. The wording of the question implies that the lines do intersect, so that can be assumed. Great care needs to be taken with the algebra – there were many addition errors and sign errors in the working to form and solve the simultaneous equations.

(c) The majority of candidates did not make appropriate use of the information given in this question, possibly because they were not familiar with the notation *AB* to represent $\left|\overrightarrow{AB}\right|$. Many candidates had a correct statement of $\overrightarrow{AB}$ and some also found a correct expression for $\overline{BD}$. The minority of candidates who understood that the question was about distances went on to form and solve the relevant equation. Although it was condoned on this occasion, candidates should be aware that it is not correct to include **i**, **j** and **k** in a solution given as a column vector.

Question 9

(a) The majority of candidates scored the mark for this part. There were several slips in the arithmetic, and some candidates did not present the answer in the form required. The omission of brackets was a common feature in the incorrect answers.

(b) The question set requires four values for the answer: two moduli and two arguments. The candidates who did this usually scored the two marks for the moduli. The arguments were more difficult because thought had to be given to the correct quadrant. It was common for candidates to give the modulus and argument of $z\omega$ and none of the values required by the question.

(c) This question requires a proof. Both aspects, the isosceles triangle and the right angle need to be addressed. Some candidates did set out with a correct approach, considering the lengths of the sides of the triangle. As it is part of a proof, sufficient working needs to be shown to confirm how the lengths were obtained. For *OB* and *OA* this can come from the work in **part (b)**. Some candidates established that *OA* = *AB* but made no comment about this demonstrating that the triangle is isosceles. Having found the lengths of all three sides, the simplest way to demonstrate the right angle is to use Pythagoras but, again, an appropriate comment is required. Some candidates were successful in using gradients to demonstrate the right angle. A minority of candidates created a circular argument, assuming that there was a right angle in order to show that there was a right angle. Although not required, many candidates drew a sketch of their triangle – many of these sketches did not show anything that looked like the correct triangle.

(d) The evidence for a correct solution should come from correct use of the arguments found in **part (b)** and correct use of the product $z\omega$ found in **part (a)**. Even if both of these were incorrect, the two method marks were available. Very few responses gave a correct proof. The majority never mentioned that they were using $\arg z\omega$ or the general result for the argument of a product.

Question 10

(a) The candidates were expected to demonstrate the given result by finding $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{\cos^3\theta}\right)$. Some candidates used the chain rule to differentiate $\cos^{-3}\theta$, others used the quotient rule combined with the chain rule. As the result is given, evidence of correct working was required. Many candidates claimed that the result obtained was $\frac{\mathrm{d}y}{\mathrm{d}x}$ rather than $\frac{\mathrm{d}y}{\mathrm{d}\theta}$.

(b) The majority of candidates separated the variables correctly. Most went on to use the result from **part (a)** to integrate the function in θ correctly. In order to make further progress the function in x needed to be split correctly. Some candidates thought that this could be integrated directly without splitting, and several claimed that $\frac{x+3}{x^2+9}=\frac{1}{x+3}$. A large minority of candidates completed the integration correctly. Many then substituted the boundary conditions correctly to obtain the constant of integration. There were often slips in the calculator work to obtain the final answer, with candidates finding values of $\cos\theta$ greater than 1.

MATHEMATICS

Paper 9709/33
Pure Mathematics 3 (33)

Key messages

Candidates need to:

- Know what is required of them when attempting a 'show that' question with a given answer, e.g. **Question 9(a)**.
- Know what is required when showing points on an Argand diagram, e.g. **Question 6(a).**
- Understand that, in this paper, the use of calculators is essentially restricted to arithmetical operations, finding the values of certain standard functions and their inverse values, the solution of simultaneous linear equations and quadratic equations. Hence, cubic and quartic equations should be factorised. Failure to factorise these will result in the rubric 'omission of essential working' being strictly applied to that part of the question as well as to any marks available for work that depends on those solutions, since these methods are explicit in the syllabus. E.g., **Question 7(c).**
- Understand when asked to differentiate a function involving sin2*x* and/or cos2*x* that it does not make sense to introduce the double angle formulae. Firstly, because the new expression(s) are more difficult to differentiate, and secondly as subsequent work will generally be straightforward if the original functions remain unchanged, whereas if candidates have used the double angle expression(s) the required approach is now much more complex and rarely leads to the correct answer. E.g., **Question 2**.

As mentioned above, calculators cannot be used in complex number questions since candidates need to show working to justify their result.

General comments

The standard of work on this paper was generally very high, with a considerable number of candidates performing well on most of the questions. It was, however, disappointing to see many candidates omitting key parts of required working, for example in **Question 11**. There, some candidates went from the integral of $\frac{1}{\sec^2\theta}$ to $\frac{1}{2}\left(\frac{1}{2}\sin 2\theta + \theta\right)$, omitting first the conversion to $\cos^2\theta$, then the introduction of the double angle formula and finally the necessary integration. Such large jumps will very likely to lead to errors, and even if that is not the case, working needs to be seen. Whilst different candidates will always display different levels of working, a good gauge of what is deemed adequate can be found in mark schemes to past question papers.

Comments on specific questions

Question 1

The majority of candidates were able to gain all four marks. The methods adopted depended upon how much algebra was undertaken before a law of logs was introduced. Those that decided to start with taking logs needed to use the power law of logs twice as well as the product law of logs once or possibly twice. Where this was performed correctly, the answer was quickly found. Those that decided to try and use indices to simplify their given equation were able to reduce the equation to one requiring only one use of a power law of logs, however many that followed this route introduced errors in their indices work. A variety of log bases was seen, including non-integer bases, all of which were allowed.

Question 2

The first two marks were generally earned; only the very weakest responses made no headway at all. The sight of $\sin 2x$ and $\cos 2x$ proved a temptation for many to unnecessarily use the double angle formulae, which led to many quite complicated equations, and rarely to a correct answer, although some were indeed seen. Those who took the simplest route were generally successful. There were very few errors or omissions of the factor '2' in the original differentiation. A few extra solutions, such as $\frac{\pi}{8}$, were offered for x and the y coordinate was quite often omitted.

Question 3

Generally, this was well done, with most candidates producing the required equations. The poorest responses started with trying to square both sides. A few who tried to express the given complex number in the form $re^{i\theta}$ were unsuccessful as they were not forming a quartic equation as required. The quadratic equation was often not shown before the correct answer appeared (see **Key messages** above). Use of a calculator was helpful here, but it was being introduced far too early; a quadratic equation in x of in y should have been produced prior to any calculator work.

Question 4

If $\ln k + \ln y = cx$ was observed, then the first three marks were most often scored. Unfortunately, far too many candidates decided to choose the quoted equation for a straight line which includes a constant 'c', hence representing something different from the c defined in the question. This is an unfortunate error, although a few candidates did managed to unravel this inconsistency. The mark for obtaining the value of k was sometimes not gained where the power value was rounded to one decimal place, i.e. $e^{1.9}$ giving $k = 7$. Sign errors also meant that some did not gain the mark for k. For those candidates that did not spot they needed to take logs, the problem was not writing 2.21 as $e^{2.21}$ and resulting in incorrect equations.

Question 5

This question proved highly challenging for the many candidates who did not divide or chose the incorrect form for their partial fractions. The consequence of this was they were restricted to a single method mark and an accuracy mark for obtaining a correct value of one of their constants using the 'cover up' rule. Whilst one can understand the choice of something like $\frac{Ax}{(x-1)}+\frac{B}{(2x+1)}$, as that can be modified into the correct partial fractions by division of the first term, there is no way the choice of $\frac{A}{(x-1)}+\frac{B}{(2x+1)}$ can produce a quadratic term in the numerator and hence this is considered a serious error.

Question 6

(a) Marks awarded for this question covered the full range. There were the problems discussed above in the **Key messages,** usually affecting the radius mark, but there where others seen as well. These usually related to the drawing of the full line as opposed to the half line, the half-line passing below the centre of the circle (so having an arg value between $\frac{\pi}{6}$ and $\frac{\pi}{4}$), together with the shading of the incorrect region of the circle.

(b) Few candidates achieved full marks here. Of those that did, nearly all opted for the application of simple trigonometry applied to the angles at the origin in the two triangles, with one triangle containing the horizontal axis and the line from the origin to the centre of the circle, and the other containing that latter line and the tangent from the origin to the circle. Unfortunately, a few candidates having chosen the correct angles muddled the sides in their trigonometrical expressions. Although graphical calculators are not allowed in this syllabus, a few candidates seemed to acquire, by some means or other, a very close approximation for the relevant coordinate. This additional information was then used in a correct procedure to produce the required answer, often equivalent to five decimal places. Acquiring this additional information in this manner is not a valid assumption and as such received no credit.

Question 7

(a) This was answered very well. It was rare to see a candidate who did not substitute $x = -7$ into their cubic equation and show the correct detailed arithmetical evaluation to be zero.

(b) Again, full marks were nearly always awarded, usually involving the method of long division. Few candidates decided to employ factorisation.

(c) Again, factorising correctly, but now with linear factors, was the norm. However, a few candidates, contrary to the rubric and what is stressed in the **Key messages**, decided to simply use their calculator. Converting their x values to the appropriate angles presented few problems although a few candidates had only two angles instead of the required four. Others, having found the correct angles in the first and second quadrant, struggled with those in the third and fourth quadrants.

Question 8

(a) Candidates usually achieved full marks here, especially in regard to the radius, although occasionally $\alpha = \tan^{-1}\left(\frac{3}{\sqrt{3}}\right)$ or $\tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$ was seen. However, several occurrences of the serious error of $\cos\alpha = 3$ and $\sin\alpha = \sqrt{3}$ were observed.

(b) By far the most common scores on this question were zero, three or five marks. Candidates who were unable to link **parts (a)** and **(b)** did not make meaningful progress here. Three marks were generally scored by candidates who saw the link between these parts, reached $\sec^2(2x+\alpha)$ and integrated correctly, but did not obtain the correct coefficient in their integration.

Question 9

(a) As this was a 'show that' question, it required a clear and detailed proof. A significant number of candidates appeared to not know what '…inversely proportional to t' meant and so were unable to begin a response. There was often no constant of proportionality and t was in the numerator. Even those that overcame these difficulties and knew how to apply the chain rule often never clearly established the general differential equation, i.e. the differential equation involving k or $\frac{1}{k}$. It is into this equation that $t = \frac{1}{10}$, $x = \frac{1}{2}$ and $\frac{dx}{dt} = \frac{-20}{37}$ should have been substituted. Regardless of whether candidates had introduced $-k$ instead of k into their proportionality constant, which is perfectly acceptable, the value of $\frac{dx}{dt}$ needed to always be negative when substituted. Without this, $\frac{k}{t}$ produced $k = \frac{1}{2}$, whilst $-\frac{k}{t}$ produced $k = -\frac{1}{2}$. Hence many retrospective amendments of their algebra were observed by candidates to try to establish the given differential equation. Most candidates never introduced a k and simply tried to work with $\frac{1}{2t}$ from the given answer, hence were limited to a maximum of one mark.

(b) Most candidates could separate correctly, although a few lost the negative sign or the 2 in trying to achieve this. Unfortunately, some had t in the numerator so just a single mark was possible. The correct integration of $-\frac{1}{2t}$ should have produced the answer $-\frac{1}{2}\ln t$ or $-\frac{1}{2}\ln 2t$, however far too many candidates either omitted the negative sign or, more often, the $\frac{1}{2}$ from the log coefficient. Establishing the value of the arbitrary constant generally presented no trouble, unlike the work required for the final mark where an expression free of the format $e^{\ln}$ is always to be expected at this level.

Question 10

(a) Candidates generally found this a rather straightforward question, with nearly all obtaining the correct unsimplified forms for the scalar product and the moduli. Unfortunately, far too many candidates at this level then saw their $\sqrt{25+a^2}$ become 5 + a, meaning none of the subsequent accuracy marks were available. Even those candidates who did not make this error were often unable to do what was required, which was to square, construct a quadratic equation and solve. As mentioned in the **Key messages**, unless one squares to establish a quadratic equation before introducing the calculator, then one is solving an equation that is not intended to be solved on the calculator; the final three marks were not intended for simply using a calculator to solve an unsimplified mathematical equation. This is not a sensible exam technique, as any error in entering the non-quadratic equation into the calculator will result in the accuracy mark for the quadratic equation, the method mark for its solution or the two accuracy marks being unavailable.

(b) This was another question which candidates found straightforward, although some failed to present an answer for the point of intersection. Candidates who correctly solved their simultaneous equations usually scored four or five marks. In such questions, candidates would be well advised, even though it is an exam and time is precious, to check their solutions in their two equations otherwise many of the following accuracy marks may not be available.

Question 11

This question saw many candidates achieve full marks. Nearly all could manage to find $\frac{dx}{d\theta}$ correctly, substitute and to express the integral in terms of a single trigonometrical function. However, in these processes, and later in the question, notation was not always consistent, with x's or A's sometimes appearing instead of θ. Candidates could not gain a mark or possibly two if such notation was not corrected at some stage in their solution. Most candidates correctly converted to $\cos^2\theta$, introduced the double angle formula and successfully completed the resulting integral. It is here that a few candidates simply omitted many of the mathematical steps in their working in progressing from the integral of $\frac{1}{\sec^2\theta}$ to $\frac{1}{2}\left(\frac{1}{2}\sin 2\theta+\theta\right)$, which resulted in some marks not being awarded.

MATHEMATICS

Paper 9709/41
Mechanics (41)

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. Such a diagram would have been particularly useful here in **Questions 2** and **5**.
- Non-exact numerical answers are required correct to three significant figures (or correct to one decimal places for angles in degrees) as stated on the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.
- In questions such as **Question 6** in this paper, where velocity is given as a function of time, calculus must be used, and it is not possible to apply the equations of constant acceleration.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1, 2, 5(a), 6(b)(i), 7(a)** and **7(b)** were found to be the easiest questions whilst **Questions 3, 5(b)** and **7(c)** proved to be the most challenging.

One of the rubric points on the front cover of the question paper, was to take $g = 10$ and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

(a) This question was answered extremely well, with almost all candidates drawing the correct velocity-time graph in **part (a)**. However, not all correctly labelled the height of the trapezium on the vertical axis with the correct value of 20.

(b) In this part, the majority of candidates used the inefficient method of calculating the area below the velocity-time graph using two triangles and a rectangle rather than applying the formula for the area of a trapezium.

Question 2

(a) This part was answered extremely well by most candidates with a good majority giving the correct value of F as either 17.3 or $10\sqrt{3}$ (from $20\sin 60$). However, it should be noted that many candidates did not make their working clear, and therefore Examiners were uncertain, at times, what answer(s) candidates were giving in either of the two parts to this question

(b) In part (b) most candidates appreciated the need to resolve both horizontally and vertically, and most went on to use Pythagoras to find the magnitude, and inverse tan to find a relevant angle. Candidates are reminded that when a direction of a resultant force is required, giving only an angle is insufficient; some indication in words e.g. above the positive x-axis, must accompany such an angle.

Question 3

The responses to this question were very varied with many candidates making little progress. Although many calculated the difference in the kinetic energy of the train correctly, most struggled to obtain the correct change in potential energy term and even fewer went on to form the correct equation based on the work-energy principle. Finally, a number of candidates did not read the question carefully and gave their answer in joules even though the question asked for the answer in kJ.

Question 4

Similar to **Question 3,** in that the responses to this part were very mixed, with a significant proportion of candidates making little progress beyond correctly calculating the correct driving force of the car using the result that $P = D \times v$. Although many attempted to apply Newton's second law to at least one part of the system (i.e. the car or trailer) or the entire system, many could not deal correctly with which force was acting on which part of the system. In these situations, individual diagrams showing the forces acting on each part of the system would have been beneficial. In some cases, candidates found the correct acceleration but failed to also find the tension in the cable too.

Question 5

(a) This part was answered extremely well with most candidates applying the equation $v^2 = u^2 + 2as$ with a correct expression for a, to find the speed when the bobsled reached the bottom of the slope. The most common error was to assume that the acceleration down the slope was 10 (therefore assuming that acceleration down the slope was the same as the acceleration due to gravity).

(b) Candidates struggled with this part and seemed to be put off by the fact that the mass of the bobsled was not given in the question. This of course turned out not to be relevant and candidates are reminded that in such situations they should define the mass to be say m, and then apply the mathematical principles accordingly (in this case, applying Newton's second law would have given $ma = mg\sin 12 - 0.03 \times mg\cos 12$ and the m's in each term would then of cancelled). Of those candidates who did apply Newton's second law correctly, most went on to find the required time by applying one (or more) of the equations of constant acceleration.

Question 6

(a) It was noticeable, that even though it was clear from the expression given for the velocity of the particle in the question that the acceleration was not constant (and therefore a calculus approach was required), many candidates attempted to use an equation of constant acceleration, or instead tried to solve the equation $v = 0$ in their attempt to show that $k = 10$. Those that realised that they had to find an expression for the acceleration, set this equal to zero (to find a relationship between t and k), and then use the fact that maximum velocity was 4.5 were usually successful.

(b) (i) This part was done extremely well with nearly all candidates verifying that the velocity was zero at the two given times.

(ii) While the responses to this part were slightly better than **part (a)** in that most candidates appreciated that they needed to integrate the given velocity expression to find an expression for the displacement, many did not realise the significance of **part (b)(i)** and instead evaluated this displacement expression between the times of 0 and 16 (without correctly considering the expression at $t = 1$ too).

Question 7

(a) This part was answered extremely well with almost all candidates correctly showing, using the equations of constant acceleration, that the speed of the particle was 15 m s^{-1} when it was at a distance of 20 m above the ground.

(b) While it was pleasing to note that most candidates correctly used the conservation of linear momentum to find the speed of Q after collision, most did not give a direction even though the velocity of Q had been requested.

(c) As expected, candidates found this final part of the last question to be somewhat demanding with many making little progress apart from finding the speed of P at impact. Only the most able candidates correctly found the time for P to reach the ground and then set up two equations for the displacements of P and Q, and then used these equations to correct find the required time and finally the height above the ground when the next collision between the two particles occurred.

MATHEMATICS

Paper 9709/42
Mechanics (42)

Key messages

- The inclusion of a well annotated force diagram has improved when answering questions involving any system of forces, when in either an equilibrium situation or when using Newton's second law. Such a diagram would have been particularly useful here in **Questions 5** and **7(a)**.
- Momentum questions would benefit from diagrams showing speeds and directions before and after collisions to ensure sign errors are eradicated when forming momentum equations.
- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.
- **Questions 1, 2(a)**, **4(a)(i)** and **4(a)(ii)** have exact answers. It should be noted that only non-exact answers should be approximated, so these answers do not need to be given to three significant figures.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1** and **3** were found to be the most accessible questions whilst **Questions 6(c)** and **7(b)** proved to be the most challenging.

In **Question 4**, the angle α was given exactly as $\sin\alpha = 0.12$. There is no need to evaluate the angle in this case, as this can often lead to inexact answers and so any approximation of the angle can lead to a loss of accuracy.

Comments on specific questions

Question 1

There were two common methods in answering this request. Both methods were seen as frequently as each other and both equally as successful. One method was to consider the kinetic energy of the cyclist at the start and end of motion, as well as the work done to overcome the resistance force. Then combining these three energy terms in a work energy equation. A common error seen was for a wrong sign with the work done against resistance. The other method seen was to use constant acceleration to find the acceleration of the cyclist and then use this acceleration in a Newton's second law equation to find the driving force of the cyclist. And finally multiplying this force by the 100 m. A common error with this method seen was for a wrong sign with the resistance force in the Newton's second law equation.

Question 2

This topic is usually a good source of marks for candidates, but a significant number of candidates found these requests a little more demanding.

(a) There where many examples of differentiating correctly to get $44 - 12t$. But a significant number of candidates were unsure what to do next. Some solved this expression equated to zero without a final set of values for t. Others thought that acceleration was positive when the velocity is zero and solved the given expression for v equated to zero, with some using the resulting t values with $44 - 12t$ to decide which gave a positive acceleration.

(b) Those that appreciated that the particle returns to point O when the displacement is zero, usually made a successful attempt at integrating v. The majority of these candidates went on to solve the displacement equated to zero correctly. The common error here was to solve the given velocity equated to zero, in the belief that when the particle returns to O, it is instantaneously at rest.

Question 3

This was a very well answered question, with a significant number of candidates being awarded five or six marks. The majority of candidates resolved in two directions, with only a minority not able to solve these equations to find θ and P.

Question 4

(a) (i) Many correct answers for k as $\frac{5}{6}$ or 0.833 were seen. The occasional 0.83 was seen from giving the answer to two significant figures rather than three significant figures as specified in the rubric for the paper for non-exact numerical answers.

(ii) Most candidates scored well here. The error seen most often was to only include the 480 N force when attempting to find the driving force produced by the car's engine, and not to include the weight component of the car.

(b) Those who realised that they had to equate the driving force F from $P = Fv$ to the given resistance kv^2 generally scored full marks. However, many did not realise this and continued to use the 480 N mistakenly thinking that the resistance remained constant.

Question 5

This question was well done by many candidates considering that it involved a significant knowledge of the mechanics behind the problem and considerable amount of calculation. The main error reported by Examiners was to have the normal reaction of the mass on the plane as $0.8g\cos 28$, omitting the component of the tension. There were also instances of approximating intermediate values in the calculation, resulting in errors in the values obtained for the tension.

Question 6

(a) Candidates should be aware that using the same variable for speeds of different particles is not a particular good thing to do. The speed of particle C is given as v ms^{-1}, so using v again for the speed of E gives an equation that involves the same variable for different speeds, with some candidates solving the equation for v. Those that did define the variable used for the speed of E as something other than v, did go on to show the given expression successfully.

(b) The given answer to this question was only fully shown by a minority of candidates. Most candidates found a correct total initial or final kinetic energy, but rarely both were found correctly. Those who did have correct expressions for both kinetic energies, went on to use them correctly to form an unsimplified quadratic equation in v. Subsequent algebraic errors, particularly in expanding $\left(\frac{15-v}{4}\right)^2$, meant few candidates achieved a correct three term quadratic equation. Even those who got this far only quoted $v = 3$, without showing how the quadratic was solved by factorising or using the quadratic formula which was required given that the request was to show that $v = 3$.

(c) (i) This part was not well answered, with very few candidates progressing beyond finding the time that A travels from X to Y as 6 seconds. Subsequent work used the distance 98 m between particles C and D, so effectively not considering the distance moved by C prior to A and B colliding to form particle D.

(ii) This part was also not well answered, with only those who scored three or four marks in the previous part being able to get a correct answer.

Question 7

(a) This part was well answered. The common approach seen was to attempt Newton's second law equations for particle P and for particle Q, and then eliminating the tension to find the required acceleration. A common error was to see $T = 5$ for the equation of motion for particle Q.

(b) This proved to be the most challenging question on the paper. The majority of candidates could not find the distance moved by either particle so that they are at the same vertical height, with 1 m being the most common height gained by particle Q. The most successful approach was to say that Q moves h m vertically. This implies that P moves h m down the slope and hence loses a vertical height of $h\sin 30$ m. Now as the two particles are initially 2 m apart, $h + h\sin 30 = 2$, so that the height gained by Q is $\frac{4}{3}$ m. Once this value was obtained, the use of energy was well done either on the system, or on individual particles when work done by the tension in the string was required. A very common misconception was to assume that P and Q had different speeds, neglecting the fact that the particles are connected and so should have the same speed.

MATHEMATICS

Paper 9709/43
Mechanics (43)

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.
- When answering questions involving Newton's second law, a complete force diagram can be helpful to ensure that all relevant terms are included in the equations formed, e.g., **Question 5(a)** and **5(b)(i)**.
- In questions where it is stated that a force is variable, candidates should be advised that no marks will be awarded for a method involving Newton's second law to try to find acceleration.

General comments

- The questions were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1**, **2**, **and 4(a)** and **4(b)** were found to be the most accessible questions, whilst **Questions 6(c)**, **7(a)** and **7(b)** proved to be the most challenging.
- In **Question 5(a)**, **5(b)(i)** and **6(b)** the angles were given exactly. There is no need to evaluate angles in situations like this and it is better not to do so as it can lead to a loss of accuracy. In **Question 5**, a significant number of candidates showed all of their working in terms of $\sin\alpha$, rather than 0.05 and 0.09 for **parts (a)** and **(b)** respectively. If the final answer given by a candidate was incorrect then it was not clear whether they had used the correct value of $\sin\alpha$ and so partial marks could often not be awarded.
- One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was pleasing to note that almost all candidates followed this instruction.

Comments on specific questions

Question 1

This question required candidates to use conservation of momentum. Most candidates found both speeds correctly. Some only found the speed of Q after the collision when the velocity of P was positive and did not realise that this velocity could be negative. Many of these then thought that the other velocity was simply – 2 ms^{-1}. A few incorrectly tried to use kinetic energy, for which no credit could be awarded.

Question 2

This question on equilibrium was very well done. A few candidates used the very efficient method of resolving parallel and perpendicular to X which produced the answer immediately. However most candidates used the far less efficient method of resolving horizontally and vertically. Most who used this latter method did correctly solve their equations to find X and the tension.

Question 3

(a) This part was again well answered, with many candidates finding the correct expression for u in terms of a. Most used the formula $s = ut + \frac{1}{2}at^2$ to find two expressions for the distances, but a few used other constant acceleration equations. The most common method was to first find the distances travelled between A and B and between B and C, and then use the fact the latter was twice the former. Unfortunately a number of candidates following this method used the initial speed for B to C of $u + 8a$ rather than simply u, which led to a common incorrect answer of $-\frac{7}{3}a$. Those whose second expression was the total distance between A and C did not have this problem.

(b) Again this part was very well answered. Candidates who had made an error in **part (a)**, but who had an expression for u in terms of a, were able to get full credit for this part. Some candidates simply gave an answer of $u + 18a$, for which no credit was awarded since the question asks for an answer in terms of a. The most common mistake was adding $10a$ rather than $18a$.

Question 4

(a) Almost all candidates realised that integration was required in order to get an expression for distance. Most then successfully substituted $t = 2$ into their expression to find the correct value of k. A few made errors in the substitution and a few did not realise that their constant was zero. A very few chose a constant acceleration method, for which no credit could be awarded.

(b) Candidates used one of three methods, usually successfully, to find the minimum velocity. The first was to differentiate the velocity to find the acceleration, set this to zero to find the value of t for which velocity is minimum and then substitute this value to find the velocity. Some, having found t, forgot that they also had to find the velocity. The second method was to use completing the square. Candidates were very successful using this method, despite the fact that it involved fractions. The third method was to use a formula which gives the minimum velocity immediately. Candidates who had got the value of k wrong in **part (a)** could still get 2 marks out of 3 for the correct value of t.

Question 5

(a) This was the first question part that caused any problems for many candidates. Although there were many correct answers, a number of candidates who did know how to proceed, mistakenly subtracted rather than added the weight components. Others missed out one of the forces, usually the tension or the weight. Some candidates omitted the acceleration due to gravity. Some candidates gave their equations in terms of sin α, but then simply wrote down the acceleration, without stating the value of α that they were using. If the answer for acceleration or tension was correct then it was assumed that they had used the correct value of sin α. However if their answer was wrong then no marks could usually be awarded since it was not clear whether their working was correct.

(b) (i) Candidates were generally rather more successful with this part, and very few added rather than subtracted the weight component. A few, having correctly found the acceleration using the system equation, forgot to find the tension. Others missed out the tension in the equation for the van, despite including it in the equation for the trailer.

(ii) Many candidates scored both marks here, and even if they had found a wrong value of the acceleration in **part (b)**, they could still get both marks here on follow through. Most candidates used the equation $v^2 = u^2 + 2as$ for the method to find the velocity. It was also possible to find the velocity using an energy method and a significant minority tried to use this method, although usually one or more of the terms were omitted and so no credit was awarded.

Question 6

(a) Most candidates scored all 3 marks here, using Newton's second law to find the driving force and then multiplying this by 7. Those who only gained partial marks usually worked out either the weight multiplied by 7, resulting in an answer of 56, or the resistance force multiplied by 7, resulting in an answer of 224.

(b) This part was answered very successfully, and even if the answer to **part (a)** was wrong, both marks could still be awarded on follow through.

(c) In this question part, many candidates scored only partial marks. Some tried to use an acceleration method, for which no marks were available since the question stated that the resistance force had variable magnitude. Candidates who did use an energy method, often made an error in trying to find the work done by the cyclist. Some candidates omitted this term altogether. Others did not find the change in potential energy correctly and yet others had a work-energy equation which included all of the correct terms, but had an error in one or more of the signs in their equation.

Question 7

Both parts of this question, which could be done by an energy method or by using Newton's second law, proved to be very challenging for many students.

(a) The first 3 marks in this part were either for finding the acceleration of the particle as it moved up the slope from *C* to *D*, or for finding the sum of the work done against friction and the change in potential energy from *C* to *D*. In either case if this was not attempted a mark could also be awarded for finding the friction force in the horizontal section from *B* to *C*. A good number of candidates were awarded these 3 marks but then rather less completed the question correctly. Many candidates did not use the acceleration in the horizontal section to find the velocity of the particle as it passed though *C* for the first time, often thinking that it was the same as the velocity as it passed through *B*.

(b) This past part proved to be even more challenging for most students, with many gaining at most 1 mark. In trying to find the value of μ, many candidates using an energy method omitted one or other of the four terms. Similarly those who used an acceleration method often did not have the correct value of u when using the equation $v^2 = u^2 + 2as$, and/or missed out one of the two acceleration terms. A reasonable number of candidates did gain the second method mark for their attempt to find the speed, either for the correct acceleration, or for the difference between the change in potential energy and the work done by friction. Despite the question asking candidates to find the value of μ, some used the value from **part (a)**.

MATHEMATICS

Paper 9709/51
Probability & Statistics 1 (51)

Key messages

Candidates need to be aware of the necessity for clear communication in their solutions. Answers should be supported by the relevant calculations linked to the appropriate stage in their workings. It is especially important to include all the required steps, including the mathematical operations, when the proof of a given result is requested. Where different scenarios are selected to fulfil the demands of a question these should be clearly identified; an organised list or table is often helpful.

Where a diagram is required it should be accurate and clear with scales chosen appropriately.

Candidates should state only non-exact answers correct to 3sf, exact answers should be stated exactly. There is no requirement for fractions to be converted to decimals. Intermediate working values correct to at least 4sf should be used throughout in order to justify a final answer correct to 3sf.

General comments

Most candidates used the response space effectively. Where there is more than one attempt at a question then the solution to be presented for marking should be identified clearly. If extra space is required the additional page should be used in the first instance.

Good solutions were clearly organised and explained. Helpful lists of scenarios, tables of results and diagrams often supported accurate answers. Many candidates were able to tackle **Question 2**, **3** and the first parts of **Question 5** and **6** while **Question 1**, **4** and **7** were more challenging. Sufficient time seems to have been available for most candidates to attempt all of the paper.

Comments on specific questions

Question 1

Many candidates found dealing with summary statistics in coded form challenging. A significant number of scripts had no response to this question. Some scripts provided a variety of calculations; the need to show stages in working and to identify the selected solution must be stressed.

(a) Good solutions found the total of all the coded values for $(x - 30)$ and divided 909 by 45. 30 was then added to find the mean of the uncoded values. Another successful approach was to consider the mean for all 45 values by adding 20×30 to 439 and 25×30 to 470 before dividing by 45 to find the mean. A common error was to find the mean of each set of statistics separately and add them whilst some candidates simply added all the given numbers and divided by 45.

(b) Here also good solutions used the summary statistics correctly finding the sum of the coded values for $(x - 30)^2$, dividing by 45 and subtracting the square of the coded mean. Some subtracted the square of the uncoded mean whilst, as in **1(a)**, others found the variances of each set of statistics separately and added them. A small minority of candidates tried to find Σx^2; this was rarely successful.

Question 2

Many clear and efficient solutions to this question were seen. Some candidates did not seem to know how to use the Normal distribution and were unable to make a productive start on this question.

(a) Solutions including a supporting diagram were more likely to be correct; this is to be encouraged. Stronger candidates performed the standardisation formulae correctly using the standard deviation appropriately and not including a continuity correction. They evaluated these to at least 3sf and used the Normal distribution tables accurately. Most then correctly subtracted the probability of a tail length less than 23 cm from the probability of a tail length less than 35 cm. Some candidates found the unrequired area and subtracted from 1 whilst others considered the 2 appropriate areas between the mean and 23 cm and the mean and 35 cm. Some simply subtracted the 2 areas, reference to an appropriate diagram would have helped to avoid this. Candidates are to be reminded to show their working. In this case the standardisation formulae must be seen, simply providing a value from the calculator is insufficient.

(b) Good solutions used the probability of 75% to obtain a z-value to use in their standardisation formula. Some used a rounded value whilst others used a value from their calculator with more than 3 sf but as this is a Critical Value 0.674 must be used. Once again a helpful diagram indicated that since 7.6 kg was less than the mean then z must be negative. Solutions containing a negative standard deviation were often seen, in some instances the sign was simply ignored.

Question 3

(a) Good solutions often stated the frequency densities before drawing the graph enabling them to obtain the method mark even if the plotting is inaccurate. A suitable scale should be chosen to allow representation of all the data and make good use of the given grid. Most successful diagrams used a scale of 2 cm to 10 cm on the height axis and 2 cm to 2 on the frequency density axis. The careful use of a ruler is essential to ensure that the lines are drawn to the level of accuracy required. Many candidates did not fulfil the requirements of the question and drew a bar chart with the original data. Some gave all the bars the same width often using the inequalities from the question as the bar ends. Graphs should be drawn with a pencil to allow for corrections to be made. The use of shading is not advisable as it can obscure the boundary lines which need to be visible. The vast majority managed to get a mark by having the correct horizontal bar ends. Axes were often unlabelled or the units for the height axis were missing.

(b) Stronger candidates stated the two classes where the upper and lower quartiles were to be found and realised that they needed to subtract the lower bound of the lower quartile class from the upper bound of the upper quartile class. Some who identified the correct classes did not use the values 175 and 160 while others gave all 4 calculations for the subtraction of the bounds. A significant number were not able to use the table to find the correct classes whilst others were unable to make a start on this question.

Question 4

Stronger candidates were able to prove the result in **part (a)** and use their answer as a probability to complete the question. Some candidates used $\frac{1}{4}$ and $\frac{3}{4}$ throughout or were unable to make any meaningful attempt at **part (b)** and **(c)**.

(a) A good number of correct solutions were seen. Most approached this part by adding the probabilities of obtaining the required 4 on the first, second or third throw with all previous throws being a different number. Others found the probability of obtaining no 4s in three throws and subtracted from 1. A small minority added the combinations of either a single 4, 2 4s, or 3 4s being thrown, however a lot of those attempts missed the combination variable. The majority of candidates understood the need for rigour in their working as they were asked to prove a given result, this is to be emphasized. Weaker solutions tried to use a probability space or to calculate the probability of all 64 outcomes.

(b) The most common method seen was to multiply the probability of no 4s in three throws by itself for the two turns. Many candidates did this for one player but didn’t account for the other player. Some used their result from **part (a)**, subtracted it from one to find the probability of getting no 4s in a turn and multiplying it by itself four times.

(c) The candidates needed to consider the possible ways in which the required scores were obtained. This was often not appreciated and correct solutions were rarely seen. Many scripts had no response to this part. Successful solutions considered the required outcomes (2, 0 or 3, 1), stated them clearly and linked them to the appropriate calculation. When finding the probability of scoring 1 or 2 points the number of ways the probabilities could be arranged, 3, was often omitted. Most who got this far realised that they needed to add the results from the 2 scenarios. Some candidates used rounded or truncated values and many solutions needed to be much clearer in their communication and organisation.

Question 5

The use of the Binomial distribution leading to a Normal approximation is a common application. A number of candidates did not identify that the question related to the Binomial distribution and were unable to proceed appropriately. A significant number of scripts had little or no attempt at **part (b)** and **(c)**.

(a) This was a standard application of the Binomial distribution. Most candidates who understood this were able to use the parameters 7 and 0.7 correctly and realised the need to show the binomial coefficients. Some were unsure of the condition requirements of finding the probability of snowing on 5, 6 or 7 days with a small proportion just giving the probability of snow on 5 days. Some used the longer method of subtracting the probability of 0, 1, 2, 3, or 4 snowy days from one. Those using this approach were more likely to use unsuitable boundaries, often omitting one or more outcomes. Weaker candidates omitted the binomial coefficients or gave no evidence of attempting the use the appropriate distribution.

(b) Of the successful candidates many used their answer from part (a) to find the probability of no white weeks and subtracted it from one. Candidates who found the probability of 1, 2 or 3 white weeks sometimes did not take the order into account or just gave 1 of the required outcomes. There were several candidates that did not use their answer from the previous part at all and instead attempted to use 0.7.

(c) Good solutions identified that the Normal distribution was an appropriate approximation in this context. The best answers started with clear, unsimplified and appropriately identified calculations for the mean and variance which were then substituted into the standardisation formula. Many realised that a continuity correction was required as the variable was discrete. Candidates are reminded that the use of the standardisation formula must be seen to gain full marks. The inclusion of a simple sketch was often seen and used to clarify the required probability area. Weaker solutions omitted the continuity correction or simply gave the probability for their z-value, not finding the required area.

Question 6

Almost all solutions used methods appropriate for Discrete Random Variables in this question. A small number of candidates worked with decimals which is less efficient and less accurate than using fractions.

(a) This part was often well done with many candidates scoring full marks. Probability tables were seen in all but the weakest solutions. The best solutions identified the correct outcomes 0, 1, 2 and 3. Omitting the probability of no heads was the most common error. Some candidates used decimal equivalents within the table, errors in rounding the probability of 3 heads to 0.0166 or 0.17 were sometimes seen.

(b) The best solutions clearly identified the outcomes being found and gave unsimplified calculations to support their answers. Whilst most who were able to tackle this part could use the probabilities correctly for the first 3 coins a surprising number doubled the probability for the other 2 coins rather than squaring them. The formation of the appropriate quadratic equation usually led to the correct answer with almost all who got this far indicating that $p = 2$ could not be a solution. Many candidates were unable to make a start worthy of gaining credit.

Question 7

Whilst many candidates were able to tackle **part (a)** the subsequent parts provided more of a challenge.

(a) This question was completed correctly by most candidates. Whilst almost all realised that the number of ways of arranging 8 items is 8! a small number did not allow for the repeated 2s and 4s.

(b) A good number of candidates started by providing a simple diagram showing a 2 at each end of a row of 8 digits. Successful solutions demonstrated the need to put the three 4s together and treat as a single item. Many appreciated that they had to fill the 6 spaces in between the 2s in 6! Ways; some neglected to take the repeated 4s into account. In many instances 4! was seen to indicate understanding that, with the 4s taken together as 1 item, there were four items to arrange between the 2s. Some candidates ignored the placement of the 2s and simply subtracted 4! from the total number of arrangements of 8 distinct items.

(c) Most candidates found this part demanding. Accurate answers were often characterised by a clear and systematic approach to identifying the possible options, for example listing all the 10 scenarios, 123, 124 etc. These were then accompanied by the corresponding combination calculations. Some tried to draw diagrams to show the use of one 2 or one 4, but they were not clear enough to get marks for scenarios. Some omitted one or more scenarios or repeated scenarios. Combinations with 8Cn were common. Many used all probabilities with 8 as the denominator, not realising this was 'without replacement'. Most didn't realise that the number 123 for example could be arranged in 6 different ways, or 221 in 3 ways. Some candidates approached the question by finding the number of scenarios with repeated digits and subtracted the probabilities of these from 1. Few complete and correct solutions were seen.

MATHEMATICS

Paper 9709/52
Probability & Statistics 1 (52)

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level and enables candidates to review their method effectively and is an essential tool when showing given statements are true. When errors are corrected, candidates would be well advised to cross through and replace the incorrect working. It is extremely difficult to interpret accurately terms that are overwritten.

There should be a clear understanding of how significant figures work for decimal values less than 1. It is important that candidates realised the need to work to at least four significant figures throughout to justify a three significant figure value. Many candidates rounded prematurely in normal approximation questions which produced inaccurate values from the tables and lost accuracy in their solutions. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. It was encouraging that more candidates used a ruler to construct box-and-whisker plots and scales that enabled the five key-values to be plotted accurately. Candidates should be aware of the axes should be labelled with both the variable and the units.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. A few candidates did not appear to have prepared well for some topics, in particular when more than one technique was required within a solution. Many good solutions were seen for **Questions 3** and **4**. The context in **Questions 2, 5** and **7** was found to be challenging for many.

Comments on specific questions

Question 1

Most candidates recognised that the geometric distribution should be used.

(a) Almost all candidates presented the correct calculation to find the required probability. A significant number of answers were stated to three decimal places rather than the required three significant figures. This has been highlighted in previous reports and candidates should be consistent in giving non-exact final answers to three significant figures. A small number of candidates calculated the probability of being successful once in seven attempts.

(b) Many good solutions were noted. These often used the less efficient process of adding the probabilities of being successful in each of the possible acceptable scenarios. The more efficient process using $1 - 0.7^5$ was seen used effectively from more confident candidates. As has been highlighted in previous reports, misinterpreting the success criteria is a very common error, and many solutions included the 6th attempt as well. A very small number of candidates used the inefficient binomial distribution approach, usually without success as they either included the probability of not being successful at all or included the 6th attempt.

(c) Most candidates found this question challenging with approximately 10 per cent presenting no solution. The most successful candidates found the probability of being successful once in the first 9 attempts and then multiplied by 0.3 to obtain the probability of being successful for the second time on the tenth attempt. A common misconception was not recognising that the first successful attempt was possible in ${}^9C_1$ ways and omitting this coefficient in the calculation. A common error was to use either ${}^{10}C_1$ or ${}^{10}C_2$ as the coefficient. As in **part (a)**, stating the final answer to three decimal places was noted.

Question 2

Many candidates found the context of this question challenging, although it was encouraging to see some very clear tree diagrams being used to support the interpretation of the information presented.

(a) Candidates should be aware that their supporting work should be such that there is no ambiguity in the method that is being presented. The best solutions identified the possible scenarios that fulfilled the criteria before showing the calculations necessary to obtain the probabilities. Where a tree diagram is present, it is acceptable to clearly identify the branches required to support calculations. Many candidates did not communicate which scenarios they were calculating, and there were terms which were inaccurate for the solution as written. A common error was to omit either P(TBR) or P(TRR). Candidates should be aware that there is no expectation to state probabilities as decimals, and it is good practice to state an exact answer when possible.

(b) Most candidates who attempted this part used the conditional probability formula appropriately. Many recognised that the denominator was the complement of **part (a)**, but any previous error was then included in this work. Candidates who calculated the denominator from first principles were generally correct. A common misconception was to simply find P(head ∩ no reds).

Question 3

This fairly standard normal distribution was attempted well by many candidates. The best solutions often included simple normal distribution curve diagram to identify the required probability areas.

(a) Almost all candidates used the normal standardisation formula correctly. Very few candidates used a continuity correction, which was not required as the data is continuous. A significant number of candidates obtained the correct probability but failed to follow the question requirement to state this as a percentage. Candidates should be encouraged to read the question again after they have finished their solution to ensure that it fully meets the demands of the question. A few candidates obtained the complement of the answer, and these solutions rarely had a sketch of the normal distribution curve to check if the value was reasonable.

(b) A fairly standard normal approximation technique was required for this question. Good solutions often used a sketch of the normal distribution curve to identify the magnitude of the anticipated z-value. Candidates who formed an appropriate equation using the normal standardisation formula were frequently successful in determining the mass required. The common error was to use the positive z-value and obtained a mass greater than the median for the small orange. Many candidates found a probability value rather than a z-value from the information, so were unable to form an appropriate equation.

Question 4

Almost all candidates were able to interpret the back-to-back stem-and-leaf accurately.

(a) The value for the median was found accurately by most candidates. As more than one item was demanded by the question, candidates should be aware of the need to identify each answer appropriately. The calculation of the interquartile range was less consistent, especially in determining the value of the upper quartile. Most candidates used the anticipated method of determining the middle value between the maximum or minimum value and the median to find the quartiles. There is an expectation that a calculation will be present or clearly implied for the interquartile range. A small number of candidates did not use the information from the stem-and-leaf key to scale their answers.

(b) The majority of candidates used an appropriate scale for their box-and-whisker plots. This enabled the key values to be plotted accurately, with the majority of values on grid lines. It was encouraging to see that a ruler was used in many plots. The constructing of a plot freehand is not appropriate at this level as an accurate representation of the data is anticipated. Many candidates only presented the plot for Ravon, where the data was presented above the grid, and did not include a plot for Petral using their own values as well. Candidates should be aware that a linear scale is required and that it should be labelled with both the variable and units (e.g. salary in $). Where the values are large, candidates should be aware that they can use a scale similar to the stem-and-leaf diagram with units of $1000.

(c) Many general, theoretical comments were noted. Candidates should be aware that comments need to be within the context of the question, and specific to the data presented. Reference to the extreme value of $36800 was expected, or to be clearly implied in the comment, and a clear statement of which of the two measures of central tendency to use was required.

Question 5

Although this was a quite text-book probability question, many candidates found it challenging. This was often because the context was misinterpreted, and replacement was assumed.

(a) As a ‘show that’ question, candidates should be aware that a fully justified and reasoned solution must be presented. Good solutions clearly identified which coins could be used to obtain $7, the probability calculations were clearly linked to each scenario and the require total obtained. Candidates who used an outcome table approach were often less successful as they either assumed that the number of coins of each value or the order that the coins were selected did not need to be considered. Again, many candidates using this approach assumed that coins were replaced and had a possible 25 outcomes. When using an outcome table in a show question, the required outcomes should be clearly identified, for example by underlining. Candidates should be aware that if an exact value is given, then only obtaining a value that has to be rounded is an indication that an error has been made.

(b) Almost all candidates attempted to form a probability distribution table. The best solutions had supporting workings for each possible outcome, using the same techniques as in **part (a)**. Although $P(X = 7)$ was given in **part (a)**, where candidates had obtained a different value, this was often used. The most common error was to include $10 as a possible outcome, especially where an outcome table had been created earlier in the question. Candidates should be aware that the total of probabilities in the table should be 1, but many other totals were noted, including those which were greater than 1. The weakest candidates created a table using the probabilities of the individual coin values. A common misunderstanding was to assume that the probabilities of all the outcomes were 0.2.

(c) Most candidates who had formed a probability distribution table in **part (b)** made good progress here. Many solutions included the full unsimplified calculations for both $E(X)$ and $Var(X)$, which is good practice. The efficient use of a calculator allows these to be evaluated without further simplification while clearly communicating the method. A small number of solutions failed to use $(E(X))^2$ in the variance calculation. As the question required two values to be found, candidates should be aware of the expectation that appropriate identification is provided.

Question 6

(a) This standard textbook question was well answered by many candidates. The best solutions provided clear calculations for the mean and variance, substituted accurately into the standardisation formula recognising that a continuity correction was required as the data was discrete and used a simple diagram of the normal distribution to clarify the probability area required. Weaker candidates either omitted the continuity correction or misinterpreted the boundary that was required.

There were a number of candidates who attempted to use a binomial distribution, which although could be evaluated would gain no credit as this is not a suitable approximation to use.

(b) Most candidates recognised that the binomial distribution was appropriate, however a number of candidates continued to use the normal distribution, possibly assuming that only one topic will be assessed in a question.

Good solutions identified the outcomes that fulfilled the success criteria, stated each required binomial term and evaluated accurately. There is no expectation that the probability of individual terms is stated in a solution. The more successful approach was to calculate 1 – P(8, 9, 10) as often P(0) was omitted from the alternative. A common misunderstanding was to find the total of the probabilities of 'fewer than 8 residents classified their service as good' and fewer than 8 residents classified their service as satisfactory'.

Again, many candidates misinterpreted the success criteria, and included 8 residents in the group.

(c) Many candidates found this part challenging, and the question was not attempted by a surprising number. The best solutions recognised that as the residents were being selected at random, the initial probabilities would be used, and that the order that the residents were selected was relevant, so multiplied by the number of possible arrangements. Some candidates multiplied by 3, possibly as there were 3 possible answers for the residents.

Weaker solutions often involved using a binomial distribution approach to find the probability that 1 resident from the 3 answered with each possibility separately and then finding the total. This was often greater than 1, which should have been a prompt to candidates that there was an error in the approach.

Question 7

This permutations and combinations question was found challenging by many, but candidates who listed logically possible scenarios or used simple diagrams to illustrate their approach often achieved good solutions.

(a) Almost all candidates obtained the expected answer. Weaker solutions usually failed to remove the repeated arrangements and omitted to divide by 2! and/or 4!.

(b) There were many different possible approaches to this question. The best solutions identified the possible placements of the Rs with the Es as a block and calculated the number of arrangements of the remaining letters. Many solutions seem to assume that the Rs and Es were identifiable, so multiplied by 2! and/or 4!. A common error was to include scenarios where it was not possible to place the Es as a block, e.g. ^ ^ R ^ ^ ^ R ^ ^ ^, and then assume that the remaining letters will still have 4! arrangements. A small number of candidates attempted a subtraction approach and made insufficient progress to gain any credit as the conditions generate a very large number of scenarios that need to be removed.

(c) Many candidates found this part challenging, and the question was not attempted by a surprising number. Again, solutions with a simple diagram to illustrate the approach were often more successful.

Two main approaches were used to answer the question. The most successful was to consider that the letters were not identifiable and to calculate the number of arrangements for the vowels and the consonants could produce either starting with a vowel, or a consonant. The denominator for the probability was then **part (a)**. The alternative was to assume they were identifiable and then the denominator was 10!, which was often a follow through from an incorrect **part (a)**. A common misunderstanding was to assume that there were 6 possible placements for the consonants if the vowels were ordered (^ V ^ V ^ V ^ V ^ V ^) so the consonants could be arranged in ${}^6P_5$ ways, but this includes arrangements where two vowels are adjacent.

Many solutions did not include $\times 2$ to allow for the first letter to be either a vowel or a consonant.

It was quite common for the final answer to be the number of arrangements that fulfilled the criteria rather than a probability. Candidates are well advised to read the question again when they finish their solution to ensure that they have met the requirements given.

MATHEMATICS

Paper 9709/53
Probability & Statistics 1 (53)

Key messages

Candidates are to be reminded to show all necessary working clearly and no marks will be given for unsupported answers from a calculator. In **Question 5(d)** a number of candidates did not show the standardisation formula. In **Question 3(b)** some candidates showed no working between the sum of products and the final answer.

Candidates are also required to explain their approach to a question. In **Question 6(b)** full marks could not be awarded if it was not clear how the numbers 840 and 120 had been derived. Also, in **Question 6(c)**, many candidates gave no indication as to where their numbers came from or how they were tackling the question. A diagram is often sufficient to inform their reader about their thinking.

General comments

Candidates need to remember that all non-exact numerical answers should be given to 3sf and that this also applies to small decimal answers. In **Question 2(b)**, the standard deviation of 0.269 was often given as 0.27 and the answers to **Question 4(c)** and **Question 4(d)** were often given as 0.0098 and 0.072.

Candidates also need to remember that the purpose of a diagram is to present information in a clear and easy to understand way. If labels are omitted, the diagram is of no use.

Comments on specific questions

Question 1

(a) This question was answered well by the majority of candidates with only a few not noticing that the die was rolled twice and almost all producing the probability distribution in a table as requested. Some were able to multiply probabilities of 1, 2 and 3 to produce the final probabilities but the majority chose to draw up a sample space in a 6 by 6 square and count up the number of times each of the x– values (2 to 6) occurred.

(b) Candidates needed to show full working to find the value of E(X) = $\frac{14}{3}$ as well as substituting into the formula for the Variance of X.

(c) Many candidates did not recognise this question as involving Conditional Probability and either gave the answer as $\frac{19}{36}$, the probability of X being an even number greater than 3 or $\frac{31}{36}$, the probability of X being greater than 3.

Question 2

(a) This part was well answered by most candidates. Almost all were able to produce the correct standardisation formula with 1.93, 1.64 and 0.25 and the majority knew to subtract the resulting probability from 1 as the probability of *z* being greater than a positive number would be less than 0.5.

(b) Most candidates used the tables the correct way round to find the *z*-value 0.44 and knew to equate – 0.44 to the standardisation formula. Most also recognised that the probability 0.75 is linked to the critical *z*-value 0.674 as listed in their tables. Strong candidates produced two equations in μ and σ and solved them simultaneously using a clear method.

Question 3

(a) This question part was answered well and most candidates produced a clear two-stage tree diagram with the outcomes and probabilities clearly labelled on each branch. The candidates who struggled with the question were often unable to establish a two-stage process and gave two separate trees for Box A and Box B. Others introduced an initial stage with probabilities of $\frac{1}{2}$ leading to outcomes Box A and Box B.

(b) Most candidates realised that they needed to sum the probabilities of two Greens and two Yellows. Those whose tree diagram was incorrectly structured were unlikely to be able to answer this question correctly.

Question 4

(a) This question was well answered with most candidates constructing a clear, labelled back-to-back stem-and-leaf diagram with the Penguins on the left, as requested. Most knew that the leaves should be carefully lined up and that there should not be any punctuation between the leaves. Producing the key to the diagram proved to be more challenging with a number omitting it altogether and others forgetting the units or replacing 'seconds' with 'metres' or 'penguins and dolphins'.

(b) Many candidates answered this part well, with only a few not identifying the median and quartiles correctly. Strong candidates knew to label the plot (Dolphins) and that the whiskers do not pass through the box.

(c) This proved to be the most challenging question on the paper. Few candidates managed to either compare the medians and comment that the Penguins were faster or the Dolphins were slower or compare the interquartile ranges and comment that the Dolphins had more consistent times or the Penguins had less consistent times. No credit was given for comparing statistical values e.g., medians, ranges, averages, interquartile ranges.

Question 5

(a) Most candidates recognised that the question required the Geometric distribution and answered the question correctly. A few gave the answer to only 2sf and forfeited a mark.

(b) This part proved to be more challenging. Strong candidates knew that there would be three days when Salah did not complete the puzzle and two days when he did and that this could occur in 4 ways. Others considered the first 4 days when there would only be one success and then multiplied that by 0.65 as Salah would definitely complete the puzzle on the fifth day. Common errors were to forget to multiply by 4 and give the answer as 0.0181 or to forget that Salah had to be successful on the fifth day and think there were 5C2 ways of fulfilling the conditions.

(c) Most candidates recognised this as a problem requiring the Binomial distribution and the most common approach was to subtract the probability of 5, 6 or 7 completions from 1. Those who opted for the slightly longer calculation, choosing to add the probabilities of up to 4 successful completions, frequently forgot to include the probability of zero. There were also notation errors with the subtraction where no bracket was used and only one of the terms to be subtracted was covered by a subtraction sign.

(d) Those who understood the meaning of using 'a suitable approximation' generally answered this question well. Having found the mean and variance of the normal approximation, most produced a correct standardisation formula including a continuity correction in the right direction. Most candidates then went on to find a probability in the appropriate area.

Question 6

(a) Candidates did very well in this part of the question, clearly showing understanding of the topic when calculating the number of arrangements with repetitions. Even weaker candidates were able to access the question and gain credit. Those that did not succeed either left it blank or omitted to deal with the repeated letters and gave an answer of 9!.

(b) Subtracting the number of arrangements with the three Rs together was the preferred method by most candidates. Generally, their solutions showed clear understanding and were well set-out.

Very few omitted to show how they had obtained the values $\frac{7!}{3!}$ and 5!. Those who chose to add the number of arrangements with no Rs together to the number with two Rs together were generally less successful with few candidates following both scenarios through. Many of the candidates who used this method calculated the number of ways for one of the two scenarios correctly, often thinking that this was the final answer to the question.

(c) This question proved to be the most challenging question on the paper with a variety of possible approaches. Many candidates did not describe what they were doing. The most commonly seen successful method was to add the number of ways that the three Rs occur together in the group of 5 (6C2) to the number of ways they occur together in the group of 4 (6C1) and divide the total by the number of ways of choosing 5 or 4 from 9 (9C5). A slightly more complicated approach was to consider the number of Es when choosing a group. A few chose a probability approach but this method was rarely successful. Others used Permutations rather than Combinations and only the strongest candidates managed to use this method successfully.

MATHEMATICS

Paper 9709/61
Probability & Statistics 2 (61)

Key messages

- In all questions, sufficient method must be shown to justify answers; unsupported correct answers will not gain full credit.
- It is important that candidates read the question carefully.
- Candidates need to work to the required level of accuracy of three significant figures. To maintain three significant figures of accuracy in a final answer all intermediate working must be to at least four significant figures.
- For answers that are required ‘in context’, quoting general textbook statements will not be sufficient.
- All working should be done in the correct question space of the answer booklet. If answers need to be continued on the additional page, all working must be clearly labelled with the correct question number.
- Clear presentation of work is of vital importance; in particular, digits must be clear and unambiguous.

General comments

Candidates did not always seem fully prepared for the demands of this paper. Questions where candidates performed well were **Questions 1**, **3a**, **4a**, **5b** and **5c**, and questions that candidates found more demanding were **Questions 3b**, **3c**, **5a**, **6b**, **6c**, and **7b**.

Candidates must note that the conclusion to a Hypothesis test must be written in context and with a level of uncertainty in the language used.

Timing did not appear to be an issue, and presentation was generally acceptable, though there were cases of poor handwriting, digits that were difficult to read, and working being given with no indication of what it represented.

Comments on specific question follow, which identify common errors, though it should be noted that there were some good and fully correct solutions seen as well.

Comments on specific questions

Question 1

This was a standard Poisson distribution question and was generally well attempted. Errors included use of an incorrect value for λ (1.2 being commonly used) and misinterpretation of ‘more than 3’. There were also cases of candidates calculating probabilities of passengers arriving at individual exits. The Poisson expression needed to be shown; there were a few cases seen of unsupported working.

Question 2

The solution to this question required finding $E(X_1 - X_2)$ and $Var(X_1 - X_2)$. Errors were made mainly when finding the variance. For candidates who made a good attempt at the question, standardising was usually correctly attempted, but errors were made in finding the correct probability area (a diagram may have helped here) and some candidates lost accuracy during working. There were some candidates who did not know how to approach this question.

Question 3

(a) Many candidates made a good attempt at finding the required confidence interval, the main errors noted were incorrect z-values, omission of $\sqrt{20}$ in their expression and a confusion between variance and standard deviation. The final answer must be an interval and not two unconnected values.

(b) The majority of candidates either omitted this part completely or thought the statement was true.

(c) This part was not well attempted. Few candidates realised that a conditional probability was required with many merely calculating 0.98^2. A large number of candidates omitted this part completely.

Question 4

(a) Finding the unbiased estimates of the population mean and variance were well attempted. The majority of candidates found the unbiased estimate of the mean correctly, but errors were sometimes made when calculating the unbiased estimate of the variance. This was often due to use of an incorrect formula (confusion between the two standard formulae was evident), and occasionally the biased variance was found.

(b) (i) This question was reasonably well attempted with hypotheses often correctly set up and a standardising equation used. There were, however, candidates who omitted to state their hypotheses and errors in the standardising equation were mainly due to the omission of $\sqrt{100}$. A comparison needed to be clearly stated to justify the conclusion; this is best shown by either an inequality statement between z-values ($1.714 > 1.645$) or alternatively between areas ($0.0432 < 0.05$), thus leading to the conclusion which must not be a definite statement and must also use the context of the question. Concluding that the mean mass had changed was not acceptable as the evidence was sufficient to show the mean mass had increased.

(ii) The given statement was not true. Many candidates realised this, but explanations were not always correct or fully reasoned. Some candidates thought it must be true because we had not been told that the distribution of the masses of the boxes was Normal.

Question 5

(a) This part was not always well answered. Many candidates were unable to state the further condition required, often merely repeating conditions already given, and comments such as the mean and variance need to be the same were often seen.

(b) This part was particularly well attempted, though some candidates gave an unsupported answer; the requirement is that all working must be shown, so the Poisson expression needs to be clearly seen. Errors included incorrect values for λ or an extra or missing term in their expression.

(c) Again, this part was reasonably well attempted. $N(168, 168)$ was usually correctly chosen as the approximating distribution. Errors included a wrong or sometimes missing continuity correction and an incorrect probability area (often an area > 0.5 rather than < 0.5).

Question 6

(a) Knowledge that the area under the curve (i.e. the quarter circle) was equal to 1 (because this was a probability density function) was the starting point here. Use of the formula for the area of a quarter circle was then required. Some candidates unsuccessfully tried integration attempts to find the area rather than using the formula, and others did not know how to approach the question.

(b) Use of the equation for a circle was required here; many candidates did not realise this and were unable to show the required expression for $f(x)$.

(c) Many candidates realised that $xf(x)$ needed to be integrated but not all of them were able to correctly carry out this integration to reach the given answer.

Question 7

(a) Some candidates used the Binomial distribution, as instructed, to carry out the test, however, some tried to use a Normal distribution. Others who correctly used B(31, 0.3) sometimes found $P(X = 4)$ rather than $P(X \leqslant 4)$ and even those who correctly compared $P(X \leqslant 4)$ with 0.05 did not always give the conclusion in the required form (in context and not definite). Hypotheses were not always given or correct, many giving them in terms of μ rather than p.

(b) Some candidates realised that the probability of a Type I error was 0.0239 but few fully justified this.

(c) (i) More candidates successfully attempted this part, correctly realising that the Poisson distribution with $\lambda = 3.65$ was required to calculate $P(Y = 4)$.

(ii) It was important that this question was answered with reference to the question rather than just quoting textbook conditions. Many candidates stated $n > 50$ and $np < 5$ but failed to justify that this was true here because n was 365 and np was 3.65.

MATHEMATICS

Paper 9709/62
Probability & Statistics 2 (62)

Key messages

- Candidates must work to three significant figures of accuracy. Final answers can be rounded to three significant figures, but candidates need to be aware that an answer such as 0.057 is only two significant figures of accuracy.
- To maintain three significant figures in a final answer all intermediate working must be to at least four significant figures.
- It is important that candidates read the question carefully.
- Clear presentation of work is of vital importance; in particular, digits must be clear and unambiguous.
- All working must be clearly shown; unsupported correct answers will not gain full credit.
- If candidates need to use the Additional page to complete their solution, the question number must be clearly indicated.

General comments

This was a reasonably well attempted paper by candidates. On the whole, candidates gave well-presented answers with relevant working shown, though there were occasions where unsupported answers were seen.

Candidates usually worked to the required accuracy, but as noted in the past, many candidates gave final answers to two significant figures (see **Question 2(b)**). There were occasions (see **Question 2c** and **3**) where it appeared that candidates had not read the question carefully. If the question states that a particular level of accuracy is required in the final answer, candidates must follow this instruction. In **Question 3**, a final answer was required to the nearest whole number; candidates did not always do this.

Timing did not seem to be an issue.

The following comments highlight particular common errors made but equally there were some very good, accurate answers seen as well.

Comments on specific questions

Question 1

(a) This question was well attempted. Most candidates used the correct approximating distribution of N(145, 145); the main error noted was an incorrect, or omitted, continuity correction. Occasionally the wrong probability area was calculated; use of a diagram could have helped candidates here.

(b) It was important that this part was answered with reference to the question rather than just quoting textbook conditions. Many candidates quoted $\lambda > 15$ but failed to justify that this was true by referencing that λ was 145. Some candidates mistakenly thought λ needed to be greater than 5 or 30 or 50, rather than 15.

Question 2

(a) Many candidates picked out the next number as 843 and did not realise that this number was out of range. The next group of three digits gave 109 which had already been used; more candidates correctly disregarded this. Trying to deal with 843 by deleting a digit (e.g. choosing 431 as the next number) was often seen, but this meant that not all values were equally likely for this choice. On the whole candidates knew how to use the list of random numbers.

(b) This part was reasonably well attempted. The majority of candidates found the unbiased estimate of the mean correctly, but errors were made when calculating the unbiased estimate of the variance, often due to use of an incorrect formula (confusion between the two standard formulae was evident), and occasionally the biased variance was found. Some candidates rounded early in the calculation causing large errors in the final answer, and some gave their final answer to only two significant figures (either because of a confusion between decimal places and significant figures, or because of a confusion about which zeros were significant).

(c) This part was not well attempted. Very few candidates realised that the variance was unrealistically small in relation to the mean. Some candidates realised this but did not complete their answer by saying that therefore there probably was a mistake in Henri's calculations. Other candidates thought that because they had an answer < 0.1 then there was not a mistake, and others commented irrelevantly on size or random nature of the sample. There were also candidates who looked at the values of Σx and Σx^2 rather than the values of the mean and variance (as directed by the question).

Question 3

Many candidates were able to set up a correct initial equation to find *z*, and many successfully solved this equation to find the correct *z*-value. Errors included confusion between the width of the interval and the limits of the confidence interval (0.244 being the lower limit not the width), and errors using totals rather than the proportions in the formula were also seen. Having found *z*, many candidates successfully found *α*, but some left their answer as 91.2 per cent, not following the instructions in the question to find *α* correct to the nearest integer. Some candidates approximated 0.956 to 0.96 and therefore lost accuracy in their final answer, whilst others merely left their answer as 95.9 per cent or 96 per cent.

Question 4

(a) Stating the required probability proved to be difficult for a large number of candidates. Many thought the probability was zero, and others were unable to give an answer at all. Many tried, mostly unsuccessfully, to calculate the probability; the instruction 'write down' means there should be no need for calculation.

(b) This part was reasonably well answered by many candidates. However, on occasion, values were given without any indication of what these values represented. The values for $E(X_1 - 2X_2)$ or $E(X_1 - 2X_2 + 3)$ and $Var(X_1 - 2X_2)$ or $Var(X_1 - 2X_2 + 3)$ needed to be calculated and clearly stated before finding the required probability. The most common error seen was an incorrect value for the variance of $X_1 - 2X_2$ or $X_1 - 2X_2 + 3$.

Question 5

(a) This was well attempted. Many candidates found the required Poisson expression, although some involved *Y* as well and used $\lambda = 5.5$ rather than 3.1, and some included an extra, unrequired, term (i.e. finding $\leqslant 4$ rather than < 4). It was important that the full Poisson expression was seen; candidates must show the method used and not give an unsupported answer.

(b) Again, this was well attempted. Po(5.5) was frequently used, and many candidates found the correct expression and probability. Again, the full Poisson expression needed to be shown to support the answer. Errors included incorrect values for λ and an incorrect interpretation of 'at least 5'.

(c) This part was generally not well attempted. Many candidates did not find a conditional probability and were only able to gain part marks for their answer. Some candidates correctly calculated P(3 goals in the first half) and P(2 goals in the second half), but instead of multiplying the two probabilities together they added them. Another, more typical error, was to find P(at least 5 goals in total) instead of P(5 goals in total), i.e. they incorrectly used their answer to **part (b)**. Some candidates who correctly realised that a conditional probability was required and formed a correct expression, rounded answers to three significant figures during their working and therefore lost accuracy in their final answer.

Question 6

(a) This question was reasonably well attempted with hypotheses often correctly set up and a standardising equation used. There were, however, candidates who omitted to state their hypotheses. Errors in the standardising equation were mainly due to the omission of $\sqrt{120}$. A comparison needed to be clearly stated to justify the conclusion; this is best shown by either an inequality statement between z-values ($-2.191 < -1.96$) or alternatively between areas ($0.0143 < 0.025$), thus leading to the conclusion. The conclusion must not be a definite statement and must also use the context of the question.

(b) This part was not well attempted. Many candidates did not attempt to find the critical value and seemed unfamiliar with the process of how to find a Type II error in this situation, though some fully correct and well-presented solutions were also seen.

Question 7

(a) This was well attempted. Most candidates attempted to integrate $\mathrm{f}(x)$ with the correct limits and were able to reach the given answer with sufficient conviction. There were occasional sign errors seen in integration attempts.

(b) This part was not quite so well attempted. Some candidates integrated between 0.83 and 0.84 which did not lead to a verification of where the median lay. Others attempted integrations using 0.835 or attempted to find the value of the median rather than use 0.83 and 0.84 as given in the question. Some candidates made a good attempt, but many were not rigorous enough, with appropriate comparisons or a valid conclusion often missing.

(c) Many candidates knew they needed to integrate $x\mathrm{f}(x)$ but were unable to carry out the required integration. Some candidates thought that $(x)(\cos x)$ or $(\cos x)(x)$ was the same as $\cos x^2$ or $\cos^2 x$. Some candidates did not attempt integration by parts, and, of those who did, many made sign errors. However, there were also some fully correct solutions seen, though some of these candidates gave a non-exact answer of 0.934.

MATHEMATICS

Paper 9709/63
Probability & Statistics 2 (63)

Key messages

In all questions, sufficient method must be shown to justify answers. All working should be done in the correct question space of the answer booklet; if answers need to be continued on the Additional page, it must be clearly labelled with the correct question number. It is important that candidates read the question carefully and refer back to it when they have completed the question to ensure they have answered it in full.

General comments

It is expected that candidates work to sufficient accuracy in the paper. This will require working to more than three significant figures in order to achieve a final answer which is accurate to three significant figures. This can be seen in **Question 6**.

Comments on specific questions

Question 1

(a) The suitable approximating distribution was the Poisson distribution Po(4), as $np = 4$ and $4 < 5$. The three terms required were the probabilities for $X = 2$, 3 and 4. Many candidates found this probability successfully.

(b) The required justification involved assessing the values of n and either np or p. Many candidates did not answer this completely.

Question 2

(a) For this 95 per cent confidence interval question it was necessary to find the corresponding z-value of 1.96. Then it was necessary to establish a correct equation involving the width of the interval or the limits of the interval such as $3.12 + z \times \frac{\sigma}{\sqrt{150}} = 3.23$. The majority of candidates found this question accessible and answered well, with around 70 per cent of responses achieving the three available marks.

(b) This question was not generally well answered. Both an answer (yes or no) and a reason were required here. The reason needed to reference the population distribution. This was not given in the initial information and hence there was a need for the Central Limit Theorem. Some candidates omitted to include the answer 'yes'. Other candidates omitted 'population'.

Question 3

For this question the new variable $5L - 10S$ was required. This had the distribution N(–10, 30) and many candidates found this correctly. Some candidates made errors when trying to find the variance, obtaining 240 instead of 30. To find the probability, standardisation of $X = 0$ was required. A sketch could be helpful in deciding whether the large or small area would give the probability.

Question 4

The information yielded the binomial distribution B(25, 0.24). The question also indicated that this should be used and not an approximating alternative. For the significance test, the tail $P(X \leqslant 2)$ was required. A significant number of candidates incorrectly used the single term $P(X = 2)$. It was necessary to write down the comparison between the calculated probability (0.0407) and the 5% significance level and many candidates did this clearly. Then the conclusion should be given in context and not in a definite form. Again, many candidates did this clearly.

Question 5

(a) To demonstrate that $a = 2$ required the use of integration of f(x) between the limits 0 and $\sqrt{2}$ and equating the result to 1. Many candidates did this clearly and correctly. A few candidates integrated xf(x), which was not worthy of credit.

(b) To find the median required the integration of f(x) between the limits 0 and m (or the limits m and $\sqrt{2}$) and the result to be equated to $\frac{1}{2}$. This yielded a quartic equation in m which could be solved by using the quadratic equation formula. This resulted in answers for m^2 so a further square root was required. Additionally, the only applicable answer had to be selected. Many candidates did all of this correctly, however some candidates incorrectly left the answer for m^2 as their final answer.

(c) To find E(X) required the integration of xf(x) between the limits 0 and $\sqrt{2}$. The exact value was required so a single term in surd form was wanted. Some candidates integrated correctly but gave an approximate answer or gave an unsimplified answer, which was not eligible for full credit given the demand of the question.

Question 6

(a) Unbiased estimates of both the population mean and variance were wanted. Some candidates omitted the mean. Other candidates substituted incorrectly in a variance formula.

(b) The vast majority of candidates gave the hypotheses correctly in terms of μ.

(c) Many candidates coped well with the unusual idea of dealing with two significance levels. For the initial standardisation of 1.535, it was necessary to use the unbiased estimate of the variance and the sample size for the distribution of sample means $\left(\frac{0.783}{200}\right)$. Both critical values of z were required (–1.645 and –2.326) for the comparisons with –1.838. Alternatively, both critical probability values (0.05 and 0.01) were required for the comparisons with 0.033.

(d) A Type I error can only occur if H_0 is rejected. Most candidates identified this correctly, however around 10 per cent of candidates were unable to begin a response to this question.

Question 7

(a) The distribution of $X + Y$ was Po(4.1). The four terms P(0, 1, 2, 3) were required. It was necessary to write these down as well as the sum and many candidates did this successfully.

(b) Many candidates only partially answered this conditional probability question, with only around 20 per cent of candidates achieving full marks. The only pairs of values for X and Y satisfying the requirements were (2, 0) and (2, 1). The two given Poisson distributions had to be used to find the total probability of these two cases (0.0957). Then for the conditional probability this value needed to be divided by the probability found in **part (a)**.

(c) There were two main ways of answering this question. One way used the distribution of the means of samples of size 60, namely N(4.1, $\frac{4.1}{60}$). Another way used the distribution of the totals of size 60, namely N(246, 246). Either of these ways produced the probability 0.351. Some candidates followed all these steps successfully. Other candidates omitted the 60 or included an extra 60. Alternatively, continuity correction factors could be included in either case giving a probability of 0.340 or 0.339.