Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8567013635

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Sketch the graph of $y = |x - 2a|$, where a is a positive constant. [1]

(b) Solve the inequality $2x - 3a < |x - 2a|$. [2]

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2 Express $\dfrac{6x^2 - 9x - 16}{2x^2 - 5x - 12}$ in partial fractions. [5]

3 The variables x and y satisfy the equation $a^{2y-1} = b^{x-y}$, where a and b are constants.

(a) Show that the graph of y against x is a straight line. [3]

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(b) Given that $a = b^3$, state the equation of the straight line in the form $y = px + q$, where p and q are rational numbers in their simplest form. [2]

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4 The equation of a curve is $ye^{2x} + y^2e^x = 6$.

Find the gradient of the curve at the point where $y = 1$. [6]

5 **(a)** It is given that the equation $e^{2x} = 5 + \cos 3x$ has only one root.

Show by calculation that this root lies in the interval $0.7 < x < 0.8$. [2]

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(b) Show that if a sequence of values in the interval $0.7 < x < 0.8$ given by the iterative formula

$$x_{n+1} = \frac{1}{2}\ln\left(5 + \cos 3x_n\right)$$

converges then it converges to the root of the equation in part **(a)**. [1]

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(c) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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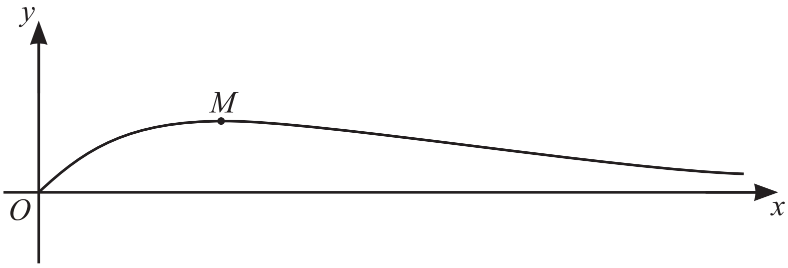

The diagram shows the curve $y = xe^{-ax}$, where a is a positive constant, and its maximum point M.

(a) Find the exact coordinates of M. [4]

(b) Find the exact value of $\int_0^{\frac{2}{a}} x\mathrm{e}^{-ax}\,\mathrm{d}x$. [5]

7 **(a)** Show that $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$. [3]

(b) Hence find the exact value of $\int_{-\frac{1}{8}\pi}^{\frac{1}{8}\pi} \left(\cos^4\theta - \sin^4\theta + 4\sin^2\theta\cos^2\theta\right) d\theta$. [6]

8 The points A, B and C have position vectors $\overrightarrow{OA} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OC} = 8\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, where O is the origin. The line l_1 passes through B and C.

(a) Find a vector equation for l_1. [3]

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The line l_2 has equation $\mathbf{r} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$.

(b) Find the coordinates of the point of intersection of l_1 and l_2. [4]

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(c) The point D on l_2 is such that $AB = BD$.

Find the position vector of D. [5]

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9 The complex numbers z and ω are defined by $z = 1 - \mathrm{i}$ and $\omega = -3 + 3\sqrt{3}\mathrm{i}$.

(a) Express $z\omega$ in the form $a + b\mathrm{i}$, where a and b are real and in exact surd form. [1]

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(b) Express z and ω in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the exact values of r and θ in each case. [4]

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(c) On an Argand diagram, the points representing ω and $z\omega$ are A and B respectively.

Prove that OAB is an isosceles right-angled triangle, where O is the origin. [2]

(d) Using your answers to part **(b)**, prove that $\tan \frac{5}{12}\pi = \frac{\sqrt{3}+1}{\sqrt{3}-1}$. [3]

10 **(a)** By writing $y = \sec^3\theta$ as $\dfrac{1}{\cos^3\theta}$, show that $\dfrac{dy}{d\theta} = 3\sin\theta\sec^4\theta$. [2]

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(b) The variables x and θ satisfy the differential equation

$$\left(x^2+9\right)\sin\theta\frac{d\theta}{dx} = (x+3)\cos^4\theta.$$

It is given that $x = 3$ when $\theta = \frac{1}{3}\pi$.

Solve the differential equation to find the value of $\cos\theta$ when $x = 0$. Give your answer correct to 3 significant figures. [8]

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Additional page

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