MATHEMATICS

Paper 9709/11
Pure Mathematics 1 (11)

Key messages

The question paper contains a statement in the rubric on the front cover that ‘no marks will be given for unsupported answers from a calculator.’ This means that clear working must be shown to justify solutions, particularly in questions involving quadratic equations or trigonometric equations. In the case of quadratic equations, for example, it would be necessary to show factorisation, use of the quadratic formula or completing the square, as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient for certain marks to be awarded. It is also insufficient to quote only the formula: candidates need to show values substituted into it. When factorising, candidates should ensure that the factors always expand to give the coefficients of the quadratic equation.

General comments

Some very good responses were seen but the paper proved very challenging for many candidates. In AS and A-Level Mathematics papers, the knowledge and use of basic algebraic and trigonometric methods from IGCSE or O-Level is expected, as stated in the syllabus.

Comments on specific questions

Question 1

Many fully correct answers to this question were seen. Successful candidates obtained $6(kx)^2\left(\frac{2}{x}\right)^2$, simplified and equated to 150, giving $24k^2 = 150$ and then $k = \frac{5}{2}$. Successful candidates then identified the correct term $4(kx)^3\left(\frac{2}{x}\right)$ and substituted $k = \frac{5}{2}$ to find the correct coefficient of 125. Some candidates simplified $(kx)^2$ to kx^2 and thus were unable to achieve full marks. This type of mistake in simplifying $(ax)^n$ to ax^n is seen each year in this paper and candidates should be careful to avoid such mistakes.

Question 2

Several candidates achieved full marks on this question by differentiating to obtain $2x + ax^{-2}$ or equivalent, equating this derivative to zero, substituting $x = -3$ and finding the value of a.

Candidates who wrote $x^2 - \frac{a}{x}$ as $x^2 - ax^{-1}$ before differentiating were usually more successful than those who attempted differentiating without rearrangement.

Question 3

Most candidates earned the first mark for finding the area of sector OBC. Successful candidates then went on to obtain $\frac{1}{2} \times 15^2 \times \frac{2}{5}\pi - \frac{1}{2} \times x^2 \times \frac{2}{5}\pi = \frac{209}{5}\pi$, or equivalent, and hence the length OA or OD. Candidates who found the correct value of OA or OD generally went on to find the correct perimeter required. Candidates should note that where a question asks for an answer in terms of π, full marks cannot be gained for decimal answers.

Question 4

Successful candidates often substituted for y in the first equation and simplified to obtain $10x^2 + 3kx - 40 = 0$ before using $b^2 - 4ac$ to show that $9k^2 + 1600 > 0$.

Some candidates attempted to solve $10x^2 + 3kx - 40 = 0$ whilst others attempted to solve $9k^2 + 1600 = 0$, not appreciating the significance of the discriminant in this context.

Question 5

(a) This proved to be a challenging question for most candidates. Stronger responses rearranged $4x - 3\sqrt{x} + 1 = \frac{11}{2}$ to find $4x - 3\sqrt{x} - \frac{9}{2} = 0$ which was then solved as a quadratic in $\sqrt{x}$.

Candidates were expected to show their method for solving the quadratic in $\sqrt{x}$ as emphasised in the Key messages.

(b) Candidates were familiar with how to find the equation of a curve given $\frac{dy}{dx}$ and a point on the line, and many candidates gained full marks on this question. Successful candidates integrated to obtain $y = 2x^2 - 2x^{\frac{3}{2}} + x + c$, substituted $x = 4$ and $y = 11$ to find the value of c and hence stated $y = 2x^2 - 2x^{\frac{3}{2}} + x - 9$.

Question 6

(a) Successful responses found the centres of the circles and then used $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the distance between the centres. Many candidates were able to find the centre of the circle $(x - 9)^2 + (y + 4)^2 - 64 = 0$. Candidates needed to rearrange $x^2 + y^2 + 6x - 10y + 18 = 0$ to find the centre of this circle and some responses showed algebraic or sign errors when doing so.

(b) Stronger responses stated $R = 4$ and $R = 8$ and then used their answer from **part (a)** to find the least and greatest distances required. Some candidates stated $R = 4$ and $R = 8$ but were unable to find the greatest and least distances because they had not found the distances between the centres in **part (a)**.

Question 7

(a) Successful candidates differentiated $12(2x + 1)^{-\frac{1}{3}}$ to obtain $\frac{dy}{dx} = -8(2x + 1)^{-\frac{4}{3}}$, substituted $x = \frac{7}{2}$ to find the gradient and found the equation of the tangent by substituting $x = \frac{7}{2}$, $y = 6$ into $y = mx + c$ or $(y - y_1) = m(x - x_1)$.

Some candidates differentiated $12(2x + 1)^{\frac{1}{3}}$ and were unable to gain all the marks. Many candidates were able to differentiate correctly and the method for finding the equation of a straight line given the gradient and a point on the line was well understood.

(b) Many candidates were familiar with how to find the area under a curve using integration. As in **part (a)**, some candidates wrote $y = 12(2x + 1)^{\frac{1}{3}}$ and integrated this, so they were unable to gain full marks.

Question 8

(a) This proved to be a challenging question for most candidates. Many responses were able to gain one mark for substituting $sin^2\beta = a^2$ or $cos^2\beta = 1 - a^2$. Successful candidates used $tan^2\beta = \dfrac{sin^2\beta}{cos^2\beta}$ and $cos\beta = -\sqrt{1-a^2}$ to obtain $\dfrac{a^2}{1-a^2} + 3a\sqrt{1-a^2}$.

(b) Many candidates were familiar with how to approach questions of this type and used $\cos^2\theta = 1 - \sin^2\theta$ to obtain $\sin^2\theta + 4\sin\theta + 1 = 0$. Solving this equation leads to $\sin\theta = \sqrt{3} - 2$ and hence $\theta = -15.5°$; some candidates then instead used $\theta = +15.5°$ and hence did not find the correct angles of 195.5° and 344.5°.

Question 9

(a) Stronger responses differentiated the given expression to obtain $5 + 12x - 9x^2$, factorised this quadratic to obtain the critical values of $-\dfrac{1}{3}$ and $\dfrac{5}{3}$ and hence concluded $x < -\dfrac{1}{3}$, $x > \dfrac{5}{3}$.

Some responses did not show a method for solving the quadratic and were therefore unable to gain full marks.

(b) Stronger responses equated their differentiated expression from **part (a)** to 9, simplified to $9x^2 - 12x + 4 = 0$, solved this quadratic and hence found $k = \dfrac{28}{9}$. Whilst many responses gained full marks on this question, many others were unsure about how to proceed.

Question 10

(a) Stronger responses stated that the first three terms of the geometric progression are $5 + d$, $5 + 4d$, $5 + 10d$, formed the equation $(5 + 4d)^2 = (5 + d)(5 + 10d)$ or equivalent, simplified to a three-term quadratic and solved it to obtain $d = 2.5$.

Candidates who stated $5 + d = a$, $5 + 4d = ar$, $5 + 10d = ar^2$ were generally unable to make any significant further progress.

(b) Successful candidates found the sum of the arithmetic progression and the sum of the geometric progression and subtracted the two values. Most candidates were familiar with finding the sum of an arithmetic progression. Some candidates used the correct formula for finding the sum of a geometric progression but used 5 rather than 7.5 as the first term.

Question 11

(a) Stronger responses obtained $\dfrac{15}{12} - 2\left(x - \dfrac{3}{12}\right)^2$ and stated the range as $y \le \dfrac{15}{12}$ or $f(x) \le \dfrac{15}{12}$.

Candidates who attempted to complete the square sometimes made sign errors or had difficulty in dealing with fractional values. Candidates who completed the square correctly were often unable to give a correct range for the function.

(b) Successful candidates stated that the reflection is in the x-axis and that the translation is $\begin{pmatrix} -\dfrac{3}{2} \\ \dfrac{15}{2} \end{pmatrix}$ or equivalent. Many responses stated the correct reflection but finding the correct translation proved to be more challenging.

(c) Many candidates correctly stated that g is a one-to-one function because each y–value is associated with a single x-value, or equivalent. Some candidates sketched the graph of $g(x) = 3 + 6x + 2x^2$ for $x \in R$ rather than $x \le 0$.

(d) This question proved to be challenging for most candidates. Candidates who attempted to find the inverse function sometimes made sign errors or had difficulty in dealing with fractional values. Candidates were awarded one mark for drawing the line $y = x$ and many candidates were able to gain this mark.

MATHEMATICS

Paper 9709/12
Pure Mathematics 1 (12)

Key messages

Candidates would benefit from spending time understanding the structure of exam papers. If information is given in the introduction to the question, then it is true and relevant for the whole question. In **Question 2**, values for the first term and common difference were given in the introduction and so were valid for both **parts (a)** and **(b)**. Some candidates did not realise this and so were unable to complete **part (b)**. However, if information is given in **part (a)** then it is only true and relevant for that part of the question. The phrase 'it is given instead' is often used to emphasise that this relates to a subsequent part of a question. Many candidates failed to appreciate this distinction in **Question 9** and incorrectly used those values found in **part (a)** in **part (b)**. This meant that no marks could be scored in **part (b)**. Similarly, many candidates appeared unsure of the required method in **part (b)(ii)** of **Question 8**. The fact that it was labelled '**(b)(ii)**' rather than '**(c)**' indicated that there was a link with **part (b)(i)**. Those who used the result found in **part (b)(i)** were very often able to complete **part (b)(ii)**.

Previous reports have highlighted the fact that on the front of the question paper, in the list of instructions, there is a statement 'You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.' Although most candidates now realise that they must do this to score full marks, there is still a significant minority who continue to omit necessary working. To score full marks for the solution of quadratic equations, for example, it would be necessary to show factorisation, use of the quadratic formula or completing the square, as stated in the syllabus. Use of calculators to solve equations and writing down the solutions is not sufficient. Neither is it sufficient to quote only the formula: candidates need to show values substituted into this. Some candidates appeared to be inventing factors after having used their calculators. Factorised quadratics must always produce the coefficients of the original quadratic when they are expanded.

General comments

The paper was generally found to be reasonably accessible for most candidates, although the first few questions proved more challenging than usual. Many very good scripts were seen, and candidates generally seemed to have sufficient time to finish the paper.

Presentation of work was mostly good, although some answers still seem to be written in pencil and then overwritten with ink. This practice often produces unclear responses and makes it difficult to mark accurately. Consequently, appropriate marks may not be awarded. Centres should strongly advise candidates to make their responses clear, utilising the Additional page if more answer space is needed.

Comments on specific questions

Question 1

This question proved to be a challenging start to the paper for many candidates, especially **part (b)**. **Part (a)** was reasonably well answered with stronger candidates often scoring all three marks, but weaker candidates often unable to score any. A common mistake was to think that b was 0.5 rather than 2. **Part (b)** was omitted by many candidates and others tried to solve the given equations rather than using the given graph to work out the number of points of intersection. Candidates may benefit from more practice on the relationship between graphs and the number of solutions to an equation.

Question 2

Part (a) was very well answered, with most candidates able to score both marks. **Part (b)** proved more challenging. Weaker responses were sometimes unable to state either the sum of the first k terms, or the sum of the first $2k$ terms. As mentioned in the Key messages, some candidates did not realise that the values given for a and d in the introduction were valid for **part (b)**. Another common error was to multiply the wrong side of their equation by 10.

Question 3

Some candidates seemed unaware of the required approach for this question and a clear understanding of the required reasoning in **part (b)** was rarely seen. Candidates would benefit from time spent understanding the first principles of differentiation. In **part (a)**, many candidates knew that the required y-coordinate of B would be $2(2+h)^2-3$ and were able to write down the required gradient, but most were then unable to simplify it correctly. In **part (b)**, many responses did not use **part (a)** as instructed, but instead differentiated the original function and substituted $x=2$.

Question 4

This question proved to be reasonably straightforward with a good number of responses able to score full marks. For both parts, some candidates wrote out the full expansion rather than focussing on the required terms. Some weaker responses did not show understanding of the phrase 'independent of x', and answers of $135x$ and $1485x$ were seen following otherwise correct working. Some candidates were unable to find a term in $\frac{1}{x^3}$ in **part (b)** and were therefore unable to make meaningful progress.

Question 5

Part (a)(i) was the best answered part of the whole paper, with 90 per cent of responses stating the correct value. Candidates need to be aware, though, that $\frac{-1}{-3}$ was not accepted if given as their final answer. Conversely, **part (a)(ii)** had one of the lowest success rates on this paper. Many candidates drew $y=-x$ as the mirror line rather than $y=x$. In **part (a)(iii)**, most responses found the inverse correctly, but most did not state its domain. Some candidates used y or f^{-1} rather than x to describe the domain. **Part (b)** was generally well answered, although some responses simply found $gf\left(\frac{1}{4}\right)$ rather than using this value to form and solve an equation.

Question 6

Most candidates seemed confident using the arc length and area formulae to form the initial equations for the perimeter and area, but many then had problems simplifying them. The main problems resulted from having to square $2r$ in the expression for the area and also having to use 2θ as one of the angles. Candidates were often able to form an equation in one variable and produce answers, although omission of the necessary working meant that some responses were sometimes unable to score full marks.

Question 7

This question was generally well answered, especially **part (a)**. Many correct answers were seen, although some candidates did not seem to understand the word 'vertex' or it's connection to the completed square form. In **part (b)**, most responses were able to equate the line and the curve and therefore find the required points of intersection. Many candidates also realised that integration was required and were able to complete this successfully. However, some responses only considered the curve and not the line, and others indicated that the area under the line would be a triangle rather than a trapezium.

It should be noted that responses which omitted crucial working, possibly as this was done in an equation solver and an integral evaluator on calculators, were unable to score any marks.

Question 8

Most candidates attempted **part (a)** but often struggled to find the correct *a* and *b* values. Candidates would benefit from more time practicing the required techniques with both algebraic and numerical values. **Part (b)** was usually completed correctly with most candidates realising that the gradient of the tangent was $-\frac{1}{2}$ and therefore the gradient of the normal would be 2. **Part (b)(ii)** was missed out by a third of candidates. As mentioned in the Key messages, the fact that it was labelled '**(b)(ii)**' rather than '**(c)**' indicated that there was a link with **part b(i)**. Those who used this link were, usually, able to find *p* and then *q* correctly.

Question 9

Overall, this was the least successfully answered question on this examination paper. **Part (b)** was missed out by almost 40 per cent of candidates. In **part (a)**, many candidates equated the line and the curve and attempted to use the discriminant, even though one point of intersection was given and a second one required. Some realised their error and re-started, but often substituted only the given *x*-value into their combined equation and so had one equation with 2 unknowns. In contrast, those who substituted both co-ordinates into the given curve were able to solve it correctly, and a good number of fully correct answers were seen. The main error made by candidates in **part (b)** was to assume that the values calculated in **part (a)** were still valid in **part (b)**. Candidates need to understand that the phrase 'it is given instead' means that this is not the case and only the information given in the introduction, before **part (a)**, is true for the whole question.

Question 10

Parts (a) and **(b)** of this question were generally well answered, although **part (c)** proved very challenging and was only completed correctly by a small proportion of candidates. Some responses mixed up **parts (a)** and **(b)** and did the integration needed for **part (b)** in **part (a)**. Some responses showed unnecessary differentiation in **part (a)** rather than using the given gradient function. In **part (b)**, many were able to integrate correctly, although some did not divide the first integral by 2 and others omitted the +*c* and consequently could not be awarded full credit. In **part (c),** many candidates appeared confused by the given equation and very few realised that because there were no solutions to it, there were therefore no stationary points on the given curve. Because the gradient was negative in **part (a)**, this then meant that it was a decreasing function.

MATHEMATICS

Paper 9709/13
Pure Mathematics 1 (13)

Key messages

Candidates would benefit from spending time understanding the structure of exam papers. If information is given in the introduction to the question, then it is true and relevant for the whole question. If it is given in a part of a question, it is only relevant to that part. In **Question 7(a)**, a value for θ was given for use in that part only and so was not valid for **part (b)**. Some candidates did not realise this and, as a result, gained very few marks in **part (b)**. The phrase '[it is] given instead' is often used to emphasise this in a subsequent part of a question. This contrasts with **Question 10** where the data required to solve both parts is given in the introduction, so can be used in both of **parts (a)** and **(b)**. Additionally, the result from **part (a)** can, and often should, be used in **part (b)**.

When formulating methods, candidates would be well advised to consider the marks available for a question or question part. In **Question 2**, some candidates chose to expand $\tan 2x$ leading to many lines of working often without a final result. With only two marks available they could have realised that a better method must be available and then looked for this method. Similarly, in **Question 6**, those candidates who used the *n*th term instead of the sum to *n* terms might have realised they obtained answers far too easily for a question worth five marks and then reconsidered their approach. Previous reports have highlighted that on the front of the question paper, in the list of instructions, there is a statement 'You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.' Although this message has been taken on board by most candidates, there is still a significant minority for whom it has not. For the solution of quadratic equations, it is necessary to show factorisation, use of the quadratic formula or completing the square, as stated in the syllabus. Using calculators to solve equations and writing down the solutions is not sufficient. Neither is it sufficient only to quote the formula: candidates need to show values substituted into it. Factors stated must always produce the coefficients of the quadratic when expanded.

General comments

The paper was accessible to nearly all candidates and many excellent scripts were seen. The final three questions were nearly always attempted showing candidates had sufficient time in which to complete the paper.

Some candidates made multiple attempts at questions. When no indication is made as to which attempt the candidate wants to be their final attempt, the last attempt is marked. If this is not the response candidates wish to be their final answer, they must make it clear which attempt they want to be marked.

Comments on specific questions

Question 1

For nearly all candidates, this was a very good first question. The *n*th term formula for an arithmetic progression was well understood and used effectively to find the 30th term. The relatively few errors which were reported were usually caused by misreads or sign errors in the algebraic manipulation.

Question 2

Most answers showed that candidates appreciated the requirement to give an exact answer, and most candidates were able to use calculators or basic skills to find the only answer in the given range.

Question 3

(a) Use of the general term of the binomial expansion produced many correct answers to this part. Apart from some observed sign errors and the omission of brackets, candidates' responses were strong for this question part.

(b) The results from **part (a)** were used effectively to produce a simple quartic equation. Those who found both solutions correctly nearly always selected the correct positive value.

Question 4

The solution of this equation by forming a quadratic equation in $\sin^2\theta$ was usually seen. Although many correct solutions for all four angles were seen, the negative solution of $\sin^2\theta = \frac{1}{2}$ was frequently omitted resulting in only two solutions appearing. No method marks were awarded to candidates who obtained the correct answers but did not include any working; as emphasised in the Key Messages, marks cannot be given for unsupported answers. When the range of the solutions is given in degrees it is expected that the solutions will be given in degrees not radians.

Question 5

(a) The name of each of three possible transformation was often given correctly. Examiners overlooked poor spelling if the candidates' answers were phonetically correct. The stretch was the transformation most often described correctly and the translation least often. Some candidates found and described the correct transformations but did not present them in the correct order. Better responses gave the description of the translation in the most economical way, i.e. as a vector. Many different sets of correct answers were possible but only two of these sets were helpful in answering **part (b)**.

(b) The effect of the stretch on the function was well understood but the reflection's effect and the translation's effects were only described correctly in a minority of answers. Some candidates used the incorrect relationship $pf(-x-q) = -pf(x+q)$, losing a correct expression in the process.

Question 6

This question was challenging for some candidates who did not make any progress with this question. Wrong starting points included: $\frac{1-r^4}{1-r^8} = \frac{17}{16}$, $\frac{1-r^7}{1-r^3} = \frac{17}{16}$ or 16 = $\frac{10(1-r^4)}{1-r}$ and 17 = $\frac{10(1-r^8)}{1-r}$.

Candidates who set up a correct equation mostly progressed accurately to form, and then solve, a correct three term quadratic in r^4, or to factorise and divide to form a simple equation in r^4. Those who arrived at $r^4 = \frac{1}{16}$ did not always get both solutions resulting in only one correct sum to infinity, while others thought the solution was $r = (\pm)\frac{1}{4}$. Some responses incorrectly stated that a sum to infinity could be found using $r = \pm 1$.

Question 7

(a) Most responses involved finding the areas of two triangles, two semicircles and the major sector. Some candidates found the area of the large circle, subtracted the areas of two segments and added the areas of the two semicircles. A higher proportion of correct answers were seen from candidates who worked in radians rather than degrees. The requirement to find the radius of the semicircles was not always appreciated and some responses incorrectly assumed the diameters of the semicircles were equal to the radius of the large circle. The formula for sector area was well known, as was the need to use an angle at the centre of $2\pi - 2\theta$.

(b) This part proved to be more difficult even for some candidates who gained full marks in **part (a)**. Using the area of a semicircle was problematic for some, and a significant number did not realise that the value of θ was now changed from that in **part (a)**. The strong hint that the perimeter could be expressed exactly was often missed and numerical values of π were sometimes used. The formula for arc length was nearly always seen correctly expressed using radians and occasionally degrees.

Question 8

(a) This was the most successfully answered question part on this paper with completion of the square for this type of function proving to be well understood by nearly all candidates.

(b) With so many correct answers to **part (a)** it was anticipated that the required value of *k* would be found from most of these. This proved to not always be the case, with *k* given as *x* or use of the value of *b* rather than *a* often seen.

(c) Change of subject of a formula was used to good effect by most candidates to obtain a correct form of the inverse function.

(d) The quickest method using $x = f^{-1}f^{-1}(29)$ was rarely seen, with most candidates attempting to express $ff(x)$ in algebraic form and equating their result to 29. This produced more opportunities for algebraic errors and the resulting quartic equation often proved too difficult to solve. Although some candidates found a correct solution, they did not realise that this method gives an additional incorrect solution of $x = 1$ and this was seldom discounted.

Question 9

(a) Most candidates knew to equate the equations and obtained a correct cubic in *x*. Some responses not worthy of full credit either omitted showing the factorisation or omitted the solution $x = 0$. Some went on to find the values of *y* unnecessarily.

(b) The required integration was done well with many responses showing full working and gaining full marks. There seemed to be an even proportion of candidates subtracting the results of two integrations and candidates subtracting the equations before integration. Some responses did not show the substitution of limits and, as such, could not be awarded full credit. A small number of responses obtained the correct answer with no working shown and therefore gained no marks.

Question 10

(a) Two main methods were used, and some very good answers were seen. Most who found the gradient and mid-point of *AB* and used these to find the perpendicular bisector of *AB* reached the required equation. Those who equated the distances of *A* and *B* from a centre (a,b) of the circle tended to make more algebraic errors, and some did not show their result, $a = 2b + 8$, to be equivalent to the given equation.

(b) Several successful methods were used to find the two possible centres. Those who found the centres usually used a general form of the circle equation effectively to obtain both answers correctly. Most correct answers came from substitution of the equation given in **part (a)** into the distance equation of *A* or *B* from the centre of the circle. However, clever use of vectors and geometry also produced correct answers. A significant number of scripts saw candidates repeating **part (a)** to find the equation of the perpendicular bisector of *AB* even though they were entitled to use the given result in this part.

Question 11

(a) The vast majority of candidates found the correct first and second differential.

(b) This part of the question proved more challenging, with many candidates who set up the correct equation $x^{-\frac{1}{2}} - 8x = 0$ unable to solve this correctly. Some of those candidates squared each term individually and arrived at the correct value for *x* from incorrect algebra. Other candidates did not deal correctly with the negative fractional index, resulting in an incorrect solution. Some candidates correctly arrived at $x^{\frac{3}{2}} = \frac{1}{8}$ but incorrectly stated the solution was $\left(\frac{1}{8}\right)^{\frac{3}{2}}$. A common error was to only find the x-coordinate of the stationary point. Candidates usually knew that the nature of the stationary point could be determined by finding the value of the second derivative, which many of the candidates did accurately for their x-coordinate. Occasionally, a candidate thought that 'increasing' or 'decreasing' was required rather than 'maximum' or 'minimum' when determining the nature of the stationary point.

(c) Responses to this part of the question were mixed. Stronger responses attempted to find both the gradients and the y-coordinates, then used these to find the equation of the tangents before equating and using $x = 0.6$ to find the value of k. It was not uncommon for there to be mistakes in the working which often occurred when finding the y-coordinate at $x = 0.25$. This resulted in one incorrect equation and therefore an incorrect value for k. Occasionally, having found the two equations correctly, some then made errors when simplifying. Some candidates correctly found the two y-coordinates but then attempted to use this in an equation involving the change in y/change in x.

MATHEMATICS

Paper 9709/21
Pure Mathematics 2 (21)

Key messages

Candidates need to ensure that they have fully met the demands of the question by reading it in full before attempting a solution. Some candidates still do not appreciate the meaning of the word 'exact' in the context of an answer. The implication is that no calculator use is required. Greater care is needed in the simplification of some algebraic expressions, with simple sign slips leading to an unnecessary loss of marks.

General comments

A wide range of responses was seen. It was evident that some candidates had very little confidence with the topic on iterative methods for the solution of equations. Most candidates appeared to have been otherwise prepared well for the examination. There were no timing issues, and most candidates had sufficient space for their solutions.

Comments on specific questions

Question 1

(a) Most candidates attempted to take logarithms as required. Some candidates wrote down the result $y = \frac{3}{2\ln a}x + \frac{k}{2\ln a}$ but did not identify the gradient from this equation, as required. This highlights the need for candidates to ensure that they have met the demand of the question.

(b) Most candidates found the gradient of the straight line, making use of the given coordinates. Stronger responses then equated this to the given result from **part (a)** and the value of *a* was found. Correct use of the equation of the straight line initially found in **part (a)** yielded the correct value of *k*. Some candidates chose to use the straight line equation from **part (a)** and formed two simultaneous equations using the given coordinates. This was less common, but equally successful. Very few candidates attempted to use the original equation and re-write the coordinates accordingly.

Question 2

The method of squaring to obtain a three-term quadratic equation was the most common method used. Fewer candidates used the method of dealing with two appropriate linear equations. Most candidates obtained the critical values $\frac{4}{5}$ and $-\frac{10}{3}$; very few candidates discounted the value of $-\frac{10}{3}$. Candidates should be reminded to check the values around the critical values to see if the inequality is fully satisfied. Alternatively, a small sketch of the lines $y = |x - 7|$ and $y = 4x - 3$, considering the relative gradients, would have clarified that there was only one point of intersection and helped with the final inequality.

Question 3

(a) Few candidates made use of the chain rule, and those that did often made errors with the coefficients of the terms involved. Most chose to rewrite the function using an identity before attempting differentiation, for example $\tan^2\left(\frac{x}{2}\right)=\frac{1-\cos x}{1+\cos x}$ or $\tan^2\left(\frac{x}{2}\right)=1-\frac{2\tan\left(\frac{x}{2}\right)}{\tan x}$. Subsequent use of the quotient rule and substitution could then be made to find the exact value required.

(b) Few candidates made any progress unless they realised that $\tan^2\left(\frac{x}{2}\right)$ needed to be written as $\sec^2\left(\frac{x}{2}\right)-1$ before integration could take place. Correct answers were seldom seen.

Question 4

(a) Most candidates were able to find the correct value of a using the factor theorem. Few errors were seen.

(b) Most candidates were able to find the quadratic factor needed to go on and fully factorise p(x). This was often done either by algebraic long division or inspection. Some candidates thought that this was sufficient and did not attempt any further factorisation. There was evidence some candidates made use of their calculator to solve the equation $\mathrm{p}(x)=0$ and 'work backwards' from the solutions, as evidenced by answers such as $(x+2)\left(x-\frac{3}{2}\right)^2$. This result is not the factorised form of p(x) and as such was not worthy of full credit.

(c) Few candidates made any attempt at this part of the question, likely not recognising the connection with **part (b)**. Of those that did make progress, most missed the negative solution.

Question 5

(a) The integration was performed well by the candidates who attempted this question part. There were occasional errors made in the application of the rules of logarithms, but candidates who applied the rules correctly usually went on to show the given result with sufficient detail.

(b) There were a large number of blank responses for this question, indicating that this is a topic with which many candidates are not confident. For the candidates who did attempt a solution, most work was completely correct, including sufficient iterations stated to the required level of accuracy.

Question 6

(a) Most candidates made a reasonable attempt at differentiation. The quotient rule was frequently used to find $\frac{\mathrm{d}x}{\mathrm{d}t}$, but errors in simplification were quite common. Most were also able to find $\frac{\mathrm{d}y}{\mathrm{d}t}$ correctly, and hence an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ as required. It was expected that an attempt be made to simplify the numerator of $\frac{\mathrm{d}x}{\mathrm{d}t}$ to a single term, and that there were no 'fractions within fractions' in the final answer.

(b) Most candidates realised that they needed to find the value of t when $x = 0$ and obtained a correct value. Unfortunately, errors in simplification in **part (a)** often prevented candidates obtaining full marks. There was also some evidence of calculator use even though an exact answer was required; candidates are reminded to reread the question once they have completed their work to ensure they have met the stated requirements.

Question 7

(a) Many candidates used correct trigonometric expansions and substitutions for the required trigonometric ratios. Use of the double angle formula for $\sin 2\theta$ was usually applied together with use of the identity $\cos^2\theta + \sin^2\theta = 1$, leading to the required result. There were the occasional sign errors, but most candidates showed sufficient detail and were accurate in their solutions.

(b) Many candidates successfully made the connection with the result from **part (a)**. Some candidates solved an equation in $\sin 2\alpha$ rather than the correct $\sin 4\alpha$. Whilst some candidates did obtain a correct solution of $6.9°$, very few continued and found the second solution in the stated range.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2 (22)

Key messages

Candidates should be aware of the required level of accuracy in their final answers, as specified by the rubric of the paper or in the question itself. It should be noted that calculations prior to giving a final answer should be conducted with greater accuracy than the level required in the answer itself. It is essential that candidates ensure that they have fully met the demands of the question. When they are asked to show a certain result, they need to show sufficient detail.

General comments

A wide range of responses was seen. It was evident that some candidates had very little confidence with the topic on iterative methods for the solution of equations. Most candidates appeared to have been otherwise prepared well for the examination. There were no timing issues, and most candidates had sufficient space for their solutions.

Comments on specific questions

Question 1

Most candidates were able to obtain a mark for correctly applying logarithms and using the power law. Many candidates appeared to be unable to rearrange the equation $8y\ln 5 = 7x\ln 6$ correctly and obtain $y = \frac{7\ln 6}{8\ln 5}x$, with some candidates mistakenly rewriting $\frac{\ln 6}{\ln 5}$ as $\ln\frac{6}{5}$. Some candidates expressed the coefficients in the equation $8y\ln 5 = 7x\ln 6$ in decimal form, which was acceptable provided a sufficient level of accuracy was used. However, many candidates were unable to obtain the final accuracy mark due to premature approximation.

Question 2

(a) A large number of candidates were unable to gain marks here as they did not identify a suitable method of solution. It was intended that candidates make use of the chain rule. Of those that did use the chain rule, errors in coefficients were common. Another valid method of solution was to make use of the double angle formula and write $4\sin^2 3x$ as $2 - 2\cos 6x$ and then differentiate. Candidates using this approach were often successful and were then able to make use of this initial work in **part (b)**.

(b) It was essential that candidates make use of the double angle formula to rewrite the integrand as $2 - 2\cos 6x$ before attempting integration. Few candidates made use of this method, and few correct solutions were seen.

Question 3

It was evident that some candidates were not confident with implicit differentiation and, as a result, were unable to gain any marks in this question. For those candidates that were familiar with the topic, many were unable to differentiate the term $6e^{-x}y^2$ as a product but were able to continue to obtain an expression for $\frac{dy}{dx}$ and hence find a value for the gradient.

Question 4

(a) Many candidates were unable to produce a correct sketch of the graph of $y = 1 + e^{2x}$, although most candidates were familiar with the form of the graph of the modulus of a function.

(b) The stated equation gave many candidates a good hint so that they considered the equation $1 + e^{2x} = -x + 4$. A correct result usually followed although there were instances of the incorrect use of logarithms. Candidates who chose to consider a squaring method were seldom successful, being unable to deal with the resulting algebraic terms successfully.

(c) It is evident that some candidates were unprepared to answer a question on this topic, as evidenced by the lack of responses to this question part. For the candidates who did attempt a solution, most work was completely correct, with sufficient iterations done to the required level of accuracy.

Question 5

(a) Most candidates were able to gain marks in this question part, showing an understanding of both the factor and remainder theorems. Sign errors and errors in simplification were usually the reason candidates did not obtain the accuracy marks.

(b) Most candidates attempted algebraic long division and, provided **part (a)** was correct, usually obtained a correct quadratic factor. Some candidates simply stopped at this point, stating that the resulting quadratic factor had no solutions when equated to zero. Candidates should be aware that the demand of the question required them to show that there was only one real root to the equation $p(x) = 0$. To obtain full marks, it was essential that candidates show that the discriminant of $2x^2 - 3x + 4$ was -23, (or use an equivalent valid method) and conclude appropriately, also stating the only real root of the equation.

(c) This question part was completed with varying levels of success. Well prepared candidates were familiar with the approach needed and many obtained a critical value of 14.5°. Fewer candidates gave the correct answer of -14.5°. Many candidates were unable to make any valid attempt at solution.

Question 6

(a) It was pleasing to see many correct solutions gaining full marks. However, it was evident that some candidates were unaware of the trapezium rule and were unable to proceed.

(b) Some candidates did not read the question carefully and considered the given area as being calculated from both $\int_0^2 \sqrt[3]{5x^2 + 7}\,dx$ and $\int_0^2 \frac{27}{2x+5}\,dx$, when only the latter was needed. Many responses were able to gain credit for a valid attempt at $\int_0^2 \frac{27}{2x+5}\,dx$, but incorrect answers of $27\ln\frac{9}{5}$ were common.

(c) It was essential that candidates show the deductive process that they used, that is their answer to **part (b)** – their answer to **part (a)**, rather than just write down an unsupported figure. Candidates should be reminded of the significance of the command word 'deduce' and the expectations in their solutions associated with this.

(d) For most candidates, the answer was an incorrect 'over-estimate'. Of the candidates who stated 'under-estimate', few were able to justify their choice correctly and with sufficient detail.

Question 7

(a) Many candidates were able to expand $4\sin\theta\sin(\theta+60°)$ correctly and obtain $2\sin^2\theta+2\sqrt{3}\cos\theta\sin\theta$. While many candidates went on to make use of the fact that $2\sqrt{3}\cos\theta\sin\theta=\sqrt{3}\sin2\theta$, most did not apply the appropriate double angle formula to $2\sin^2\theta$. No further progress could be made until this was completed. As a result, very few fully correct solutions were seen.

(b) Very few correct solutions were seen. Many candidates, having no final response to **part (a)**, did not attempt this part. Other candidates who attempted to make use of their incorrect result from **part (a)** usually did not deal with the double angle correctly. Some candidates did not appreciate the meaning of the word 'hence' and attempted to solve the given equation by other incorrect means.

MATHEMATICS

Paper 9709/23
Pure Mathematics 2 (23)

Key messages

Candidates need to ensure that they have fully met the demands of the question by reading it in full before attempting a solution. Some candidates still do not appreciate the meaning of the word 'exact' in the context of an answer. The implication is that no calculator use is required. Greater care is needed in the simplification of some algebraic expressions, with simple sign slips leading to an unnecessary loss of marks.

General comments

A wide range of responses was seen. It was evident that some candidates had very little confidence with the topic on iterative methods for the solution of equations. Most candidates appeared to have been otherwise prepared well for the examination. There were no timing issues, and most candidates had sufficient space for their solutions.

Comments on specific questions

Question 1

(a) Most candidates attempted to take logarithms as required. Some candidates wrote down the result $y = \frac{3}{2\ln a}x + \frac{k}{2\ln a}$ but did not identify the gradient from this equation, as required. This highlights the need for candidates to ensure that they have met the demand of the question.

(b) Most candidates found the gradient of the straight line, making use of the given coordinates. Stronger responses then equated this to the given result from **part (a)** and the value of a was found. Correct use of the equation of the straight line initially found in **part (a)** yielded the correct value of k. Some candidates chose to use the straight line equation from **part (a)** and formed two simultaneous equations using the given coordinates. This was less common, but equally successful. Very few candidates attempted to use the original equation and re-write the coordinates accordingly.

Question 2

The method of squaring to obtain a three-term quadratic equation was the most common method used. Fewer candidates used the method of dealing with two appropriate linear equations. Most candidates obtained the critical values $\frac{4}{5}$ and $-\frac{10}{3}$; very few candidates discounted the value of $-\frac{10}{3}$. Candidates should be reminded to check the values around the critical values to see if the inequality is fully satisfied. Alternatively, a small sketch of the lines $y = |x - 7|$ and $y = 4x - 3$, considering the relative gradients, would have clarified that there was only one point of intersection and helped with the final inequality.

Question 3

(a) Few candidates made use of the chain rule, and those that did often made errors with the coefficients of the terms involved. Most chose to rewrite the function using an identity before attempting differentiation, for example $\tan^2\left(\frac{x}{2}\right)=\frac{1-\cos x}{1+\cos x}$ or $\tan^2\left(\frac{x}{2}\right)=1-\frac{2\tan\left(\frac{x}{2}\right)}{\tan x}$. Subsequent use of the quotient rule and substitution could then be made to find the exact value required.

(b) Few candidates made any progress unless they realised that $\tan^2\left(\frac{x}{2}\right)$ needed to be written as $\sec^2\left(\frac{x}{2}\right)-1$ before integration could take place. Correct answers were seldom seen.

Question 4

(a) Most candidates were able to find the correct value of a using the factor theorem. Few errors were seen.

(b) Most candidates were able to find the quadratic factor needed to go on and fully factorise p(x). This was often done either by algebraic long division or inspection. Some candidates thought that this was sufficient and did not attempt any further factorisation. There was evidence some candidates made use of their calculator to solve the equation $\mathrm{p}(x)=0$ and 'work backwards' from the solutions, as evidenced by answers such as $(x+2)\left(x-\frac{3}{2}\right)^2$. This result is not the factorised form of p(x) and as such was not worthy of full credit.

(c) Few candidates made any attempt at this part of the question, likely not recognising the connection with **part (b)**. Of those that did make progress, most missed the negative solution.

Question 5

(a) The integration was performed well by the candidates who attempted this question part. There were occasional errors made in the application of the rules of logarithms, but candidates who applied the rules correctly usually went on to show the given result with sufficient detail.

(b) There were a large number of blank responses for this question, indicating that this is a topic with which many candidates are not confident. For the candidates who did attempt a solution, most work was completely correct, including sufficient iterations stated to the required level of accuracy.

Question 6

(a) Most candidates made a reasonable attempt at differentiation. The quotient rule was frequently used to find $\frac{dx}{dt}$, but errors in simplification were quite common. Most were also able to find $\frac{dy}{dt}$ correctly, and hence an expression for $\frac{dy}{dx}$ as required. It was expected that an attempt be made to simplify the numerator of $\frac{dx}{dt}$ to a single term, and that there were no 'fractions within fractions' in the final answer.

(b) Most candidates realised that they needed to find the value of t when x = 0 and obtained a correct value. Unfortunately, errors in simplification in **part (a)** often prevented candidates obtaining full marks. There was also some evidence of calculator use even though an exact answer was required; candidates are reminded to reread the question once they have completed their work to ensure they have met the stated requirements.

Question 7

(a) Many candidates used correct trigonometric expansions and substitutions for the required trigonometric ratios. Use of the double angle formula for $\sin 2\theta$ was usually applied together with use of the identity $\cos^2\theta + \sin^2\theta = 1$, leading to the required result. There were the occasional sign errors, but most candidates showed sufficient detail and were accurate in their solutions.

(b) Many candidates successfully made the connection with the result from **part (a)**. Some candidates solved an equation in $\sin 2\alpha$ rather than the correct $\sin 4\alpha$. Whilst some candidates did obtain a correct solution of $6.9°$, very few continued and found the second solution in the stated range.

MATHEMATICS

Paper 9709/31
Pure Mathematics 3 (31)

Key messages

Candidates should:

- take time to think about the most efficient method to use
- think about standard results and how these could be relevant
- check that their solution answers the question.

General comments

There was only a small entry for this paper. Some of the candidates demonstrated a good understanding of the topics examined, but many scored low marks, even on the more accessible topics.

There were fewer blank responses compared to previous series, with more candidates showing knowledge of most of the specification.

The candidates scored well on numerical methods (**Questions 5(b)** and **5(c)**), partial fractions (**Question 7(a)**) and solving the differential equation (**Question 10(b)**). In some places, they adopted methods that made the solution unnecessarily complicated, such as using long division rather than the factor and remainder theorems (**Question 1**) and not using the most obvious trigonometrical identity (**Question 4(a)**). Particular areas candidates can improve on include sketching (**Question 5(a)**), integration of trigonometric functions (**Question 6(b)**), the vector equation of a straight line (**Question 9**) and forming a differential equation (**Question 10(a)**).

Comments on specific questions

Question 1

Those candidates who were familiar with the factor theorem and the remainder theorem were quick to form the pair of simultaneous equations required and had few difficulties in solving them. In contrast, the responses which attempted to use algebraic long division often found this challenging and rarely reached the point where they had the necessary remainders to form the required simultaneous equations. It is not incorrect to use long division, but the method is more complicated than necessary for this task.

Question 2

The majority of candidates recognised the need to use integration by parts. Most obtained an expression of the correct form but with the correct coefficients rarely seen. This was often due to errors in differentiating $\ln 3x$. The limits were used correctly, but due to errors in integrating and in simplifying the log terms, fully correct responses were rare.

Question 3

Responses which recognised the need to use implicit differentiation usually obtained the correct derivative for the right-hand side of the equation. The left-hand side was more challenging, but the stronger candidates often completed this correctly. Some candidates then rearranged their derivative to obtain an expression of the form $\frac{dy}{dx} = ...,$ which was not necessary and made the solution more complicated than needed.

Substituting $x = 1, y = 0$ into the unsimplified derivative gives a simple equation in $\frac{dy}{dx}$.

Question 4

(a) There are two simple approaches to this question. One is to factorise the left-hand side of the identity using a difference of two squares and the other is to use the substitution $\sec^2\theta = 1 + \tan^2\theta$. The second was the more popular option, but both lead to the solution. Some candidates attempted to produce an expression in $\sin\theta$ and $\cos\theta$, which usually resulted in a complicated expression which did not lead to the required form.

(b) A small number of candidates used the result from **part (a)** to form and solve a quadratic in $\tan^2 2\alpha$. To obtain the final answer, candidates needed to consider both solutions of the equation and to remember that their equation gives values of 2α, so there is a further step necessary to obtain the values for α. Fully correct solutions were rare.

Question 5

(a) This question demonstrated that many candidates need to improve upon the quality of their sketches. Some candidates showed an understanding of the exponential curve and some gave a good representation of $y = \ln(1 + x)$. Other candidates must remember to show both curves on the same sketch and remembered to indicate that the point of intersection represents a solution of the equation.

(b) The majority of candidates were aware of the 'sign change' method, and many used this effectively.

(c) Those candidates who used an initial value of 8 found that the iteration converged very quickly. There were several fully correct responses. Other responses did not work to the required level of accuracy or omitted the final conclusion.

Question 6

(a) Most candidates knew that they needed to differentiate to locate M. Some differentiated the given equation as a product, which was usually the more successful method, and some expanded the bracket before differentiating. The question instructs candidates that they are looking for a value of x in the interval $\frac{1}{2}\pi < x < \frac{3}{4}\pi$, but few candidates noticed this, and fully correct responses were rare.

(b) A few candidates recognised that they needed to rewrite $\sin^2 2x$ as $\frac{1-\cos 4x}{2}$, but it was more common to see an incorrect solution involving $\sin^3 2x$ or $\cos^3 2x$.

Question 7

(a) The majority of candidates used the correct form for the partial fractions and there were several fully correct answers. Incorrect answers were usually due to arithmetic errors.

(b) Many candidates obtained the correct value for the coefficient of x^3 in the expansion of $(1+2x)^{-1}$. The expansion of $(2+x^2)^{-1}$ was more challenging, with several responses incorrectly dealing with the 2 at the start of the bracket. Some candidates gave the expansion up to and including the term in x^3. This involved additional work, but so long as the coefficient of x^3 was correct the other terms were ignored.

Question 8

(a) The candidates who understood that they needed to write $\frac{1}{z}$ as $\frac{1}{1+y\mathrm{i}}$ and to multiply the top and the bottom by $1-y\mathrm{i}$ usually obtained the correct answer.

(b) Demonstrating the given answer requires the correct answer from **part (a)**. However, two of the three marks were for correct use of the results from **part (a)**. These marks depended on correct algebra, but also on having an answer to **part (a)**.

(c) Most candidates drew the correct vertical line and understood that the second locus was a circle. Responses which were not awarded full credit were normally due to the circle being located incorrectly or having the incorrect radius.

(d) This part of the question required candidates to draw together the previous parts of the question. This was a novel question, but there were many correct responses seen.

Question 9

(a) This was a straightforward question about determining a vector line equation from basic information. Most responses included some of the key processes in how to form the equation, but many overlooked the requirement that the equation should commence $r = \ldots$

(b) There were a few correct solutions, but many candidates did not appear to be confident working with vectors. Candidates should be aware that it is necessary to form simultaneous equations using the components of the line equation for questions such as this.

(c) Those candidates who knew how to use the scalar product were able to form an equation in a. Some solutions were incorrect as they did not use the directions of the lines, and some due to algebraic or numerical errors.

Question 10

(a) There were only a few correct solutions seen for this part of the question. Several candidates did not form a correct equation using the information about proportionality, and several did not attempt to form an equation using related rates of change.

(b) There was little work to do to separate the variables in this differential equation, and several candidates had a correct strategy for solving the equation. The question does ask for an expression for t, so candidates who did not form an equation with t as the subject were unable to gain full credit.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3 (32)

Key messages

Candidates should:
- ensure their responses meet all the demands of the question
- take care in the details of their algebra and in the accuracy of their arithmetic calculations
- ensure that, if the question asks for a particular method, this request is followed
- note that diagrams can be helpful, especially in vector questions
- not replace a solution by writing over it, as the result may be illegible to the examiner. This applies equally to pencil working overwritten in ink.

General comments

This paper proved to be challenging for many candidates, but also was very accessible for many others. While there were many candidates scoring fewer than 10 marks, the number of candidates scoring 60 marks or more was higher than in previous series.

The paper was a mixture of familiar questions: **Question 1** (binomial expansion), **Question 3** (square roots of a complex number), **Question 6** (linear graph), **Question 7** (trigonometric identities) and **Question 8** (parametric form) should all have been recognised and were accessible. **Question 4** (index notation) and **Question 5** (complex numbers) proved to be particularly difficult. There was no obvious pattern to which questions the candidates found difficult – some weaker candidates scored close to full marks in **Question 11** and some otherwise strong candidates made limited progress with it.

Incorrect use of mathematical notation continues to be a problem, with brackets frequently omitted, and many responses treated terms like tan and ln as algebraic objects. There were many very basic algebraic errors: in particular, the incorrect cancelling $\dfrac{4e^{2x}(e^{2x}-3e^{x}+2)-2e^{2x}(2e^{2x}-3e^{x})}{\left(e^{2x}-3e^{x}+2\right)^{2}}=\dfrac{4e^{2x}-2e^{2x}(2e^{2x}-3e^{x})}{\left(e^{2x}-3e^{x}+2\right)}$ was common. In **Question 3** and **Question 7(b)**, several candidates were unable to obtain full marks from only considering positive square roots of a number.

Comments on specific questions

Question 1

There were a good number of fully correct solutions seen. The most successful method was to start by taking a factor of $9^{\frac{1}{2}}$ from the bracket. Some candidates then omitted multiplication by 3 after they had expanded the bracket. The incorrect statement $(9-3x)^{\frac{1}{2}}=9(1-3x)^{\frac{1}{2}}$ was common and there were many sign errors seen. Some candidates who attempted to use the general form for the expansion of $(a+b)^{n}$ got no further than the initial substitution, being unable to give a value for ${}^{\frac{1}{2}}C_{1}$ or ${}^{\frac{1}{2}}C_{2}$.

Question 2

(a) The quality of the sketches was lower than in previous series. Many curves seen were notably different to the curves expected. Some candidates were unclear about the horizontal scale they were using, often confusing $\cot 2x$ with $\cot x$ or $\cot \frac{1}{2}x$. Curves of the correct shape sometimes crossed the asymptote at $\frac{1}{2}\pi$ or appeared to be heading for an asymptote much less than $\frac{1}{2}\pi$. A small number of candidates took an alternative approach and attempted to sketch $\tan 2x$ and $\cos x$.

(b) The most successful method was to start with the iterative formula and show that, at the root, $\cot 2x = \sec x$. Candidates who worked in the reverse direction frequently stopped at $x = \frac{1}{2}\tan^{-1}(\cos x)$ and did not state the iterative formula. Quite a few candidates thought that **part (b)** was an invitation to find the root and embarked on the iterative process, reaching the value 0.3747. This does not answer the question and candidates should ensure that, as highlighted in the Key messages, their responses meet the demands of the question.

Question 3

The majority of candidates were familiar with the method required in this question. There were some errors in squaring and sign errors seen, particularly when comparing the imaginary parts. Responses which got as far as comparing both real and imaginary parts usually obtained the quartic equation correctly. The final mark could often not be awarded due to candidates not considering both the positive and negative square roots or an incorrect combination of signs.

A significant minority of candidates could not get started with the question. The most common errors were to start by squaring $6 - 8\mathrm{i}$ or to find the expansion of $(x + \mathrm{i}y)^4$.

Question 4

Many candidates scored no marks for this question. In the majority of cases, this was due to starting with the incorrect use of logarithms; use of the incorrect formula $\log(a + b) = \log a + \log b$ was the most common first step.

Those candidates who started by using $5^{x+2} = 5^x \times 5^2$ usually obtained the correct answer. There were only a few responses which did not give the answer to the required level of accuracy.

Question 5

(a) Very few candidates were sufficiently familiar with the properties of a number written in the form $r(\cos\theta + \mathrm{i}\sin\theta)$ and the associated form $re^{\mathrm{i}\theta}$. Only a small minority of candidates spotted that this was a very straightforward question merely requiring subtraction of two arguments, albeit with one of them negative. There were many blank responses. Some responses showed an attempt to multiply top and bottom of the fraction by the conjugate of the denominator, which was fine, but the significance of the result and how this related to what they had been asked to find was often not fully appreciated.

(b) Several candidates were aware that *u** is a reflection of *u*, but they often omitted which line it was reflected in or gave an incorrect line. The line $y = x$ was a common incorrect answer. Few candidates seemed aware that the argument of *u** is simply the negative of the argument of *u*. Although both marks here were accessible to a candidate aware of the basic properties of a number and its conjugate, there were many blank responses seen.

Question 6

Candidates who started by forming a pair of simultaneous equations in *a* and *b* often became confused between *y* and $\ln y$. Equations involving ln 2.24 and ln 8.27 were common. Candidates who started by finding the gradient of the line usually equated this to $\ln b$ and went on to find *b* correctly. Finding the value of *a* proved to be more challenging. Some responses starting with a correct equation lost the sign and obtained $\ln a = 1.20...$. Some who had $\ln a = -1.20...$ thought that the answer for *a* would be $-e^{1.20}$. There was some confusion between significant figures and decimal places.

Question 7

(a) Most candidates recognised that they need to start by using the formula for $\tan 2x$, which was usually stated correctly. Some candidates overlooked that they need to double this, and some doubled both the numerator and the denominator. Careful manipulation of the left-hand side usually resulted in the correct expression. Common errors included $\tan^3 x \times \tan^2 x = \tan^6 x$ and $-\tan x\left(1-\tan^2 x\right) = -\tan x - \tan^3 x$. Both of these were often followed by forced algebraic manipulation of incorrect working to obtain the given answer.

(b) Most candidates recognised that they needed to use the result from **part (a)** to obtain $4\tan^4 2\theta - 2\tan^2 2\theta - 3 = 0$. From here, several responses went on to obtain the correct solutions. The common errors were to overlook the negative square root of 3, to replace 2θ with θ and never halve the solutions to obtain the required answers and to state $\tan^2 2\theta = -1 \Rightarrow \tan 2\theta = \sqrt{-1} = 1$. Several candidates obtained fortuitous answers by rewriting $4\tan^4 2\theta - 2\tan^2 2\theta - 3 = 0$ as $\tan^2 2\theta\left(4\tan^2 2\theta - 2\right) = 3$ and concluding that $\tan^2 2\theta = 3$. There were some decimal answers seen despite the question asking for exact values.

Question 8

(a) The basic method of differentiating each of x and y with respect to t and then using the chain rule was widely understood. In differentiating y, the (–)2 was often omitted. In differentiating x, the $2t$ was often replaced by t or x. Several candidates did not obtain a derivative of the correct form, often settling on $2\sec^2 2x$. Despite these errors, most candidates claimed to have obtained the given answer. Very few candidates with errors in their working attempted to identify these and thus could not be awarded full credit.

(b) There were several fully correct solutions. A few responses showed errors in simplifying their answers to the required form. Some candidates substituted the given value of t correctly but then found the equation of the tangent rather than the normal. Some candidates were not aware of the values of $\tan\frac{1}{4}\pi$ and $\cos\frac{1}{4}\pi$, and some responses stated decimal values which were very different from the true values of these.

Question 9

Three common errors were seen in this question: candidates paid insufficient attention to the directions of the vectors (they would find $\overrightarrow{BA}$ in place of $\overrightarrow{AB}$ and there were many errors in copying vectors), minus signs were frequently lost in the middle of working and many candidates did not appear to be aware that the vertices of a polygon are named sequentially (several worked with *ABDC* in place of *ABCD*).

(a) There were several fully correct solutions. Many errors occurred between stating a correct equation in $\overrightarrow{OD}$ and stating the answer. There were several errors due to using $\overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OB}$. A significant minority believed that $\overrightarrow{AB} = \begin{pmatrix} 2\times 0 \\ 1\times 4 \\ -3\times 1 \end{pmatrix}$.

(b) Most responses showed an attempt to solve this using the intersection of two lines. There were several fully correct solutions, but several went wrong due to lost signs and other numerical slips. The most common error was to use the sides *AD* and *BC*, rather than the diagonals *AC* and *BD*. A few responses even tried to find the point of intersection of *AB* and *DC*. Very few responses showed a diagram of the trapezium, which might have helped candidates find a correct method. A small number of candidates solved the problem by using similar triangles.

(c) This question asks for the use of a scalar product, so otherwise correct solutions using the cosine rule were unable to be awarded any credit. The majority of candidates understood the correct process for using the scalar product and many fully correct solutions were seen. Some errors were due to use of the incorrect vectors and some were due to slips in the arithmetic. Candidates should be aware that in order to find the angle at *B*, they need to use $\overrightarrow{BA}$ and $\overrightarrow{BC}$ or $\overrightarrow{AB}$ and $\overrightarrow{CB}$.

Question 10

(a) Few candidates gave a correct solution to this part. For many responses, the only mark which could be awarded was for the derivative of V with respect to r. Correctly dealing with the derivative of V with respect to t proved the main difficulty for many, with many attempts looking at the components $\frac{\mathrm{d}V}{\mathrm{d}t} = 40\pi$ and $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.8\pi r$ separately, but not combining them. Some responses showed a misunderstanding of the request and tried to work from the given answer.

(b) There were several clear and concise solutions seen. Some solutions started correctly but did not go far enough to earn full credit. Several quotients involved r^2 and r^3. Some solutions were untidy, with multiple crossings out, largely because many candidates appeared not to have a structured approach to algebraic division. A small number of candidates used the remainder theorem to obtain the remainder.

(c) The majority of candidates separated the variables correctly. Those candidates with a correct solution in **part (b)** were then able to solve the differential equation quite quickly. Some candidates who had not been successful in **part (b)** completed the division correctly here and solved the equation. Responses which did not have a correct form for the quotient and remainder were not able to make progress beyond the first two marks. The question asked for an expression for t, so responses which did not get as far as an equation with t as the subject could not score the final mark.

(d) The responses with a correct equation in t usually scored this mark.

Question 11

(a) The marks for the correct use of the quotient rule were often scored but there were many responses which did not use this formula correctly. The denominator was often not squared or was missing entirely. The omission of brackets was a common issue, causing errors in the later work.

Most responses did score the mark for setting their derivative equal to zero and many then, despite errors, still managed to retain an equation that allowed them to obtain an exact value for x.

Several candidates tried to make the problem look simpler by substituting for e^x. What followed was differentiation with respect to e^x, and this was rarely accompanied by use of the chain rule to obtain $f'(x)$.

Some candidates started by trying to apply partial fractions to split the given fraction. The majority of these attempts took no account of the fact that the degree of the numerator equals that of the denominator. A few correct solutions using partial fractions were seen, but they involved much more work than necessary to obtain the derivative.

(b) There were some concise and fully correct solutions to this question. However, many candidates did not work through the substitution process correctly, so they did not appreciate that the partial fractions required were straightforward, as was the integration that followed. Many tried to apply partial fractions before they attempted the substitution. This could have worked if they had then completed the substitution correctly and used partial fractions a second time. Many candidates worked on a more complicated fraction than necessary, and they did not choose an appropriate form of partial fractions for their fraction. This resulted in a lot of unproductive work.

MATHEMATICS

Paper 9709/33
Pure Mathematics 3 (33)

Key messages

Candidates need to:

- know what is required in their response when attempting a question with a given answer – **Questions 7(a)** and **10(a)**
- know what is required when sketching on an Argand diagram – **Questions 1(a)** and **1(b)**
- know how to apply the chain rule in differentiation − **Question 11(a)**
- think in both trigonometrical relation and complex number questions about which method best suits that problem, as numerous different approaches are often possible − **Questions 4** and **5.**

As mentioned above, candidates need to show working to justify their result in complex number questions, so reliance on a calculator is not advised here.

General comments

The standard of work on this paper was very variable, with some of the short initial questions proving far more difficult than expected. Furthermore, many candidates were still not showing sufficient working in their responses, for example **Questions 7(a)** and **10(a)**. In **Question 7(a)**, some candidates went from an expression in $\cos t$ and $\sin t$ to the final given answer in $\sin 2t$ with no mention of what double angle formula they were using. Such large jumps are very likely to lead to errors and, even if that is not the case, working needs to be seen to gain the marks, especially in questions with the answer stated. Whilst different candidates will always display different levels of working, a good gauge of what is deemed adequate can be found in past question papers and their respective mark schemes.

Comments on specific questions

Questions 1

(a) (b) Many candidates struggled with this question. The common partial attempts were complete circles of radii 2 and 4, although in some cases neither radius was stated or indicated. Candidates often shaded the region between 0 and $\frac{1}{4}\pi$, and between 0 and $\frac{1}{2}\pi$. This question had nothing to do with regions and was simply about points satisfying two conditions: one condition being on the modulus and the other on the argument defined over an interval. The solution to **part (a)** should have been an arc of the circle centred at the origin and of radius 2 between 0 and $\frac{1}{4}\pi$ and nothing else, for example a line drawn from the centre of the circle to the circle itself at $\frac{1}{4}\pi$. Likewise, the solution to **part (b)** should have been an arc of a circle centred at the origin and of radius 4 between 0 and $\frac{1}{2}\pi$.

Question 2

(a) Candidates adopted one of two different approaches, one far more successful than the other. Those who assumed convergence of the iterative formula, say to the limit x, had little trouble showing that this limit was the solution to the equation $f(x) = 0$. However, those candidates who started from x being the solution of $f(x) = 0$, were usually not successful in recovering the given iterative formula.

(b) Most candidates scored full marks. Of the few responses which did not obtain the final mark, this was caused by not showing convergence, not working to 4 decimal places or not quoting the converged root to 2 decimal places.

Question 3

(a) This was another question that saw many candidates score full marks. However, it was common to see $P = 3$ being used instead of $\ln P = 3$.

(b) Many candidates became confused in doubling their value of P, taking $P = 3$ and $2P = 6$. In fact, the value of P initially is e^3 and $2e^3$ when doubled. Although any value of P above e^3 was acceptable, values below this were not as they arose from a negative time.

Question 4

Many candidates struggled with this question, as highlighted in the Key messages. The obvious approach is to cross multiply, then either solve the linear equation for z and subsequently rationalise the denominator, or to introduce $z = x + iy$ and consider real and imaginary parts separately. Far too many candidates showed a lack of understanding of complex numbers by taking real and imaginary parts of an equation involving z as opposed to one in x and y. The most common approach saw candidates rationalising $z + 3i$ in the denominator on the left-hand side of the equation, resulting in $z^2 + 9$ or a similar such non-linear expression involving x^2, y^2 and xy terms. Having done this, few responses were able to make much progress, even after many attempts and pages of working.

Question 5

(a) This identity proof generated several different approaches. The most common successful approach was using the difference of two squares, together with the double angle formula for cosine, allied with recognition that $4\sin^2\theta\cos^2\theta = \sin^2 2\theta$. However, candidates who directly expressed $\cos^4\theta - \sin^4\theta$ as $\cos 2\theta$ could not receive full credit as the "show that" demand in the question requires complete working. A few of the common errors observed were that $\cos^4\theta - \sin^4\theta = \cos^2 2\theta$ and including/excluding factors of $\frac{1}{2}$ and 2 in the $\sin 2\theta$ identity.

Candidates who chose to work from the right-hand side often ended up with a mixture of different sign errors, thus preventing much further credit. However, most candidates gained at least one mark on this question for a single correct use of a double angle formula. Better responses were able to show the proof in just a few lines of working, but for a significant number their written answers were not structured or clear.

(b) The majority of candidates recognised the relevance of **part (a)** in **part (b)**. A minority did not realise the disguised quadratic was in $\cos 2\alpha$ not $\cos\alpha$, and thus did not solve for the required variable. Of those who solved successfully for one solution, the vast majority found both without issue.

Question 6

(a) The majority of candidates were unable to gain credit for this question. Many left their answers in terms of position vectors rather than coordinates, which was explicitly required in the question. Some candidates unnecessarily solved simultaneous equations to find the point of intersection, even though both lines shared the same position vector. Candidates should pay close attention to the terminology used in the question: a 'state' question should not involve complex calculations.

(b) This part was done well, with candidates recognising that they needed to use the scalar product to find the cosine of the angle. Some did not obtain the final mark by giving their answer in an unsimplified form and leaving it as, for example, $\frac{8}{\sqrt{5}\sqrt{30}}$. Some candidates used the position vector for P and therefore gained no marks. A minority of candidates used the cosine rule to find the required cosine value. The most common errors seen were misreads of the direction vectors or arithmetic errors in the dot product.

(c) Most candidates earned the first mark, which was awarded for the two lengths required to find the area of the triangle. However, many struggled to secure the next two marks because they were unable to appreciate that the phrase ‘exact area’ required avoiding the use of a decimal approximation for $\cos\theta$. Instead, they proceeded to calculate the area of the triangle using this approximated angle, leading to incorrect results. The simplest approach, which was rarely seen, was to use the identity $\sin^2\theta = 1 - \cos^2\theta$ to find the value of $\sin\theta$ and then proceed to find the area by using $\frac{1}{2}ab\sin C$ or draw a ratio triangle to evaluate $\sin\theta$. Some candidates used the vector product to compute the area and some even used the dot product to find the perpendicular height and used $\frac{1}{2}$ × base × height. Many who did this did not manage to obtain full marks, usually due to a careless mistake or not working in exact values.

Question 7

(a) Most candidates had the correct approach and differentiated at least one of x and y correctly with respect to t. Common errors included incorrectly taking the derivative of $\cot t$ to be $\frac{1}{\cos^2 t}$. It was also common for candidates with incorrect derivatives to manipulate their expressions and claim to have reached the given answer. Weaker responses often presented derivatives with various combinations of x, t and θ. Of those that got down to $\frac{-1}{6\cos^2 t\sin^2 t}$, a significant proportion then just stated the given answer without any justification of the change in numerator or denominator. Candidates should be reminded that ‘show that…’ questions require full working with correct notation to arrive at the conclusion as stated in the question paper.

(b) Most candidates recognised the connection between **part (a)** and **part (b).** The majority were successfully able to derive the required gradient of the normal, and the co-ordinates needed to establish the normal. The most common errors seen were forgetting to take the negative reciprocal for the normal gradient and not putting the answer in the required form. Candidates who used the gradient of the tangent were unable to gain any marks for this part.

Question 8

(a) Many candidates spent a considerable amount of time and effort on this question only to fail to score any marks. One relatively quick and efficient approach is substituting key values after equating numerators. Some candidates seem to want to avoid this method, intent instead on employing the method of equating coefficients. Whilst perfectly acceptable, this approach suffers from the fact that if a candidate gets one coefficient wrong then the other one follows likewise. Other approaches do not have this weakness.

The presence of a did confuse some candidates and some changed the $7a^2$, substituting an x or a value for a. Others made an error with the calculation of $(a - 2x)$, despite having stated $x = -3a$. To confirm their success, candidates should check their answer quickly by recombining their two terms. One or two candidates miscopied in going from **part (a)** to **part (b)**.

(b) Many responses did not correctly deal with the a’s and the need to get the bracket into the form (1 +/−..). Some did the reciprocal of the a but not of the 3, and some forgot to bring in the original $2a$ and a after they had performed the expansions. Quite a few errors were seen with the sign of the 'x term in the expansion of $(a - 2x)^{-1}$. However, many good solutions were observed and for those who stayed accurate throughout, the process was manageable. Several responses received partial credit having got the expansions correct but incorrectly combined their expansions. It was common to see 215 instead of 217 or to have lost some a’s in the denominators.

(c) The vast majority of candidates found this part challenging and sometimes omitted this altogether. Very few correct responses were seen.

Question 9

(a) Large numbers of candidates reached the correct answers and correctly identified the quotient and remainder. However, not all really understood the significance of this part and did not connect it properly with **part (b)**.

(b) This part was generally well answered. Where responses started off with the form found in **part (a)**, the integration was generally correct and simplified to the given answer. Some responses did not receive full credit due to insufficient detail, and those who started with the wrong setup usually scored nothing other than the first mark.

Question 10

(a) Few fully correct responses were seen here, with many candidates finding even the first mark challenging to earn. Candidates need to concentrate more on the information given and correctly identify what each part of the question is telling them and to what variables it relates. Many were too driven by the final answer to identify correctly what they needed. Many responses did not state any chain rule equations but produced the answer by putting numbers together; responses scoring full marks were not widely seen.

(b) Candidates who recognised a differential equation and separated variables correctly did extremely well and almost always obtained the required answer, dealing well with both the –3 coefficient of h and the ln term. It was also very rare to see the constant omitted. Responses which did not separate variables correctly could not score any marks.

Question 11

(a) Many candidates completed the differentiation involving the chain rule accurately and obtained the correct form. A good number of these who got through to the required quadratic were able to solve it correctly. At this stage, some chose the correct answer for a, some offered two answers for a, some chose just 1.35 and some stated $1.35 + \pi$. Obtaining only 4.93 would score the final mark.

(b) Six marks were available here for a relatively straightforward integration by substitution, with the substitution given in the question. If candidates followed the method and structure for solving this sort of question they usually scored very well. Many dealt deftly and clearly with the negative and the inverted limits. Some candidates did not obtain the final mark, usually from stating a negative area.

MATHEMATICS

Paper 9709/41
Mechanics (41)

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. Such a diagram would have been particularly useful here in **Questions 3** and **6**.
- Non-exact numerical answers are required correct to three significant figures (or correct to one decimal place for angles in degrees) as stated on the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.
- In questions such as **Question 8** in this paper, where acceleration is given as a function of time, then calculus must be used, and it is not possible to apply the equations of constant acceleration.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1, 2(a), 3, 4(a)** and **7(a)(i)** were found to be the easiest questions whilst **Questions 4(c), 5** and **6** proved to be the most challenging.

In **Questions 6** and **7(b)**, the angles were given exactly as $\sin\theta = \frac{7}{25}$ and $\sin^{-1}(0.15)$ respectively. There is no need to evaluate the angle in these cases and in doing so can often lead to inexact answers (and so any approximation of the angle can lead to a loss of accuracy).

One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

This question was answered well with most candidates setting up the equations of motion for both particles separately and then solving simultaneously to find the required magnitude of the acceleration and the tension in the string. The most common error was to either give a negative acceleration or to set up two equations that were not consistent with each other (e.g. having the same sign for T in both equations).

Question 2

(a) Almost all candidates correctly used the principle of conservation of energy to find the speed of the particle at B.

(b) Approaches to this part were almost equally split between those who continued to use an energy approach, and those who used Newton's Second Law and the equations of constant acceleration. Generally, both groups were equally successful with the only common error being sign errors in either the application of the work-energy principle or Newton's Second Law.

Question 3

This was the best answered question on the entire paper with almost all candidates correctly resolving horizontally and vertically to find the values of P and θ.

Question 4

(a) Almost all candidates sketched the correct velocity-time graph for the bus's journey from A to B.

(b) It is important that candidates read the question carefully as many in this part correctly calculated the time when the bus was either accelerating or decelerating, but very few correctly added these two expressions together and subtracted from the given value of 240 to obtain the required expression, in terms of a, for the length of time that the bus was travelling at constant speed.

(c) Responses to this part were varied with several candidates leaving this part blank or not appreciating that to work out the distance from A to B then the area below the velocity-time graph needed to be found. Those candidates who had correctly calculated the required expression in **part (b)** were usually successful in this part. Finally, those candidates who could recall and correctly apply the formula for the area of a trapezium were generally more successful than those who needed to split this area into a rectangle and a triangle.

Question 5

(a) Most candidates were successful in obtaining at least one mark in this part for applying the constant acceleration formula $s = ut + \frac{1}{2}at^2$ at least once, but only the most able could set up a correct equation for when the two particles collided e.g. $100(T-1) - 5(T-1)^2 = 80T - 5T^2$ and then solve this to obtain the given result that $T = 3.5$. The most common errors were to either use the same expression for the time in both equations or to use $T+1$ instead of $T-1$.

(b) Surprisingly this part was left blank by a good number of candidates with many not realising that all that was required was to use $s = ut + \frac{1}{2}at^2$ again with the given value of T (or $T - 1$) and $u = 80$ (or 100).

(c) Very few candidates scored full marks here and many left this part blank. Not all appreciated that there were three parts to finding the required time. The first part was to find the velocity of each particle at the point of collision (which was best done using $v = u + at$), then apply conservation of linear momentum to find the speed of the combined particle after impact, and then use a complete method to find the required time (the most efficient way of doing this was to again use $s = ut + \frac{1}{2}at^2$).

Question 6

This was a question in which many candidates struggled, and it was clear that many were unsure where to begin with this type of extended response. Examiners commented that it was very unclear at times what candidates were doing and what mechanical principles were being applied. It would be extremely beneficial if candidates stated which principle (e.g. resolving parallel to the plane) they were applying at each stage of their working.

One common error was to assume that the normal contact force of the plane on the particle was a component of the weight only when in fact it was a combination of this component and a component of the horizontal force P. A second common error was to have the frictional force acting in the wrong direction; as the least possible value of P was required this meant that the frictional force would be acting up (and not down) the plane.

Finally, candidates are reminded that in questions such as this that it is advisable to be working with exact trigonometric expressions until the end of the problem and to only use their calculators at this final stage to work out the numerical value of P from an exact expression.

Question 7

(a) (i) This part was done extremely well with almost all candidates correctly showing that $k = 40$.

(ii) This part was also well done by candidates with many applying $P = Dv$, and then correctly applying Newton's 2nd Law to find the required acceleration. The most common errors were to either misread the fact that speed had changed to 45 (from 48 in the previous part) or to not include the resistive force in their application of Newton's 2nd Law.

(b) In this part many candidates appreciated the need to apply both $P = Dv$ and Newton's 2nd Law with $a = 0$, but a number failed to include the correct weight component as $1200 \times g \times 0.15$ and some included the acceleration from **part (a)(ii)**. Many candidates who correctly obtained a quadratic equation in v went on to solve it correctly.

Question 8

The responses to this final question were mixed; Examiners reported seeing several perfect responses (that scored all 7 marks) to some that were either blank or little in the way of progress was made. Most candidates appreciated the need to use calculus (rather than the constant acceleration formulae) so many scored at least 2 of the 7 marks for correctly integrating the given expression for the acceleration twice to obtain corresponding expressions for the velocity and displacement of the particle (it was surprising how many candidates though believed incorrectly that this expression needed differentiating too). It was relatively common (and pleasing) to see candidates correctly interacting with the fact that the velocity at $t = 0$ was 4.2 to correctly calculate the first constant of integration and realising too that they needed to solve their expression for the velocity set equal to zero to find the two required times when the particle was at instantaneous rest. However, it was only the most able candidates who recognised that they needed to integrate their expression for v between these two values of t to find the required distance.

MATHEMATICS

Paper 9709/42
Mechanics (42)

Key messages

- In previous reports, it has been mentioned that when answering questions involving any system of forces, a well annotated force diagram could help candidates to include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. Force diagrams are now being seen more often than in previous examination series.
- In questions such as **Question 6** on this paper, where acceleration is given as a function of time, calculus must be used, and it is not possible to apply the equations of constant acceleration. When integrating, a constant of integration must be considered as this constant may not necessarily be zero.
- Non-exact numerical answers are required correct to three significant figures or angles correct to one decimal place as stated on the front of the question paper. Candidates are strongly advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 2, 3(a)** and **7(a)** were found to be the most accessible questions whilst **Questions 4, 5,** and **7(b)** proved to be the most challenging.

In **Question 7(a)**, the angle θ was given exactly as $\sin\theta = 0.6$. It was pleasing to see that many candidates used this exact value rather than evaluate the angle which would have led to a loss of accuracy.

One of the rubric points on the front cover of the question paper was to take g = 10 and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

(a) A well answered question by a significant majority of candidates by using the fact that the gradient of the line between $t = 4$ and $t = 10$ is equivalent to deceleration. The most common error seen was finding a negative gradient of the line and equate it to $\frac{5}{3}$ m s^{-2} leading to $V = -10$, when the diagram has V as a positive value and described in the text as a speed.

(b) Another well answered question by using the area between the graph and the t-axis is equivalent to distance. The main error seen was to use a value of –3 for the distance moved between $t = 10$ and $t = T$, using displacement rather than distance. A minority of candidates wrongly assumed that the time accelerating and decelerating between $t = 10$ and $t = T$ were equal.

Question 2

Given that the resistance force is not described as being constant, the only valid approach to answering the request is to use energy. Most candidates who used energy gained some credit for a kinetic energy term or a potential energy term. When forming a work energy equation, errors were seen in signs with the four terms, or occasionally a term was omitted. Those who assumed that the resistance force was constant, gained some credit but not full marks.

Question 3

(a) This was a well answered question by most candidates.

(b) This question was only correctly answered by a minority of candidates. The two main errors seen was to omit the weight component when forming an equation, or for the cyclist to be accelerating even though the request was for the steady speed the cyclist could maintain.

Question 4

This question proved to be challenging for most candidates. Difficulty arose from candidates labelling the tensions in the three distinct strings with the same label, usually T, when these tensions were different.

(a) This is essentially a three-force problem with forces acting at the point A. Once this is realised, there are a variety of approaches that could be used. The simplest of which is a triangle of forces to get an isosceles right-angled triangle so that the tension in AB is equal to the weight of the 0.2 kg particle. The most common approach was to resolve in two directions, usually horizontally and vertically, to get two equations involving T_{AB} and T_{AP}.

(b) This is also a three-force problem with forces acting at the point B. For those who attempted this question, the most common approach was to resolve in two directions to form two equations in T_{BP} and θ and solve simultaneously.

Question 5

This question proved to be the most challenging for the majority of candidates.

(a) The approach to this question is to get expressions in terms of t for the distance that P and Q move, usually $s_P = 2t - \frac{1}{2}gt^2$ and $s_Q = \frac{1}{2}gt^2$, and then use the fact that the particles are 1 m apart so that $s_P + s_Q = 1$ leads to the particles meeting after 0.5 seconds. This time then can be used in the equation $v = u + at$ to find speeds of 3 m s^{-1} and 5 m s^{-1}. Candidates who did get a correct time, invariably gave a negative value for at least one of the speeds.

(b) The first part of this question is to use conservation of momentum, but there was a great deal of confusion about the directions that P and Q are travelling at the time they meet. In fact, the particles are travelling in the same direction. It was quite common to see that the speed of P found from using the conservation of momentum, to be given as the final answer. Once this speed is found, a further calculation using constant acceleration is needed, but firstly the distance that the particles are above the ground at collision is required.

Question 6

This topic is usually a good source of marks. However, a significant number of candidates assumed the constant of integration to be zero with a resulting loss of marks.

(a) This was usually successfully answered for the candidates who included a constant of integration. The most common incorrect answer seen was $v = -t^{\frac{3}{2}}$ leading to a velocity of –1 m s^{-1}.

(b) A constant of integration was seen here but was either not evaluated, or used $t = 0$ to evaluate it, even though $t = 0$ is not within the domain of this stage of the motion.

(c) Those who obtained correct expressions for the velocities in each stage of the motion answered this part well, integrating and applying the correct limits correctly.

Question 7

(a) This was a straightforward pulley question. Most candidates correctly wrote down the two Newton's second Law equations, sometimes leading to a negative acceleration which then failed to score the associated mark unless the magnitude, as requested, was stated. One error seen was to omit the friction in the equation for particle A. Another error was to use the coefficient of friction as though it was friction. Overall, most candidates scored well on this question.

(b) This question proved challenging for most candidates. The speed of A (or B) at the time when B reaches the ground needed to be calculated first. As B remains on the ground, A is now moving with the string slack, so a new acceleration needs to be calculated so that the distance that B moves subsequently can be calculated and added to the 0.25 m. Often, candidates assumed that the acceleration remained the same once the string was slack.

MATHEMATICS

Paper 9709/43
Mechanics (43)

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.

- In questions where the total work done over a period of time is given, candidates should be advised that a method involving acceleration is not appropriate and an energy method should be used.

- Candidates should be advised that the wording at the start of a question, then applies to the whole of that question, and not just the first part of the question.

General comments

- The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. The first five questions together with **Question 6(b)** and **7(a)** were found to be the most accessible questions, whilst **Questions 6(a), 6(c), 6(d)** and **7(b)** proved to be the most challenging.

- In **Question 3** the angle was given exactly. There is no need to evaluate the angle in situations like this, and it is better not to do so, as it can lead to a loss of accuracy. A significant number of candidates showed all of their working in terms of $\sin\alpha$, rather than 0.08. If the final answer given by a candidate was incorrect then it was not clear whether they had used the correct value of $\sin\alpha$ and so partial marks could sometimes not be awarded.

Comments on specific questions

Question 1

This question was well answered with many candidates gaining all four marks. Candidates had to form an energy equation relating the work done by the athlete to the work done against friction and the change in kinetic energy. The most common mistake was to think that the athlete did work at a constant rate. Candidates who thought this, divided the work done by the athlete by the distance to get the force (which they thought was constant) to use in Newton's second law. Although this method produced the right answer it was not correct as the work done by the athlete was given rather than the force exerted by the athlete, and so the method could only gain 2 marks.

Question 2

This four-force problem was again generally well answered with many candidates gaining at least five marks out of six. Many candidates did not gain the final mark, either getting the direction wrong (often giving the direction of the resultant force) or stating that the angle was 4 degrees or 86 degrees but not specifying in which direction it was. Some gave a general indication such as north-west without specifying an angle. A few candidates had not resolved correctly and had sine rather than cosine or vice versa. Some candidates only gave the horizontal and vertical components and went no further, and some missed one of the forces (usually the 24 Newton force).

Question 3

(a) Many candidates answered this question well, but more errors were made than in previous questions. The most common error was to omit the fact that the car is traveling up a slope and simply multiply the resistance force by the speed (for which no marks were available). Some candidates thought that the angle was $\sin 0.08$, instead of $\sin^{-1} 0.08$.

(b) This question proved more challenging. Candidates had to use the new power to find the acceleration of the car. A considerable number of candidates omitted either the resistance force or the fact that the car was travelling up a slope or sometimes both in their answer. Such candidates could still score a mark for the follow through of their power from **part (a)**. It should be noted that the wording at the start of the question before **part (a)** applies to the whole question, not just **part (a)**. A large number of candidates rounded the answer to two significant figures, perhaps thinking that the zero in front of the decimal point was a significant figure. A few candidates made a sign error in resolving up the slope and others again used an angle of sin 0.08.

Question 4

In this question candidates had to use conservation of momentum to find the new velocity or velocities and then find the greater change in kinetic energy. Although most found two velocities, some only found the lower one as they presumably realised that this would produce the greater loss in kinetic energy. Most candidates did find both possible velocities and then went on to find the loss in kinetic energy for both, usually correctly. Some candidates found the loss for each particle, but did not combine them, and some found both losses but then did not state which was the greater and so could not gain the final mark. A few candidates made errors in using conservation of momentum and a few found the velocity of 3 ms^{-1}, but then thought that the other option was that the particles coalesced, which was incorrect.

Question 5

(a) Candidates had to resolve forces to find the normal reaction R, and then relate the horizontal component of the tension to μR. Most candidates found no difficulty in doing this. A few made a sign error in R, but more candidates thought that R was equal to the weight, missing out the component of the tension perpendicular to the plane. These candidates could still get a mark if they equated $T\cos 30$ to their friction. A few tried to include acceleration, which they could not do since the particle was at rest.

(b) This part was rather similar to **part (a)**, but this time there was acceleration involved. Candidates often correctly used Newton's second law to find the required tension and those who thought that R was equal to the weight could still get a mark for the equation $T\cos 30 - F = 12 \times 0.2$, with F replaced by 0.5×120.

Question 6

(a) In this part, candidates had to integrate the acceleration $a = 0.6t$ to find the velocity, and then substitute $t = 4$. Many tried to use constant acceleration equations instead to show that the velocity was 4.8 ms^{-1}, which was an incorrect method. Some candidates did integrate correctly but then simply stated that when $t = 4$ was substituted, the value obtained was 4.8, but did not actually show the substitution and so did not get the mark.

(b) Candidates had to draw a velocity time graph for this part of the question. Although some produced a fully correct graph, many others produced a graph where the first section was straight, so producing a trapezium. This was the case even with some candidates who had a correct **part (a)**. Some candidates who had used constant acceleration equations, produced a fully correct graph. A few candidates thought that the particle started slowing down at $t = 11$ rather than $t = 15$, and some had an incorrect maximum speed or no figures on their diagram at all.

(c) In this part, almost all candidates found the gradient of the velocity time graph correctly. Many wrongly found the constant term to be 4.8 rather than 19.2, by using $t = 0$ rather than $t = 15$. Candidates who substituted the two points (15, 4.8) and (20, 0) almost always found the equation correctly.

(d) Candidates had to find the total distance travelled by the particle in this part. Those who had a trapezium in **part (b)** found the area of this trapezium (often as two triangles and one rectangle) and so their final answer (of 74.4 m) was incorrect. Those candidates who had a correct graph in **part (b)** usually found the area correctly.

Question 7

(a) This question involved finding the tension in a string joining two particles over a pulley, and the time taken for one of the particles to reach the plane. Most candidates answered this well and produced a fully correct response. A small minority of candidates made an error in one or more of the Newton's second law equations.

(b) This question was the most challenging question in the paper. Many candidates correctly found the speed of the particles when the particle *B* reached the plane. However, the remaining stages in finding the overall time for which the particle is at least 3.25 m above the plane proved to be more challenging. There were two popular methods of finding this time. The most popular was to use $s = ut + \frac{1}{2}at^2$ with $s = 0.25$, $v = \sqrt{10}$ and $a = -10$ to find the two times at which the particle was 3.25 m above the plane, and then subtract them. Several candidates who used this method had $a = 10$ or had a different value of s, often 2.25 or 3.25. Many candidates who tried to use this method did not subtract the two times but gave one or other as the total time. The other method was to find the speed of the particle when its height was 3.25 and then use $v = u + at$ with $u = \sqrt{5}$, $v = 0$ and $a = -10$ to find the time to the highest point, and then double it. Most candidates who used this method came to the correct answer. There were also a number of 'hybrid' methods, which sometimes came to the correct answer. Those who tried to use an energy method rarely had any success at all. A considerable number of candidates had a fully correct method but due to premature approximation did not find the final answer to three significant figures.

MATHEMATICS

Paper 9709/51
Probability and Statistics 1 (51)

Key messages

Candidates need to be aware of the necessity for clear communication in their solutions. Answers should be supported by the relevant calculations linked to the appropriate stage in their workings. Where different scenarios are selected to fulfil the demands of a question these should be clearly identified; an organised list or table is often helpful.

Where a statistical diagram is required, accuracy and clarity are vital. Scales should be chosen appropriately and the axes labelled fully.

Candidates should state only non-exact answers correct to three significant figures, exact answers should be stated exactly. There is no requirement for fractions to be converted to decimals. Intermediate working values correct to at least four significant figures should be used throughout in order to justify a final answer correct to three significant figures.

Candidates should be aware that numerical accuracy is expected, and the efficient use of a calculator should enable the initial calculation to be stated and then fully evaluated as the next stage of the process. Some simple multiplication and addition errors were noted.

General comments

Most candidates used the response space effectively. Where there is more than one attempt at a question, the solution to be presented for marking should be identified clearly. If extra space is required, the additional page should be used in the first instance.

Good solutions were clearly organised and explained. Helpful lists of scenarios, tables of results and diagrams of the Normal distribution often supported accurate answers.

Many candidates were able to tackle **Questions 3** and **5** while **Questions 6** and **7** were more challenging. Sufficient time seems to have been available for most candidates to attempt all of the paper however some topic areas of the syllabus did not appear to have been prepared well for.

Comments on specific questions

Question 1

Most candidates recognised that the geometric approximation was appropriate. However, inconsistent interpretation of the success criteria limited overall success.

(a) Candidates who used the longer $P(X = 0,1,2,3,4,5,6,7)$ approach tended to be more successful than those who attempted to use the $P(X < 8)$ formula as this was often interpreted as $P(X \leqslant 8)$ leading to an incorrect final probability.

(b) Good solutions considered the second 6 as being fixed on the 8th throw and then identified the possibilities for the placement of the first 6. A simple 'diagram' with the first 6 placed in the 7 different positions was often helpful. Some candidates did not include the probability of the second 6 in their calculation, but the most common error was failing to use '× ${}^7C_1$' in the solution. Over 10 per cent of candidates submitted no attempt for this part.

Question 2

Many candidates found this question challenging, with the requirement to establish the value of constant k before completing the probability distribution table.

(a) Good solutions included clear workings to show the possible values of X substituted into the probability formula and created an equation in k which was then used to complete the probability distribution table. Weaker solutions often omitted 0, or having obtained a value for k did not generate a numerical probability distribution table. Some more able candidates stated a value for k without the supporting work that is expected at this level. The values in some probability distribution tables did not sum to 1, which should be a standard check that candidates are aware of.

(b) Where there was an appropriate probability distribution table present, candidates were usually successful in finding $\mathrm{E}(X)$, but found the variance formula more challenge, with less numerical accuracy or failing to use $(\mathrm{E}(X))^2$. The techniques required are quite standard, and candidates could be encouraged to develop a scaffolded approach to answering this topic. Because **2(a)** was often incomplete, over 15 per cent of candidates did not attempt this part.

Question 3

(a) Some excellent cumulative frequence graphs were noted. These included plotting clearly at the upper boundary of the classes using a 'x', joining the points with a freehand curve, labelling both axes appropriately, including the units on the time axis. Weaker candidates used a ruler to join the points, which is inappropriate at this level, or plotted the values at the mid-point of the range. A few candidates attempted to draw a histogram.

(b) Many candidates were able to use their graphs effectively to find the median, and with less accuracy the interquartile range. As the question states that the graph must be used, there should be a clear indication on the graph where values are being obtained, with the best solutions using a ruler to draw horizontal/vertical lines to obtain the readings. Some candidates did show some marking on the graph, but many did not communicate their working process. An unexpected error was using 35 minutes on the time axis to obtain the median, rather than finding the time of the 'middle' value for the appropriate terms.

(c) Candidates who used a table format to collate the required data were often more successful. Some candidates used the space near the original data table before **3(a)** to find both the class midpoints and the class frequencies, but were not always careful in transferring their values to further calculations. Many good attempts were noted, with clear working to show how the estimated mean formula was being used. A surprising number of candidates made slight slips in either calculating the frequencies or the mid-points for just a single class. A small number of candidates presented their final answer as an improper fraction, which is not appropriate in this context.

Question 4

The best solutions included a tree diagram in **4(a)** to clarify the information presented, and then use this to support their work throughout the question.

(a) The best solutions used a tree diagram to identify the four possible ways of obtaining marbles of the same colour, stating clearly the scenarios when evaluating the individual probabilities before summing to present the answer as a proper fraction. Some candidates did convert the fraction to a decimal which is not required. Weaker solutions failed to identify the possible scenarios to communicate their thinking. A major misconception was not including the probability of obtaining the initial head or tail in the scenario, even when the marble probabilities clearly identified which had been obtained.

(b) There were many good attempts at calculating the conditional probability seen, including from candidates who had not used the 'coin probability' in **3(a)**. Some accuracy errors in the calculation from the correct expression were seen. Weaker solutions again omitted the 'coin probabilities'.

Question 5

The use of the Binomial distribution leading to a Normal approximation is a common application. A large number of candidates did not interpret the success criteria accurately. A significant number of scripts had little or no attempt at **part (b)**.

(a) This was a standard application of the normal distribution. Most candidates were able to form at least one of the normal standardisation formulas correctly, although some introduced a continuity correction even though mass is a continuous variable. Good solutions then used a simple sketch of the normal distribution to identify the required probability area to form the correct calculation. Weaker solutions often simply found the difference between the two probabilities obtained. It was unexpected that some candidates assumed that the required probability area was symmetrical.

(b) Good solutions identified that the Normal distribution was an appropriate approximation in this context. The best answers started with clear, unsimplified and appropriately identified calculations for the mean and variance which were then substituted into the standardisation formula. Many realised that a continuity correction was required as the variable was discrete. Candidates are reminded that the use of the standardisation formula must be seen to gain full marks. The inclusion of a simple sketch was often seen and used to clarify the required probability area. Weaker solutions omitted the continuity correction or simply gave the probability for their z-value, not finding the required area.

Question 6

Many candidates found this question challenging, with 10 per cent not making any attempt. It is possible that candidates did not read the entire question, as **6(b)** was a fairly standard question for the Binomial distribution, but was omitted by over 40 per cent of candidates.

(a) Most solutions included an attempt to form at least one standardisation formula equated to a z-value. Candidates should be aware that 90 per cent is a critical value, and as such the value stated in the tables provided must be used. Weaker solutions equated with either the original probabilities, or used the tables incorrectly and treated the given probabilities as z-values and found the linked probability. Most candidates who formed the two simultaneous equations were able to solve them accurately.

(b) The best solutions interpreted the success criteria accurately, stated the individual terms within the calculation and evaluated accurately. A common error was to include obtaining 8 in the required probability. A small number of candidates did not use the probabilities given in the question. Most candidates used the 1 – P(8,9,10) approach, but accurate solutions using the alternative, but longer, P(0,1,2,3,4,5,6,7) were also noted. Candidates should be aware that they should work to at least four significant figures if a three significant figure answer is anticipated. There did appear to be some confusion between four significant figures and four decimal places within the calculations presented.

Question 7

Whilst many candidates were able to tackle **7(a)**, the following parts provided more of a challenge.

(a) This question was completed correctly by most candidates. Whilst almost all realised that the number of ways of arranging 9 letters was 8! because the Ts were together, a small number did not allow for the repeated Es and Ls. The weakest attempts did not apply the Ts condition and simply divided by an additional 2! As the Ts were repeated.

(b) A good number of candidates started by providing a simple diagram showing a T at each end of a row of 9 letters. Most successful solutions then excluded the Es and considered the number of arrangements that the remaining five letters could make and then the number of ways that the Es could be inserted so that they were not together. More confident candidates used ${}^6C_2$ within their calculation. Candidates who attempted to use a subtraction approach were less successful, failing to identify an appropriate 'total arrangements' term initially, often using either their answer to **7(a)** or 9!. Again, candidates who used a simple diagram to represent the criteria were often more successful.

(c) Most candidates found this question demanding. Accurate answers were often characterised by a clear and systematic approach to identifying the possible options, calculating the total number of selections without restrictions that could be made and then calculated the percentage accurately. Common errors were the omission of TEE, the inclusion of TTEE or not including the number of ways that the T and Es could be selected initially. A significant number of candidates found a value for the number of selections but failed to make any attempt at finding the required percentage, or used an incorrect divisor, typically their answer to **7(a)** or 9!. Few complete and correct solutions were seen.

MATHEMATICS

Paper 9709/52
Probability and Statistics 1 (52)

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level and enables candidates to review their method effectively and is an essential tool when showing given statements are true. When errors are corrected, candidates would be well advised to cross through and replace the term. It is extremely difficult to interpret accurately terms that are overwritten.

There should be a clear understanding of how significant figures work for decimal values less than one. It is important that candidates realised the need to work to at least four significant figures throughout to justify a three significant value. Many candidates rounded prematurely in normal approximation questions which produced inaccurate values from the tables and lost accuracy in their solutions. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. It was encouraging that more candidates formed the back-to-back stem-and-leaf diagram more accurately, although the omission of units was still common in the key provided.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. It is good practice to look quickly through the paper initially to identify the syllabus content for each question, as this should help candidates manage their time more efficiently. A few candidates did not appear to have prepared well for some topics, in particular when more than one technique was required within a solution. Many good solutions were seen for **Questions 4** and **6**. The context in **Questions 1, 2** and **7** was found to be challenging for many.

Comments on specific questions

Question 1

Although this probability question was quite standard, many candidates did not use the raw data accurately when determining the required values.

(a) Good solutions used the data table and stated $\dfrac{\text{number of students in Year 1 who played netball}}{\text{number of students who played netball}}$.

However, many solutions used either the total number of candidates in Year 1 or the total number of candidates in the table as the denominator. A small number of solutions misinterpreted the question and found P(N | X).

(b) Again, good solutions used the data table and stated $\frac{\text{number of students in Year 1 who played netball}}{\text{number of students in Year 1}}$. However, many solutions used either the total number of candidates who played netball or the total number of candidates in the table as the denominator. A small number of solutions misinterpreted the question and found $P(X \mid N)$.

(c) The majority of candidates used the relationship $P(X) \times P(N) = P(X \cap N)$ to determine independence. The instruction 'determine' in the question requires candidates to justify their conclusion, so clear communication using correct notation is expected and numerical values must be present to compare. In many solutions, some or all of these features were omitted and so gained no credit. There were a number of inaccurate probabilities stated from the data table and candidates should be reminded to always be careful with simple arithmetic. The alternative approach required comparing either **1(a)** with $P(X)$ or **1(b)** with $P(N)$, and where attempted this was usually successful with the required relationship stated initially, e.g. $P(N \mid X) = P(N)$, the values substituted and the conclusion stated.

(d) Most candidates recognised that this part was a standard binomial approximation and used an appropriate approach for the task. The best solutions stated the unsimplified calculation for P(0,1,2) and used a calculator to evaluate accurately, providing an answer accurate to more than three significant figures before rounding to three significant figures. Weaker solutions often included the values of the individual terms before summing, which allowed rounding errors to occur. A very small number of candidates used the alternative approach and calculated 1 – P(3,4,5,6,7,8) but these were often unsuccessful. As in previous years, a common error was to misinterpret the success criteria and include P(3) in the solution. Candidates are well advised to practice interpreting the common success criteria accurately as part of their preparation.

Question 2

Many candidates found the context of this question challenging, although it was encouraging to see more diagrams showing the possible scenarios within solutions. This is good practice in 'permutation and combination' questions.

(a) Good solutions stated the initial calculation and evaluated accurately. A very small number of candidates did round their exact answer to three significant figures, which is inappropriate in this component when an exact numerical value has been found. Weaker solutions did not consider the effect of the repeated As and omitted dividing by 2!.

(b) Most candidates found this part challenging. The most successful approach was to identify the three possible scenarios which fulfilled the success criteria, and then determine the number of arrangements possible for each. As in **1(d)**, interpreting the success criteria inaccurately was a common error with the omission of A ^ ^ A ^ ^ ^ ^ ^ as a possible scenario. Many solutions found the number of arrangements possible with the As in their initial position, but did not continue to identify the different places that the As could be positioned and still fulfil the success criteria. A few candidates multiplied their answers by two, as the position of the As could be interchanged, but should then have divided their value by two because of the repeated As.

Question 3

The context of this question was found challenging by many, with a common misconception being that the outcomes must be no greater than 6. The successful solutions often generated an outcome space initially.

(a) Candidates who included an outcome space were often more successful in identifying all the possible outcomes for the probability distribution table. Good solutions set up the probability distribution table so that it could be used to support the calculations required in **3(b)**. A number of tables had probabilities that did not sum to 1, or included odd numbers which should not have been possible with the random variable conditions. The inclusion of 1, but no other odd variable was unexpected.

(b) Over 15 per cent of candidates made no attempt at this part, even when a good attempt at **3(a)** was present. Candidates should be aware that as this is syllabus content, clear supporting working is expected at all stages. Where errors were present in **3(a)**, only the unsimplified calculations are able to gain credit. A small number of candidates did not use the variance formula accurately and subtracted E(X). It is good practice not to state final answers as improper fractions in most contexts. Good solutions often included supporting work for E(X) and Var(X) as part of the 'probability distribution table' in **3(a)**. However, the final solution for Var(X) must be stated in the answer space for **3(b)**.

Question 4

This normal distribution question was answered well by most candidates.

(a) This part was answered well by most candidates. The best solutions clearly stated the standardisation formula with an appropriate evaluation. The probability was found by using the provided tables accurately and the criteria interpreted correctly to find the appropriate probability area. A clear calculation was stated, using a probability to at least four significant figures, and the answer interpreted to state an integer number of trees having a height less than 18.2 m. As in previous years, some candidates did not attempt to find the expected number of trees, and it is recommended that candidates do re-read the question to confirm that they have completed the task set before moving on. Candidates should be aware that they are interpreting the value found, rather than simply approximating or stating to a given number of significant figures. A small number of candidates introduced a continuity correction, which was not necessary as heights is a continuous variable.

(b) The majority of candidates used an appropriate standardisation formula with a variable for the denominator. The majority of errors seen were equating this to either the original probability or using the tables incorrectly and not finding a z-value. The context was such that an improper fraction was not appropriate for the final answer.

Question 5

Although this was a quite a standard probability question, many candidates found it challenging, often because the requirements were not considered in a logical manner.

(a) Many good solutions were seen, with the initial criteria applied accurately. A common misconception was that the selection of the violinists, guitarists and pianists were separate outcomes, and the values were added instead of multiplied. A few candidates multiplied the correct expression by 3! or 6, which appears to be considering that the order the type of musician was selected was significant. The solutions of some candidates suggested that the question had not been read fully, as ${}^{21}C_7$ was presented as the answer, which is the number of ways that a group of seven can be chosen from the class with no restrictions.

(b) Candidates who listed possible scenarios in a logical order were often more successful in this part. Good solutions then clearly linked the calculations with the scenarios before finding the total. Weaker solutions often omitted one or more possible scenario. Again, some candidates considered the scenarios as a combination of separate outcomes and added their values.

Question 6

(a) Many excellent stem-and-leaf diagrams were seen. The best used a ruler to ensure that the stem was vertical, which then enables the vertical alignment of the leaves to be achieved more easily. Most candidates followed the instruction to place the Falcons on the left and very few diagrams were seen with 'punctuation' used between terms. The most frequent error was to omit the units from the key, which should be included at this level.

(b) Many fully correct solutions were seen. Candidates should be aware that when more than one value is required in a question, these should be clearly identified to obtain credit. Most candidates correctly identified the median as the 8th value, but the Upper Quartile and Lower Quartile are expected to be the median of the two set of remaining values, so here the 12th and 4th values.

(c) Many candidates found this part challenging, and the question was not attempted by a number of them. The best solutions recognised that the data given needed to be combined into the mean and variance formulae with a group size of 30 and presented clear calculations. Weaker solutions often found the mean and variance of the groups separately and then either combined both values, or found the mean average of the values. This is a common misconception which candidates can be supported to avoid. Some poor arithmetic was noted in this question.

Question 7

This question using the geometric and normal approximations was found challenging by many and a large number of incomplete solutions were noted.

(a) Many good solutions to this style of question were seen. The best included a clear supporting calculation. A few candidates used the complements of the given probabilities or omitted to include the probability of scoring a goal on the 5^{th} attempt. A misinterpretation of the success criteria was calculating scoring 5 goals in total instead of scoring a goal on the 5^{th} attempt.

(b) There were two common approaches to this question. The most successful recognised that P(3,4,5,6,7) needed to be found and extended the work in **7(a)** to obtain the result. Candidates who attempted the more efficient process of calculating $P(X \leqslant 7) - P(X \leqslant 2)$, often failed to interpret the success criteria accurately and omitted P(3) or failed to evaluate the two required probabilities consistently, frequently omitting '1 – ' from $P(X \leqslant 2)$. Rounding intermediate answer values was noted in this question, which often resulted in an inaccurate final answer.

(c) Most candidates found this part challenging, and the question was not attempted by a surprising number. Again, solutions which listed possible scenarios were often more successful. The best solutions recognised in each scenario that the position of the first goal was variable and included an appropriate multiplier in each scenario. Weaker solutions assumed that the first goal was scored on the first attempt. A common incorrect approach was to find the probability of scoring a goal on the fifth attempt and adding to the probability of scoring at least one goal in four attempts. An alternative approach seen from some better candidates was to use a binomial approximation and calculating P(2,3,4,5) from 5 attempts, which was sometimes more efficiently found as 1 – P(0,1).

(d) This standard normal approximation question was omitted by over 10 per cent of candidates. Candidates who did attempt the question often scored well, as clear working was shown even if their final answer was incorrect. Good solutions included clear calculations to find the mean and variance, used a continuity correction when forming both standardisation formulas and included a simple sketch of the normal distribution to identify the required probability area. The interpretation of the success criteria was often inconsistent, with often one continuity correction applied incorrectly. A number of candidates, having formed the standardisation formulas assumed that the required probability area was symmetrical about the mean and did not use both of their values. Candidates should be aware that incorrect terminology will be penalised, for example calculating the variance and stating that it is the standard deviation.

MATHEMATICS

Paper 9709/53
Probability and Statistics 1 (53)

Key messages

When answering complex questions candidates are reminded that they should make their working clear and explain their approach to the reader. This was particularly true in **Question 5c** and **Questions 6b, c** and **d**. In **Question 5c** they needed to explain what the numerator and denominator of their probability fraction represented. In **Questions 6b, c** and **d,** where they were adding or subtracting different totals, they needed to explain what those totals represented. This was particularly true in **Question 6b** where there was a variety of acceptable approaches. No part marks can be awarded for incorrect final answers if the subtotals are not explained.

Another issue in this paper was premature approximation. Premature approximation is penalised every time and some candidates lost several marks across the paper. In **Question 1b** a rounded z-value of 1.56 produced an inaccurate final answer, as did a rounded value for E(X) in the calculation of the standard deviation in **Question 4c** and rounded values of the individual binomial terms in **Question 5a**. Candidates need to understand that all numbers they work with should be correct to four or more significant figures if the final answer is to be correct to three significant figures.

General comments

Being able to use tables for the Normal Distribution is an essential requirement for this syllabus and most candidates appreciate that they need to show the complete standardisation formula with substituted values for the mean and standard deviation. Those who produced the final answer from a calculator in **Questions 1b** and **3** with no standardisation formula written out lost unnecessary marks.

The style of grouping of the data in **Question 4** caused a variety of challenges. Candidates who listed the calculated cumulative frequencies were far more likely to produce an accurate graph than those who plotted the points without having written down the totals. Candidates need to plan their graphs carefully. If they recognise that the boundary values are not integers, they will be more likely to plot the points accurately if they allocate thick graph lines to the boundary values. Those who aimed to plot points between graph lines rarely managed to plot them all accurately. Candidates need to remember that a graph should tell a story and if the axes are not fully labelled (‘cumulative frequency’ and ‘height in cm’ in this case), the graph is meaningless.

Comments on specific questions

Question 1

(a) Most candidates knew to cube the probabilities of both owning an electric car (0.3) and not owning an electric car (0.7), and the majority knew that the ‘either...or...’ meant they needed to add the two cubed values. However, a significant number gave two separate values, not realising that they needed to sum them.

(b) Candidates seemed very comfortable with tackling this question. The question asked for an approximation and only the weaker candidates ignored this instruction and tried unsuccessfully to use a binomial calculation. Most candidates remembered the continuity correction, and almost all took it in the correct direction. A few forfeited an easy mark by prematurely rounding the z-value which resulted in a narrowly inaccurate final answer.

Question 2

(a) This question was answered well with the majority producing a clear table, often after having investigated the outcomes in a sample space diagram. An unfortunate number misunderstood the question and dealt with the sum rather than the product of the scores and a special case was made for them, allowing them one of the three available marks. Careful candidates often wisely included a check that their probabilities added to 1. Most candidates knew and were able to apply the formula for E(X). Those who had worked with sums rather than products in **part (a)** were eligible for the follow through mark. The most common error seen was candidates dividing the correct calculation by either 6 or 36.

Question 3

(a) This question was answered well with candidates knowing that we needed to see the full standardisation formula with 170, 176 and 4.8 substituted correctly. Careful candidates often supported their working with a diagram.

(b) This second part of the question proved to be more challenging. Strong candidates knew to subtract their answer to **part (a)** from 1 and to add this to 60 per cent or 0.6 to find the probability of an adult male having a height less than k cm (0.7056). They then used their tables to find the z-value corresponding to this probability and equated that z-value to $\frac{(k-176)}{4.8}$ to find k. However, many solutions were very confused with z-values and probabilities being interchanged and added together with little understanding.Those who did use a correct process often gave an incorrect or truncated version of the required z-value (0.541).

Question 4

(a) The style of grouping of the heights in this question seemed to be unfamiliar to many candidates and it was insisted on seeing points plotted above 9.5, 19.5 etc. (not 10, 20 …). Strong and careful candidates clearly listed the cumulative frequencies rather than relying on their Examiner having to read them off the graph, and they knew to plot the points above the upper boundaries of each group with the initial point being at (9.5, 0) and to join the points to make a curve without the use of a ruler. A number of graphs had points plotted above the mid-points or lower boundaries instead of upper boundaries. Axis had to be labelled, and the horizontal axis had to start on or before 9.5. Those who chose to allocate 9.5, 19.5 etc. to the thick graph lines usually produced the most accurate graphs.

(b) This question asked for the graph to be used, and we needed to see evidence of this, preferably a line drawn across from 45 on the cumulative frequency axis.

(c) The majority of candidates knew that they needed to find the mid-point of each group to estimate the mean and standard deviation, with only a few using other measurements and most correctly found an accurate estimate of the mean. Many were less familiar with the formula for the standard deviation, and they often resorted to a calculator for the answer. The calculations were bulky, and it was allowed for a shortened calculation, as long as the first, last and one of the middle terms in the summation could be seen.

A significant number of candidates used the approximated value of 38.9 for E(X) in their calculation of the standard deviation and produced an inaccurate final answer, usually 10.7. They need to understand that, if the final answer is to be correct to three significant figures, all input numbers must be correct to at least four significant figures.

Question 5

(a) This was a familiar style of binomial question and was tackled confidently by the majority of candidates. A few candidates confused it with a geometric distribution and gave a response that lacked binomial coefficients. The wording of ‘no more than’ was generally well interpreted with only a few omitting the probability of either 2 or zero. Lack of necessary working was rarely seen with candidates appreciating that they need to show all the binomial terms in full. Just a few lost accuracy and a mark through premature rounding before they had added the three terms.

(b) This was another well answered question with the majority of correct responses using the method of subtracting the probability of none of the first 6 chocolates being in red foil from 1. However, a significant number chose the longer method of summing six geometric terms. Those who attempted the method of subtracting two probabilities from 1 were generally less successful, often including an extra incorrect probability of 0.25 in their calculation. Whilst this question could be answered with use of a binomial distribution within it, this was not the correct general approach and those who tried this method were generally unsuccessful

(c) This was a difficult question to interpret and there were a number of errors seen as a result. The most common was in assuming independence between the second chocolate being the first one in gold foil and the fifth one being the first unwrapped. Many recognised the correct conditional probability calculation and strong candidates clearly explained what they were doing. A large number calculated the denominator correctly, the probability of the 5^{th} chocolate being the first unwrapped. Finding the numerator, the intersection of the second chocolate being the first gold wrapped and the 5^{th} being the first unwrapped, was more taxing. Those who attempted to find the single five factor product were generally more successful than those who considered the four scenarios that satisfied the conditions and needed to sum four five factor products. A few candidates spotted the shortcut in the question, realising the selection was based on the first two choices alone. The first one had to be red wrapped and the second gold wrapped given that they were both wrapped.

Question 6

(a) This part of the question was answered well by most candidates with only a few not taking account of the repeats of the Ps and Ss and dividing by 2!2!.

(b) Candidates found this part of the question more challenging. The most successful candidates subtracted the number of ways with Ps at either end or Ss at either end from the total number of ways calculated in the previous part of the question. They needed to remember that there would still be a repeated letter in the middle seven. Those who tried to consider all the situations when there was not a repeat made the question much more complex than it needed to be. They often did not make it clear what they were trying to do which made awarding marks difficult. Candidates who tried to deal with all the five non-repeated letters separately, often gave up part way through the question.

(c) Strong candidates added the number of ways the letters could be arranged with the two Ps between the Ss (6!) to the number of ways they could be arranged with the two Ps outside the Ss (5! X 5P2). Less successful candidates split the problem into more parts and usually looked either at the positioning of the Ss or the positioning of the Ps. This made the problem significantly more challenging and few of these achieved full marks. Candidates need to appreciate that they need to make their method clear to their reader or marker. Unless there is a clear explanation for working, credit cannot be awarded.

(d) This part of the question proved to be more accessible than the previous two parts and most candidates made a good attempt. Strong candidates recognised that if the Ps were in the group of five it meant that the Ss were in the group of four and vice versa. This meant they only had two situations to consider rather than four. If they did not appreciate this, they tended to produce a numerator that was double the correct value. A small number of candidates attempted a probability approach and were rarely successful. A few others attempted to use Permutations rather than Combinations where the denominator of their probability fraction was either 90720 or 9!. This made the question unnecessarily complex by considering arrangements rather than selections and was rarely successful.

MATHEMATICS

Paper 9709/61
Probability and Statistics 2 (61)

There were too few candidates for a meaningful report to be produced.

MATHEMATICS

Paper 9709/62
Probability and Statistics 2 (62)

Key messages

- In all questions, sufficient method must be shown to justify answers.
- When the answer is given in the question, candidates need to show convincingly their full and clear working leading to the answer.
- Candidates should be encouraged to read questions carefully and extract the relevant information.
- Candidates should be encouraged to reread the question when they have completed their solution to ensure that all the requirements have been fulfilled (for example answering 'in context' if required).
- Candidates need to leave their final non-exact answers to at least three significant figures unless stated otherwise. This means they need to keep four significant figures or more in the working leading to the answer.
- All working should be done in the correct question space of the answer booklet. If a candidate does not have enough space to complete their answer, they should either use the additional page, or ask for an extra sheet of paper, and label it clearly with the question number.
- Arithmetical errors, especially sign errors are very common. Candidates should get into the habit of checking their work carefully.
- Mathematical notation and vocabulary should be used appropriately.
- Conclusions to Hypothesis tests should be written in context and with a level of uncertainty in the language used.

General comments

Candidates, in general, were able to apply their knowledge in the situations presented. Questions that were done well were **Question 1** and **Question 5b**, whilst **Question 2** and **3b** and were found to be more demanding. On the whole candidates showed the required amount of working.

Presentation was generally good, and there did not appear to be any issues with timing.

There were some very good scripts, but equally there were scripts where candidates appeared to be unprepared for the demands of the paper.

The following comments, on individual questions, reflect some common errors, but there were also some complete and fully correct solutions.

Comments on specific questions

Question 1

This question was well attempted with most candidates using the correct Poisson distribution with mean 4.5. Calculation of 1 – P(0,1,2,3) was required; common errors seen were the inclusion of P(4) in this expression, use of an incorrect value for λ and occasional errors in calculation. Most candidates gave evidence of their method of calculation and showed the full Poisson expression used.

Question 2

(a) Many candidates did not know how to find the value of μ, not realising that the mean was the mid value of 245 and 263. A common error was to find 245 + 263 and divide by 50 instead of 2. A lack of understanding of confidence intervals was evident in many cases.

(b) Again, this was not particularly well attempted, though some candidates were able to find a correct expression for the width of the confidence interval and some used an appropriate expression using their mean found in **part (a)**. Errors included an incorrect z-value in their expression. Some candidates did not use information from the confidence interval and attempted to use the standard deviation formulae.

(c) Only a small number of candidates gave a fully correct explanation that the given fact would not make a difference to their calculation in **part (b)**, by recognising that n was large (implying that the Central Limit Theorem had been used) or by stating the Central Limit Theorem explicitly.

Question 3

(a) Many candidates gave correct hypotheses, but errors were made in finding the required probability of $X \geqslant 23$ using B(25,0.8). Some candidates found $X > 23$ and occasionally $X = 23$. A valid comparison with 0.1 was required followed by a conclusion which was in context and was not a definite statement; many candidates used phrases such as 'we can conclude that …' without using the necessary level of uncertainty in the language used. Some candidates attempted to use a Normal approximation despite clear instructions in the question to use the Binomial distribution.

(b) This part was found challenging with many candidates stating that 30 was large enough for a normal distribution to be used rather than appreciating that the initial model was only based on 25 employees so using it for 30 employees would not be suitable.

Question 4

(a) This part was reasonably well attempted. Common errors included omission of $\sqrt{140}$ when standardising, using continuity corrections and not keeping to the required level of accuracy.

(b) Again, many candidates knew how to approach this question and set up a standardising equation, though some used 0.986 rather than $\phi^{-1}(0.986)$, or made sign mistakes in their equation and others did not keep to the required level of accuracy.

Question 5

(a) The interpretation of 'more than 2 but not more than 5' proved difficult for some candidates, with a particularly common error being to calculate P(3,4) rather than P(3,4,5). Most candidates correctly used Po(3.7).

(b) Many candidates used Po(6.3) and reached the correct answer of 0.126. Use of a Normal approximation was occasionally seen, but on the whole candidates did well on this question.

(c) This part was found more challenging. Candidates who found the approximating distributions N(37,37) and N(26,26) usually went on to find N(–11,63) and the correct probability, though errors finding the variance were made. There was confusion between 10 sweets and 10 bags, highlighting the importance of reading the question carefully. Full marks could be gained for a correct method with or without the appropriate continuity correction.

Question 6

(a) Many candidates did not answer in context and merely said that *a* and *b* were the limits of *X*, or similar, without reference to the time taken.

(b) This part was reasonably well attempted. Errors included a sign error when integrating, and reversed limits of integration. On the whole sufficient working was shown.

(c) Some candidates assumed that *b* was 2 rather than showing it, and others used the correct method but were unable to rearrange their equation in *b* correctly.

(d) There was a mixed response here, and again sign errors when integrating and incorrect limits were noted.

Question 7

(a) Candidates did not always read the question carefully and many defined a Type I error here rather than give its probability. Others gave an answer of 98 per cent or less than 2 per cent, and some omitted this part.

(b) Marks were gained by many for finding unbiased estimates for the population mean and variance. The biased variance was occasionally used and there was also some confusion seen between different formulae for the unbiased estimate of the population variance. Some candidates stated correct hypotheses, but others omitted to include them or did not use μ. Errors when standardising included omission of $\sqrt{100}$ and confusion between the variance and standard deviation. The comparison with 2.054 (or equivalent) was not always clearly stated, and 2.54 was often seen. The conclusion should be in context and not a definite statement; again, there were candidates who used phrases such as 'we can conclude that …' without using the necessary level of uncertainty in the language used.

(c) Many candidates omitted this part, and many defined a Type II error and did not comment on the claim.

MATHEMATICS

Paper 9709/63
Probability and Statistics 2 (63)

Key messages

It is expected that candidates work to sufficient accuracy in the paper. This may require working to more than three significant figures in order to achieve a final answer which is accurate to three significant figures. This can be seen in **Question 3(b)** and in **Question 5(b)**.

General comments

Clear writing of words and numbers was desirable. Many candidates gave clear presentations of their method and calculation steps. However, it was difficult to distinguish between certain letters and numbers in the work of some candidates. For example, in **Question 4** the 'a' and the '9' could look very similar thus making marking difficult.

Comments on specific questions

Question 1

(a) To find the 96 per cent confidence interval it was necessary to find the appropriate value of z, namely 2.054 or 2.055. Many candidates found this value. Some candidates used an incorrect value such as 1.751 or 2.54. It was essential that $\sqrt{150}$ was included in the calculation and that the answer was given as an interval.

(b) The explanation to this part required both an answer (number or equivalent) and a reason. The distribution used in the calculation related to the mean height.

Question 2

To compare the total mass of 3 small bags to the mass of 1 large bag it was necessary to consider a new variable of the form $3S - L$ with a normal distribution $N(-3, 2.1)$ and to consider $3S - L > 0$. After standardising the '0' it was necessary to select the small probability. A sketch could help in deciding on whether the small or large probability was appropriate. Use of $S_1 + S_2 + S_3$ instead of 3S would be of help in finding the correct mean and variance.

Question 3

(a) Many candidates found the unbiased estimates correctly. Answers were accepted as a decimal or as a fraction. Some candidates used an incorrect formula for the variance.

(b) It was necessary to change from the given probability 0.234 to 0.766 and then find the corresponding z value 0.726. Many candidates omitted at least one of these steps or made errors in trying to find the new values. Having found this z-value it could be equated to the standardised form for x in the distribution of means of samples. This required the use of $\sqrt{75}$. Some candidates omitted this. Some accuracy could be lost if the values for mean and variance found in **part (a)** were prematurely rounded. An inaccurate answer of 3.22 was often seen.

Question 4

(a) As many candidates demonstrated, the property to use here was that the area under the curve of $f(x)$ was equal to 1. So, integration of $f(x)$ between the limits 2 and 3 was required. The integration steps, the substitution of the limits and the simplification of the terms then needed to be clearly shown.

(b) As the answer was also given in this part of the question, the various steps needed to be clearly shown. The method required the correct integration of $x\,\mathrm{f}(x)$ between the limits 2 and 3. This involved $\ln(x)$ and $\frac{18}{x}$ or equivalent and the clear substitution of the limits.

Question 5

(a) As the scientist wished to test whether the true value of μ was different to that given in the article, a two-tailed test was suitable. To answer the question fully it was necessary to give both the type of test and a brief reason.

(b) In order to carry out the significance test it was necessary to state the hypotheses, standardise the sample mean, compare the z-value with the critical value and state the conclusion. The standardisation required the use of the normal distribution of means of samples of size 50 and therefore the use of $\sqrt{50}$. The critical z-value for this 1 per cent two-tailed test was –2.576. The main alternative comparison could be between the probabilities 0.0099 and 0.005 or equivalent. The conclusion needed to be stated in context, not definite and with no contradictions. Many candidates showed all these steps correctly. Some candidates used an incorrect critical value such as 2.326 or an incorrect probability such as 1 per cent. Other candidates gave a definite conclusion, omitting a suitable phrase such as ‘there is insufficient evidence that’.

Question 6

(a) For the number of customers arriving at desk A in a 15-minute period the new Poisson parameter 2.7 was required. For more than two customers the method was to find the probability 1 – P(0, 1, 2). It was necessary to write down the relevant terms.

(b) For the number of customers arriving at desks A and B in a 5-minute period, the new Poisson parameters 0.9 and 1.05 were required and hence 1.95 for the total number. It was necessary to write down the relevant terms for P(0, 1, 2, 3).

(c) For customers arriving at desk B in a t-minute period the Poisson distribution with parameter $0.21t$ could be used. Then for a 90 per cent probability for the inspector the inequality $1 - e^{-0.21t} \geqslant 0.90$ needed to be solved. This required re-arrangement and the taking of $\ln(t)$. To demonstrate that sufficient accuracy was being used the value 2.3026 was required here and then 10.96 (or 10.97) and then 11 minutes. Working with inequalities was also given marks. Some candidates showed these steps successfully. Other candidates attempted to take $\ln(t)$ of inappropriate expressions or did not show sufficient accuracy.

Question 7

(a) In the past, the value of the Poisson parameter was 3.3 for 1 year and hence for 2 years was 6.6. For a Type I error the probability of rejecting H_0 was required. Thus, calculation of the sums of the early terms of the Poisson distribution were required. P(0, 1, 2) = 0.0399676 and P(0, 1, 2, 3) = 0.105. The first of these was less than 0.05 whilst the second of these was greater than 0.05. Hence the critical range was $X \leqslant 2$ and the probability of a Type I error was 0.0400 (to 3 sf). These various terms, expressions and values were required to score full marks.

(b) To complete the test the hypotheses needed to be stated. This might have been done in **part (a)**. The calculation work for the probabilities might have also been done in **part (a)** and the results quoted here. The results gave the comparison 0.04 < 0.05 for the 5 per cent test and then the appropriate conclusion.

(c) A type II error would occur when $X > 2$ with the Poisson parameter 1.2 for the 2 years. To find this probability it was necessary to find 1 – P(0, 1, 2). Some candidates used 0.6. Other candidates omitted the ‘1–’.

(d) For the 30-year period the Poisson parameter would be 18. As $18 > 15$ the appropriate approximating distribution was the normal distribution N(18, 18). This change required a continuity correction (10.5) and standardisation and selection of the relevant probability area. Many candidates answered this correctly. Some candidates omitted the correction factor or used an incorrect factor such as 9.5. Other candidates chose the incorrect area. A sketch could help with the choice.